Gerber-Shiu Function in the Compound Poisson Risk Model Perturbed by Diffusion under a Barrier Strategy

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Abstract—People who work in the field of actuarial science pay more and more attention to the risk model with dividend strategy, it has become one of the hot topics in the current actuarial science research. In this paper, we want to study the Gerber-Shiu expected discounted penalty function due to oscillation which written as $\phi_{b,d}(u)$, considering a classical compound poisson risk model perturbed by diffusion in the presence of a constant dividend. The integro expression of the Gerber-Shiu function is derivated by the strong markov property and also $\phi_{b,d}(u)$ is continuous and twice continuously differentiable. then we obtain the integro-differential equation of the Gerber-Shiu function by $It\hat{o}$ formula, It is unique to research the ultimate ruin probability due to oscillation compared with other articles. Finally, we give the explicit expression of solution of integro-differential equation satisfied by $\phi_{b,d}(u)$ when the claim sizes are exponential distribution.

Keywords-Barrier strategy; Diffusion process; Compound poisson process; Gerber-Shiu function; Integro-differential

I. INTRODUCTION

Since the Swedish actuary Filip Lundberg published his doctoral paper about ruin theory in 1903, ruin theroy has been the core content of risk theroy. The classical risk model founded by Filip Lundberg and Swedish mathematician Harald Cramer has been studied in a number of papers and books. Gerber and Shiu introduced the Gerber-Shiu function in the classical risk model in 1998, the edge distribution and joint distribution of laplace transform of ruin time, ruin probability, the instantaneous surplus before ruin and the deficit at ruin researched early, also can be got by selecting penalty function appropriately. The introduction of Gerber-Shiu discounted penalty function largely promoted the development of risk theory. As the classical risk model is too idealistic, in fact there are a lot of distractors, so it's necessary to study the risk model perturbed by diffusion. Wan (2007) studied the Gerber-Shiu discounted penalty function in compound poisson risk model perturbed by diffusion under threshold dividend strategy; Li, Wu (2009) also studied the aggregate discounted dividend mean in this risk model; Gao,Liu (2010) consided the mean of aggregate dividend discounted and Gerber - Shiu discounted penalty function in compound poisson risk model perturbed by diffusion under constant interest rate and threshold dividend strategy; Peng, Hou (2012) considered the absolute ruin issue in above risk model.

Dividend strategy was first proposed by De Finetti (1957), Barrier dividend strategy was originally proposed by Gerber in 1969.A barrier strategy is considered by assuming that there is a horizontal barrier of level $b(b \geq u)$ such that when the surplus reaches level b, dividends are paid continuously so that no dividends are paid when the surplus is less than b. Tan, Yang (2006) studied the problem of stochastic dividends in binomial risk model under constant value of dividend strategy, And when the discount factor is 1, they get the progressive expression of Gerber - Shiu discounted penalty function; Eric, David (2009) studied the moment of aggregate discounted dividend in perturbed markov risk model under above strategy, and got the explicit expressions of aggregate discounted dividend when claim amount is rational family; Wang, Yin (2010)studied the perturbed compound poisson risk model with interest on loans and constant dividends under absolute ruin. And also obtained Gerber-Shiu discounted penalty function satisfies the integral - differential equations and boundary conditions.

On the basis of Peng, Hou(2012), this paper continue to study the problems associated with Gerber -Shiu expected discounted penalty function in perturbed classical risk model under a barrier strategy.

II. THE MODEL

Consider the following classical surplus process perturbed by a diffusion.

$$U(t)=u+ct-\sum_{i=1}^{N(t)}X_i+\sigma B(t),t\geq 0 \qquad (1)$$

Where $u\geq 0$ is the initial surplus, $c>0$ is the premium rate per unit time, $\sigma>0$ is the diffusion coefficient, $\{B(t),t\geq 0\}$ is a standard wiener process that is independent of the aggregate claims process $S(t)=\sum_{k=1}^{N(t)}X_i$, $\{N(t),t\geq 0\}$ is a poisson process with parameter $\lambda(\lambda>0)$,denoting the total number of claims from an insurance portfolio. $X_1,X_2,...,$ which are independent of $\{N(t),t\geq 0\}$, are positive i.i.d. random variables with common distribution function $P(x)=1-\bar{P}(x)$ density function $p(x)$,and the laplace transform $\hat{p}(s)=\int_0^\infty e^{-sx}p(x)dx$.

Definition 2.1 The surplus process of ruin time

$$T_b=\inf\{t,U_b(t)\leq 0\}$$

Definition 2.2 The expected discounted dividend function

$$V(u;b)=E\left[D_{u,b}\middle|U(0)=u\right]$$

Where $D(u;b)=\int_0^{T_b}e^{-\delta t}dD(t)$ is the present value of all dividends, $\delta>0$ is discount factor.

We take Gerber-Shiu expected discounted penalty function $\phi_b(u)$ into two parts: one caused by a claim and another due to oscillation ,denoting $\phi_{b,s}(u)$ and $\phi_{b,d}(u)$ respectively.

$$\phi_b(u)=\phi_{b,s}(u)+\phi_{b,d}(u),\ 0\leq u\leq b. \qquad (2)$$

$$\phi_{b,s}(u)=E\Big[e^{-\delta T_b}\omega\big(U(T_b-),|U(T_b)|\big)\cdot I\big(T_b<\infty,U(T_b)<0\big)\big|U(0)=u\Big]. \qquad (3)$$

$$\phi_{b,d}(u)=E\left[e^{-\delta T_b}I\big(T_b<\infty,U(T_b)=0\big)\big|U(0)=u\right] \qquad (4)$$

Here δ , T_b are interpreted as the force of interest and ruin time, $\omega(x_1,x_2),x_1\geq 0,x_2\geq 0$ is non-negative measurable function, x_1 , x_2 respectively represent the surplus immediately before ruin and the deficit at ruin, $I(\cdot)$ is the indicator function. As $\phi_{b,s}(u)$ caused by claim has been discussed in many literatures, this paper we mainly study $\phi_{b,d}(u)$ due to oscillation.

III. GERBER-SHIU EXPECTED DISCOUNTED PENALTY FUNCTION

Let $a>0$,define $\tau_a=\inf\{s:|B_s|=a\}$, for $x\in[-a,a]$,then we have

$$H(a,t,x)=\frac{1}{\sqrt{2\pi t}}\sum_{k=-\infty}^{\infty}\left\{\exp\left[-\frac{(x+4ka)^2}{2t}\right]-\exp\left[-\frac{(x-2a+4ka)^2}{2t}\right]\right\} \qquad (5)$$

$$h(a,t)=\frac{a}{2\sqrt{2\pi t^3}}\sum_{k=-\infty}^{\infty}\left\{(4k+1)\exp\left[-\frac{a^2(4k+1)^2}{2t}\right]+(4k-3)\exp\left[-\frac{a^2(4k-3)^2}{2t}\right]-(4k-1)\exp\left[-\frac{a^2(4k-1)^2}{2t}\right]\right\} \qquad (6)$$

According to the conclusion of Revuz, Yor (1991), we know that $P(B_s\in dx,\tau_a>s)=H(a,s,x)dx$ and $p(\tau_a\in ds)=h(a,s)ds$, then it will be easy to prove that $h(a,t)$ in a , t and $H(a,t,x)$ in a , t , x at least twice continuously differentiable.

Theorem3.1 For $0<u<b$, we can derivate $\phi_{b,d}(u)$ satisfies the following integral expression**:**

$$\begin{aligned}\phi_{b,d}(u)&=e^{-(\lambda+\delta)t_0}\int_{-a}^{a}\phi_{b,d}(u+ct_0+\sigma y)H(a,t_0,y)dy\\&+\frac{1}{2}\int_0^{t_0}e^{-(\lambda+\delta)t}h(a,t)\big[\phi_{b,d}(u+ct+\sigma a)\\&+\phi_{b,d}(u+ct-\sigma a)\big]dt\\&+\int_0^{t_0}\lambda e^{-(\lambda+\delta)t}dt\int_{-a}^{a}H(a,t,y)dy\\&\cdot\int_0^{u+ct+\sigma y}\phi_{b,d}(u+ct+\sigma y-x)p(x)dx.\end{aligned} \qquad (7)$$

Where $0<t_0\leq\frac{(b-u)}{(2c)},0<a\leq\frac{(b-u)\wedge u}{2\sigma}$.

Proof. Consider $T=t_0\wedge\tau_a\wedge T_1$, for $t\in(0,T)$, there is $0<U_b(t)<b$, $P(T\leq T_b)=1$ $a.s.$ $T_b=T+T_b\text{o}\theta_T$, thus we can get the following expression by the strong markov property of $\{U(t)\}$.

$$\begin{aligned}\phi_{b,d}(u)&=E\left[e^{-\delta T_b}I\big(T_b<\infty,U(T_b)=0\big)\big|U(0)=u\right]\\&=E\left[E^u\left[e^{-\delta T_b}I\big(T_b<\infty,U(T_b)=0\big)\big|\mathrm{F}_T\right]\right]\\&=E\left[E^u\left[e^{-\delta(T+T_b\text{o}\theta_T)}I\big(T+T_b\text{o}\theta_T<\infty\big)\big|\mathrm{F}_T\right]\right]\end{aligned}$$

$$= E\left[e^{-\delta T}E^{u}\left[e^{-\delta T_b}I(T_b<\infty)\text{o}\theta_T\left|\mathrm{F}_T\right.\right]\right]$$
$$= E\left[e^{-\delta T}\phi_{b,d}\left(U(T)\right)\right] \quad (8)$$

Further, according to the size of t_0 、 τ_a and T_1 , $\phi_{b,d}(u)$ can be divided into three parts:

$$\begin{aligned}\phi_{b,d}(u) &= E\left[e^{-\delta t_0}\phi_{b,d}\left(U(t_0)\right)I(t_0<\tau_a)I(t_0<T_1)\right]\\ &+E\left[e^{-\delta\tau_a}\phi_{b,d}\left(U(\tau_a)\right)I(\tau_a\le t_0)I(\tau_a<T_1)\right]\\ &+E\left[e^{-\delta T_1}\phi_{b,d}\left(U(T_1)\right)I(T_1\le t_0)I(T_1\le\tau_a)\right]\\ &= I_1+I_2+I_3. \end{aligned} \quad (9)$$

By independence, we can obtain

$$\begin{aligned} I_1 &= E\left[e^{-\delta t_0}\phi_{b,d}\left(u+ct_0+\sigma B(t_0)\right)I(t_0<\tau_a)I(t_0<T_1)\right]\\ &= e^{-(\lambda+\delta)t_0}\int_{-a}^{a}\phi_{b,d}(u+ct_0+\sigma y)P\left(B(t_0)\in dy\right)P(\tau_a>t_0)\\ &= e^{-(\lambda+\delta)t_0}\int_{-a}^{a}\phi_{b,d}(u+ct_0+\sigma y)P\left(\tau_a>t_0,B(t_0)\in dy\right)\\ &= e^{-(\lambda+\delta)t_0}\int_{-a}^{a}\phi_{b,d}(u+ct_0+\sigma y)H(a,t_0,y)dy. \end{aligned} \quad (10)$$

$$\begin{aligned} I_2 &= E\left[e^{-\delta\tau_a}\phi_{b,d}\left(u+c\tau_a+\sigma B(\tau_a)\right)I(\tau_a\le t_0)I(\tau_a<T_1)\right]\\ &= E\left[e^{-\delta\tau_a}\phi_{b,d}(u+c\tau_a+\sigma a)I(\tau_a\le t_0)I(\tau_a<T_1)\right.\\ &\qquad\left.\cdot I\left(B(\tau_a)=a\right)\right]\\ &+E\left[e^{-\delta\tau_a}\phi_{b,d}(u+c\tau_a-\sigma a)I(\tau_a\le t_0)I(\tau_a<T_1)\right.\\ &\qquad\left.\cdot I\left(B(\tau_a)=-a\right)\right]\\ &= \int_0^{t_0}e^{-(\lambda+\delta)t}\phi_{b,d}(u+ct+\sigma a)P\left(\tau_a\in dt,B(\tau_a)=a\right)\\ &+\int_0^{t_0}e^{-(\lambda+\delta)t}\phi_{b,d}(u+ct-\sigma a)P\left(\tau_a\in dt,B(\tau_a)=-a\right) \end{aligned} \quad (11)$$

By the conclusion of port and stone (1978), We can obtain

$$\begin{aligned} P\left(B(\tau_a)=a,\tau_a\in dt\right) &= P\left(B(\tau_a)=-a,\tau_a\in dt\right)\\ &= \frac{1}{2}P(\tau_a\in dt)=\frac{1}{2}h(a,t)dt. \end{aligned} \quad (12)$$

$$\begin{aligned} I_2 = \frac{1}{2}\int_0^{t_0}e^{-(\lambda+\delta)t}h(a,t)\left[\phi_{b,d}(u+ct+\sigma a)\right.\\ \left.+\phi_{b,d}(u+ct-\sigma a)\right]dt. \end{aligned} \quad (13)$$

$$\begin{aligned} I_3 &= E\left[e^{-\delta T_1}\phi_{b,d}\left(u+cT_1+\sigma B(T_1)-X_1\right)I(T_1\le t_0)\right.\\ &\qquad\left.\cdot I(T_1\le\tau_a)I\left(X_1<u+cT_1+\sigma B(T_1)\right)\right]\\ &= \int_0^{t_0}\lambda e^{-(\lambda+\delta)t}dtE\left[\phi_{b,d}\left(u+ct+\sigma B(t)-X_1\right)\right.\\ &\qquad\left.\cdot I(\tau_a\ge t)I\left(X_1<u+cT_1+\sigma B(T_1)\right)\right]\\ &= \int_0^{t_0}\lambda e^{-(\lambda+\delta)t}dt\int_{-a}^{a}E\left[\phi_{b,d}(u+ct+\sigma y-X_1)\right.\\ &\qquad\left.\cdot I\left(\tau_a\ge t,B(t)\in dy\right)I(X_1<u+ct+\sigma y)\right]\\ &= \int_0^{t_0}\lambda e^{-(\lambda+\delta)t}dt\int_{-a}^{a}H(a,t,y)dy\\ &\qquad\cdot\int_0^{u+ct+\sigma y}\phi_{b,d}(u+ct+\sigma y-x)p(x)dx \end{aligned} \quad (14)$$

Then we take I_1 、 I_2 、 I_3 into above expression, the conclusion of theorem can be derivated.

For $x_1,x_2\ge 0$,make $\delta=0,\omega(x_1,x_2)=1$, (4) will be written as the expression of the ultimate ruin probability due to oscillation, shown as follows:

$$\psi_{b,d}(u)=E\left[I\left(T_b<\infty,U_b(T_b)=0\right)\middle|U_b(0)=u\right],0\le u\le b$$

Remark 3.1 For $0<u<b$,we can obtain $\psi_{b,d}(u)$ satisfies the following integral expression by a similar derivation of Theorem 3.1,

$$\begin{aligned} \psi_{b,d}(u) &= e^{-\lambda t_0}\int_{-a}^{a}\psi_{b,d}(u+ct_0+\sigma y)H(a,t_0,y)dy\\ &+\frac{1}{2}\int_0^{t_0}e^{-\lambda t}h(a,t)\left[\psi_{b,d}(u+ct+\sigma a)+\psi_{b,d}(u+ct-\sigma a)dt\right]\\ &+\int_0^{t_0}\lambda e^{-\lambda t}dt\int_{-a}^{a}H(a,t,y)dy\\ &\qquad\cdot\int_0^{u+ct+\sigma y}\psi_{b,d}(u+ct+\sigma y-x)p(x)dx \end{aligned} \quad (15)$$

Theorem3.2 Suppose $p(x)$ is continuously differentiable on $(0,\infty)$, $\omega(x,y)$ in x is continuous bounded, then $\phi_{b,d}(u)$ is continuous and twice continuously differentiable on $(0,b)$.

Proof. First, we need to prove $\phi_{b,d}(u)$ is continuous on $(0,b)$, for any $\varepsilon>0$, as long as we can prove $\phi_{b,d}(u)$ is continuous on $(\varepsilon,b-\varepsilon)$ that will be perfect. Assume $t_0<\dfrac{\varepsilon}{2c}$, $0<a<\dfrac{\varepsilon}{2\sigma}$, I_1 、 I_2 、 I_3 can be expressed respectively as follow by using variable substitution:

$$I_1=e^{-(\lambda+\delta)t_0}\int_{u+ct_0-\sigma a}^{u+ct_0+\sigma a}\phi_{b,d}(y)H\left(a,t_0,\frac{y-u-ct_0}{\sigma}\right)\frac{1}{\sigma}dy. \quad (16)$$

$$\begin{aligned} I_2=\frac{1}{2c}\int_u^{u+ct_0}e^{\frac{-(\lambda+\delta)(t-u)}{c}}h\left(a,\frac{t-u}{c}\right)\left[\phi_{b,d}(t+\sigma a)\right.\\ \left.+\phi_{b,d}(t-\sigma a)dt\right]. \end{aligned} \quad (17)$$

$$\begin{aligned} I_3 &= \int_0^{t_0}\lambda e^{-(\lambda+\delta)t}dt\int_{-a}^{a}H(a,t,y)dy\\ &\qquad\cdot\int_0^{u+ct+\sigma y}\phi_{b,d}(x)p(u+ct+\sigma y-x)dx. \end{aligned} \quad (18)$$

It's easy to get $\phi_{b,d}(u)$ is bounded by the assumption of $\omega(x,y)$ is bounded. Then according to the continuity of H 、 h and f , we know that I_1 、 I_2 、 I_3 in u is continuous on $(\varepsilon,b-\varepsilon)$, and $\phi_{b,d}(u)$ in u is continuous on $(\varepsilon,b-\varepsilon)$. Further, according to the continuity of $\omega(x,y)$ and $f(x)$ in x , and continuous differentiability of

H 、 h, we can get $\phi_{b,d}(u)$ is twice continuously differentiable.

Remark3.2 Suppose density function $p(x)$ is continuously differentiable on $(0,\infty)$, then we can derivate $\psi_{b,d}(u)$ is continuous and twice continuously differentiable on $(0,b)$.

Theorem3.3 Assume $\phi_{b,d}(u)$ is twice continuously differentiable on $(0,b)$, we can obtain $\phi_{b,d}(u)$ satisfies the following integro-differential equations:

$$\frac{\sigma^2}{2}\phi''_{b,d}(u)+c\phi'_{b,d}(u)=(\lambda+\delta)\phi_{b,d}(u)-\lambda\int_0^u\phi_{b,d}(u-x)p(x)dx. \quad (19)$$

Proof. Let us make $\varepsilon,t>0$ and $\varepsilon<u<b-\varepsilon$, $T_t^\varepsilon=\inf\{s>0,u+cs+\sigma B(s)\notin(\varepsilon,b-\varepsilon)\}\wedge t$, there we notice $\phi'_{b,d}(u)$ and $\phi''_{b,d}(u)$ is bounded on $(0,b)$, and also $\int_0^{s\wedge T^\varepsilon}\sigma\phi'_{b,d}(u+cv+\sigma B(v))dB(v)$ is the Martingale, denote $T=T_t^\varepsilon\wedge T_1$, then we can get

$$\begin{aligned}\phi_{b,d}(u)&=E\left[e^{-\delta T}\phi_{b,d}(U(T))\right]\\&=E\left[e^{-\delta T_t^\varepsilon}\phi_{b,d}\left(U\left(T_t^\varepsilon\right)\right)I(t<T_1)\right]\\&+E\left[e^{-\delta T_{T_1}^\varepsilon}\phi_{b,d}\left(U\left(T_{T_1}^\varepsilon\right)\right)I(t\ge T_1)\right]. \quad (20)\end{aligned}$$

$$\begin{aligned}\phi_{b,d}(u)&=e^{-\lambda t}E\left[e^{-\delta T_t^\varepsilon}\phi_{b,d}\left(u+cT_t^\varepsilon+\sigma B\left(T_t^\varepsilon\right)\right)\right]\\&+\int_0^t\lambda e^{-(\lambda+\delta)s}E\left[I\left(T_s^\varepsilon=s\right)\right.\\&\left.\cdot\phi_{b,d}\left(u+cs+\sigma B(s)-X_1\right)\right]ds\\&+\int_0^t\lambda e^{-\lambda s}E\left[I\left(T_s^\varepsilon<s\right)e^{-\delta T_s^\varepsilon}\right.\\&\left.\cdot\phi_{b,d}\left(u+cT_s^\varepsilon+\sigma B\left(T_s^\varepsilon\right)\right)\right]ds\\&=I_1(t)+I_2(t)+I_3(t). \quad (21)\end{aligned}$$

We can get the expression by *Itô* formula shown as follow:

$$I_1(t)=e^{-\lambda t}E\left[e^{-\delta T_t^\varepsilon}\left(\phi_{b,d}(u)\right)+\int_0^{T_t^\varepsilon}\left(c\phi'_{b,d}(u)+\frac{\sigma^2}{2}\phi''_{b,d}(u)\right)\right]ds. \quad (22)$$

As $\lim_{t\to0}P\left(T_t^\varepsilon=t\right)=1,\lim_{t\to0}P\left(T_t^\varepsilon<t\right)=0$ we know

$$\lim_{t\to0}\frac{I_1(t)-\phi_{b,d}(u)}{t}=\frac{1}{2}\sigma^2\phi''_{b,d}(u)+c\phi'_{b,d}(u)-(\lambda+\delta)\phi_{b,d}(u). \quad (23)$$

$$\lim_{t\to0}\frac{I_2(t)}{t}=\lambda\int_0^u\phi_{b,d}(u-x)p(x)dx. \quad (24)$$

$$\lim_{t\to0}\frac{I_3(t)}{t}=0. \quad (25)$$

So we have
$$\frac{\sigma^2}{2}\phi''_{b,d}(u)+c\phi'_{b,d}(u)=(\lambda+\delta)\phi_{b,d}(u)-\lambda\int_0^u\phi_{b,d}(u-x)p(x)dx$$

Then the theorem is proved.

Remark3.3 Assume $\psi_{b,d}(u)$ is twice continuously differentiable on $(0,b)$, we obtain $\psi_{b,d}(u)$ satisfies the following integro-differential equations:

$$\frac{\sigma^2}{2}\psi''_{b,d}(u)+c\psi'_{b,d}(u)-\lambda\psi_{b,d}(u)+\lambda\int_0^u\psi_{b,d}(u-x)p(x)dx=0. \quad (26)$$

IV. THE SOLUTION OF INTEGRO-DIFFERENTIAL EQUATION

In this section, we will discuss the claim amount is exponential distribution, the expression of the solution of $\phi_{b,d}(u)$ satisfies the homogeneous integro-differential equations. Assume $p(x)=\kappa e^{-\kappa x}$, $x\ge0$ .By laplace transform, then $\hat{p}(s)=\kappa/(s+\kappa)$. Li (2006) shows the generalized Lundberg equation:

$$\left[\sigma^2s^2/2+cs-(\lambda+\delta)\right](s+\kappa)+\lambda\kappa=0. \quad (27)$$

The generalized Lundberg equation has a positive root ρ, two negative roots $-R_1,-R_2$, the solution of (19) can be expressed as follow:

$$\begin{aligned}\phi_{b,d}(u)&=\frac{\kappa-R_1}{R_2-R_1}\left(1+\frac{\xi}{\rho+R_1}\right)e^{-R_1u}\\&+\frac{\kappa-R_2}{R_1-R_2}\left(1+\frac{\xi}{\rho+R_2}\right)e^{-R_2u}\\&-\frac{\xi(\kappa+\rho)}{(\rho+R_1)(\rho+R_2)}e^{\rho u},\quad(0<u<b) \quad (28)\end{aligned}$$

Where $\xi=v_1'(b)/v_2'(b)$.

Example 1 In generalized Lundberg equation, if we make $c=1.1$, $\lambda=1$, $\kappa=1$, $\sigma=0.5$, $\delta=0.05$, $b=10$, then the root of (27) can be expressed:

$$\rho = 0.1812, \quad -R_1 = -0.2264, \quad -R_2 = -9.7548.$$

These results are substituted into the (28), we can get:

$$\phi_{10,d}(u) = 0.08e^{-0.2264u} + 0.9183e^{-9.7548u} + 0.0017e^{0.1812u},$$

Where $0 \le u \le 10$.

ACKNOWLEDGMENT

The authors are indebted the referee for constructive comments, which significantly improved the presentation of our results. L. P. Jia gratefully acknowledges support of the National Social Science Foundation of China (No. 14BJY200), this research would not have been possible without it..

REFERENCES

[1] Wan, N. "Dividend payments with a threshold strategy in the compound poisson risk model perturbed by diffusion"[J]. Insurance: Mathematics and Economics, 2007, 40(3) pp.509-523.

[2] Li, B.; Wu, R. "A note on the perturbed compound poisson risk model with a threshold dividend strategy"[J]. Acta Math.Appl.Sin. Engl.Ser., Vol 25, February 2009, pp.205- 216.

[3] Gao, S.; Lm, Z. "The perturbed compound poisson risk model with constant interest and a threshold dividend strategy" [J]. Comput. Appl. Math. Vol 233, September 2010, (9) pp.2181-2188.

[4] Peng, D. Hou, Z. T. and Liu, Z. M. "The perturbed poisson risk model with constant interest and a threshold dividend strategy under absolute ruin"[J]. Acta Mathematicae Applicatae Sinica, Vol 35, May 2012, pp.855-866.

[5] De Finetti, B.. "Su un'impostazione alternativa dell teoria colletiva del rischio" [J]. Transactions of the XV International Congress of Actuaries. Vol. 2, 1957, pp. 433–443.

[6] Tan, J.; Yang, X.."The compound binomial model with randomized decisions on paying dividends"[J]. Insurance: Mathematics and Economics, Vol 39, January 2006, pp. 1-18.

[7] Eric, C. K.; David, L. "Perturbed MAP risk models with dividend barrier strategy" [J]. J. Appl. Prob, Vol 46, February 2009, pp. 521-541.

[8] Wang, C. W. and Yin, C. C. "On the perturbed compound poisson risk model under absolute ruin with debit interest and a constant dividend barrier" [J]. Acta Mathematica Scientia, Vol 30, January 2010, pp.31-41.

[9] Revuz, D. and Yor, M. Continuous Martingales and Brownian Motion. Springer, Berlin. 1991, 105-106.

[10] Port, S. and Stone, C. Brownian Motion and Classical Potential Theory. Academic Press, New York. 1978.

[11] Li, S. "The distribution of the dividend payments in the compound poisson risk model perturbed by diffusion"[J]. Scandinavian Actuarial Journal, vol. 13, 2006, pp.73-85.