A new linguistic-valued aggregation operator to multiple attribute group decision making

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Abstract

In this paper, a new kind of linguistic-valued aggregation operators, namely, a linguistic-valued weighted aggregation (LVWA) operator, is proposed to multiple attribute group decision making with linguistic-valued information. An example of evaluating university faculty for tenure and promotion is illustrated how to use the LVWA approach to multiple attribute group decision making.

Keywords: Aggregation, Multiple attribute group decision making, Linguistic-valued weighted aggregation (LVWA) operator, Linguistic-valued lattice implication algebra, Lattice theory

1. Introduction

Multiple attribute group decision making (MADM) addresses the problems of choosing an optimum choice that has the highest degree of satisfaction by multiple experts' assessments from a set of alternatives that are characterized in terms of their attributes. Generally, multiple attribute group decision making problems follow a common scheme composed by the following phases:

(1) Evaluation phase: Experts are asked to give preference values to each attribute of each alternative.

(2) Aggregation phase: It combines individual preference values to obtain a collective preference value for each alternative.

(3) Exploitation phase: It orders the collective preference values to obtain the best alternatives.

In the first phase, experts are asked to provide their preferences on each attribute of each alternative. Usually, the information is expressed by means of numerical values such as exact value, interval values, fuzzy numbers, etc. However, in real world, human beings are constantly making decisions under linguistic environment. For example, when evaluating the "comfort" or "design" of a car, linguistic terms like "good ", "fair", "poor" are usually used; evaluating a car's speed, linguistic labels like "very fast", "fast", "slow" can be used, and evaluating students' performances in their courses, linguistic labels like "bad", "medium", "good" can be used. As a result, it is necessary to consider aggregations of linguistic information.

To date, several methods have been proposed for dealing with linguistic information. These methods are mainly as follows:

(1) The method based on the extension principle, which makes operations on fuzzy numbers that support the semantics of the linguistic labels [14]-[15].

(2) The method based on symbols, which makes computations on the indexes of the linguistic terms [16]; Both the above methods develop some approximation processes to express the results in the initial expression domains, which produce the consequent loss of information and hence the lack of precision [17].

(3) The method based on a fuzzy linguistic representation model, which represents the linguistic information with a pair of values called 2-tuple, composed by a linguistic term and a number [17]-[21]. Together with the model, the method also gives a computational technique to deal with the 2-tuple without loss of information.

(4) The method, which computes with words directly [1]-[3].

In this paper, we use the 4th method to aggregate linguistic-valued information for group decision making. At present, a number of researches have recently focused on group decision making with linguistic preference. Herrera et al. developed a consensus model for group decision making under linguistic assessments [7] and combined the linguistic ordered weighted averaging (LOWA) operator with linguistic preference relations and the concept of dominance and non-dominance to show its use in the field of group decision making based on the LOWA operator [8]. Later, Herrera et al. presented a consensus model in complete linguistic framework for group decision making guided by consistency and consensus measures [9]. Xu proposed an uncertain linguistic ordered weighted averaging (ULOWA) aggregation operator and uncertain linguistic hybrid aggregation (ULHA) operator, and developed an approach to multiple attribute group decision making with uncertain linguistic information based on the ULOWA and ULHA operators [10]. In [11], Xu proposed some aggregation operators including the uncertain linguistic ordered weighted geometric (LUOWG) operator, and induced uncertain linguistic ordered weighted geometric (IULOWG) operator to group decision making.

Although there are many aggregation operators to aggregate linguistic information, they can only be used to aggregate linearly ordered linguistic information. Note that there exist incomparable linguistic terms, such as *approximately true*, *possibly true*, and *more or less true*. So it is necessary to find an algebra for modeling the ordering relation of the natural language terms.

Lattice theory is a well-developed branch of an abstract algebra for modeling the ordering relation in the real world. Lattice-valued algebra for modeling linguistic values would be a possible choice. To establish theories and methods to simultaneously deal with fuzziness and incomparability of processed object itself and uncertainty in the course of information processing, Xu et al. combined a lattice with implication algebra and established the lattice implication algebra [6]. It provides a necessary foundation for the processing of incomparable information. In addition, there are some research works on incomparable information processing. An evaluation method with incomparable information is presented in [13]. Lattice-valued linguistic-based decision making method is discussed in [22]. A model for handling linguistic terms in the framework of lattice-valued logic is presented in [4]. In this paper, a new aggregation operator LVWA is proposed. Based on the LVWA operator, an approach to solve multiple attribute group decision making with incomparable linguistic-valued information is established.

The paper is organized as follows: Section 2 includes some basic definitions of lattice implication algebra and linguistic-valued lattice implication algebra. Section 3 introduces the LVWA operator and discusses its properties. Section 4 proposes an approach for multiple attribute group decision making based on the LVWA operator with a linguistic-valued lattice implication algebra preference set. Section 5 illustrates how to use the proposed approach. Section 6 concludes the paper.

2. Preliminaries

In this section, we recall some basic concepts about lattice implication algebra and linguistic truth-valued lattice implication algebra.

2.1 Lattice implication algebra

Definition 2.1.1 Let $(L,\vee,\wedge,')$ be a bounded lattice with an order-reversing involution "′" and the universal bounds O, I, $\rightarrow: L\times L\rightarrow L$ be a mapping. $(L,\vee,\wedge,',\rightarrow,O,I)$ is called a quasi-lattice implication algebra if the following axioms hold for all $x,y,z\in L$:

(I$_1$) $x\rightarrow(y\rightarrow z)=y\rightarrow(x\rightarrow z)$;

(I$_2$) $x\rightarrow x=I$;

(I$_3$) $x\rightarrow y=y'\rightarrow x'$;

(I$_4$) $x\rightarrow y=y\rightarrow x=I$ implies $x=y$;

(I$_5$) $(x\rightarrow y)\rightarrow y=(y\rightarrow x)\rightarrow x$.

Theorem 2.1.1 Let $(L,\vee,\wedge,',\rightarrow,O,I)$ be a quasi-lattice implication algebra. Then

(1) (l_1) holds if and only if (l_4) holds,

(2) (l_2) holds if and only if (l_3) holds,

where

(l_1) $(x\vee y)\rightarrow z=(x\rightarrow z)\wedge(y\rightarrow z)$;

(l_2) $(x\wedge y)\rightarrow z=(x\rightarrow z)\vee(y\rightarrow z)$;

(l_3) $x\rightarrow(y\vee z)=(x\rightarrow y)\vee(x\rightarrow z)$;

(l_4) $x\rightarrow(y\wedge z)=(x\rightarrow y)\wedge(x\rightarrow z)$.

Definition 2.1.2 A quasi-lattice implication algebra is called a lattice implication algebra, if (l_1) and (l_2) hold.

Theorem 2.1.2 Let L be a lattice implication algebra. Then for any $x,y,z\in L$:

(1) If $y\le z$, then $x\rightarrow y\le x\rightarrow z$;

(2) If $x\le y$, then $x\rightarrow z\ge y\rightarrow z$;

(3) $O\rightarrow x=I$;

(4) $I\rightarrow x=x$;

(5) $x\le y$ if and only if $x\rightarrow y=I$;

(6) $x\rightarrow y\ge x'\vee y$.

Example 2.1.1 (Boolean algebra) Let $(L,\vee,\wedge,')$ be a *Boolean lattice*. For any $x,y\in L$, define

$$x\rightarrow y=x'\vee y,$$

then $(L,\vee,\wedge,',\rightarrow)$ is a lattice implication algebra.

Example 2.1.2 (Lukasiewicz implication algebra on [0, 1]) Its operations on [0, 1] are defined respectively as follows:

$$x\vee y=\max\{x,y\},$$
$$x\wedge y=\min\{x,y\},$$
$$x'=1-x,$$
$$x\rightarrow y=\min\{1,1-x+y\},$$

then $([0,1],\vee,\wedge,',\rightarrow)$ is a lattice implication algebra.

Example 2.1.3 (Lukasiewicz implication algebra on finite chains) Consider the set $L=\{a_i\,|\,i=1,2,\cdots,n\}$. For any $1\le j,k\le n$, define

$$a_j \vee a_k = a_{\max\{j,k\}},$$
$$a_j \wedge a_k = a_{\min\{j,k\}},$$
$$(a_j)' = a_{n-j+1},$$
$$a_j \to a_k = a_{\min\{n-j+k,n\}},$$

then $(L, \vee, \wedge, ', \to)$ is a lattice implication algebra.

2.2 Linguistic-valued lattice implication algebra

Definition 2.2.1 Let $(L_i, \vee_i, \wedge_i, \to_i, I_i, O_i)$ $(i = 1, 2, \cdots, n)$ be a family lattice implication algebra. Then

$$\prod_{i=1}^{n} L_i = \{(a_1, a_2, \cdots, a_n) | a_i \in L_i\}$$

is called a direct product of n lattice implication algebra. Define the operators $\vee$, $\wedge$, $'$, $\to$ on $\prod_{i=1}^{n} L_i$ as follows: for any $(a_1, a_2, \cdots, a_n)$, $(b_1, b_2, \cdots, b_n) \in \prod_{i=1}^{n} L_i$,

$$(a_1, a_2, \cdots, a_n) \vee (b_1, b_2, \cdots, b_n) = (a_1 \vee b_1, a_2 \vee b_2, \cdots, a_n \vee b_n),$$
$$(a_1, a_2, \cdots, a_n) \wedge (b_1, b_2, \cdots, b_n) = (a_1 \wedge b_1, a_2 \wedge b_2, \cdots, a_n \wedge b_n),$$
$$(a_1, a_2, \cdots, a_n) \to (b_1, b_2, \cdots, b_n) = (a_1 \to b_1, a_2 \to b_2, \cdots, a_n \to b_n),$$
$$(a_1, a_2, \cdots, a_n)' = (a_1', a_2', \cdots, a_n').$$

Theorem 2.2.1[6] Let $L = L_1 \times L_2 \times \cdots \times L_n$ and L_i $(i = 1, 2, \cdots, n)$ be a lattice implication algebra. Then $\prod_{i=1}^{n} L_i$ be a lattice implication algebra.

Corollary 2.2.1 Let $L = L_1 \times L_2$, and L_i $(i = 1, 2)$ be a finite-chain-type lattice implication algebra. Then L is a lattice implication algebra.

Definition 2.2.2 Let ML={ b_1 , b_2 } be a linguistic-valued set, and b_1 and b_2 be antonym, and $b_1 \le b_2$ such as "poor" and "good", "false" and "true" etc. Define the operators on ML are the same as in Example 2.1.1, then we know that ML is a lattice implication algebra, called a meta linguistic-valued lattice implication algebra.

Example 2.2.1 Let ML={good, poor}. The operators on ML are defined as those in Example 2.1.1, then ML is a linguistic-valued lattice implication algebra.

Definition 2.2.3 Let MW=$\{a_i | i = 1, 2, \cdots, n\}$, and a_i $(i = 1, 2, \cdots, n)$ be modifiers, used to modify the meta language. Define an order on MW as follows: $i \le j$ if and only if $a_i \le a_j$. The operators $\vee$, $\wedge$, $'$, $\to$ on MW as follows:

$$a_j \vee a_k = a_{\max\{j,k\}},$$
$$a_j \wedge a_k = a_{\min\{j,k\}},$$
$$(a_j)' = a_{n-j+1},$$
$$a_j \to a_k = a_{\min\{n-j+k,n\}},$$

then (MW, $\vee$, $\wedge$, $'$, $\to$, a_1, a_n) is a lattice implication algebra, called a lattice implication algebra with modifiers.

Example 2.2.2 Let MW= {absolutely (Abbr. to Ab), highly(Abbr. to Hi), very(Abbr. to Ve), quite(Abbr. to Qu), exactly(Abbr. to Ex), almost(Abbr. to Al), rather(Abbr. to Ra), somewhat(Abbr. to So) , slightly(Abbr. to Sl)} be a modifactory word set. Then the chain Ab $\ge$ Hi $\ge$ Ve $\ge$ Qu $\ge$ Ex $\ge$ Al $\ge$ Ra $\ge$ So $\ge$ Sl is a lattice implication algebra with operations as given in Definition 2.2.1. .

Definition 2.2.4 Let MW= $\{a_1, a_2, \cdots, a_n\}$ be modifiers lattice implication algebra, ML be a meta lattice implication algebra. Then the direct product MW $\times$ ML of MW and ML is a lattice implication algebra. We call it a linguistic-valued lattice implication algebra, the operations are defined as those in Definition 2.2.1.

Example 2.2.3 Let MW= {absolutely, highly, very, quite, exactly, almost, rather, somewhat, slightly} be a modifactory word set, and ML={good, poor}. Then MW $\times$ ML={absolutely good, highly good, very good, quite good, exactly good, almost good, rather good, somewhat good, slightly good, absolutely poor, highly poor, very poor, quite poor, exactly poor, almost poor, rather poor, somewhat poor, slightly poor}. Then (MW $\times$ ML, $\vee$, $\wedge$, $'$, $\to$, O (slightly poor), I (absolutely good) is a linguistic-valued lattice implication algebra. In the following, we will use this linguistic-valued lattice implication algebra as a preference information set S.

3. A linguistic-valued aggregation operator for multiple attribute group decision making

Yager introduced an ordered weighted averaging (OWA) operator, which is defined as follows [5].

An OWA operator of dimension n is a mapping OWA: $R^n \to R$ that has associated an n vector $w = (w_1, w_2, \cdots, w_n)^T$ such that $w_j \in [0,1]$, $j = 1, 2, \cdots, n$,

$\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\text{OWA}_w(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} w_j b_j$$

where b_j is the jth largest of the a_j.

However, the OWA operator can only be used in the situations where the input arguments are the exact numerical values. In our real world, human beings are constantly making decisions under linguistic environment lack of knowledge, and the decision maker's limited attention and information processing capabilities. Hence, it is necessary to research on linguistic-valued information aggregation. In the following, we shall investigate a linguistic-valued weighted aggregation operator, which can be used in situations where the aggregated arguments are given in the form of uncertain linguistic values.

Definition 3.1 Let LVWA: $S^n \to S$, if

$$\text{LVWA}_w(a_1, a_2, \cdots, a_n) = \mathop{\wedge}_{j=1}^{n} (w_j \to a_j)$$

where S is an evaluation set which is a linguistic-valued lattice implication algebra and includes both comparable and incomparable natural linguistic terms, $w = (w_1, w_2, \cdots, w_n)^T$ is the weighting vector of linguistic-valued $a_i \in S$ ($i = 1, 2, \cdots, n$), and $w_j \in S$, $j = 1, 2, \cdots, n$. Then the LVWA is called the linguistic-valued weighted aggregation (LVWA) operator.

Remark 3.1: It follows from Examples 2.1.1-2.1.3 that Boolean lattice, interval [0, 1], and finite chain can be a lattice implication algebra, so the LVWA operator can be used to aggregate linear preference information.

Remark 3.2: Yager's aggregation method in [23] is a special case of the proposed method.

The LVWA operators have the following properties:

Theorem 3.1 (Monotonicity) Let $A = (a_1, a_2, \cdots, a_n)$ be an ordered argument vector and $C = (c_1, c_2, \cdots, c_n)$ be another ordered argument vector such that for each j, $a_j \geq c_j$. Then LVWA(A) $\geq$ LVWA(C).

Proof. Since LVWA (A) = $\mathop{\wedge}_{j=1}^{n} (w_j \to a_j)$,

and LVWA (C) = $\mathop{\wedge}_{j=1}^{n} (w_j \to c_j)$,

the result follows directly from the property $a_j \geq c_j$.

Theorem 3.2 (Commutativity) $\text{LVWA}(a_1, a_2, \cdots, a_n) = \text{LVWA}(a_1', a_2', \cdots, a_n')$,

where $(a_1', a_2', \cdots, a_n')$ is any permutation of the elements in $(a_1, a_2, \cdots, a_n)$.

Proof. Suppose that $w = (w_1, w_2, \cdots, w_n)^T$ is the weighting vector of linguistic-valued a_i ($i = 1, 2, \cdots, n$), then

$$\text{LVWA}_w(a_1, a_2, \cdots, a_n) = \mathop{\wedge}_{j=1}^{n} (w_j \to a_j);$$

$$\text{LVWA}_w(a_1', a_2', \cdots, a_n') = \mathop{\wedge}_{j=1}^{n} (w_j' \to a_j').$$

Hence,

$$\text{LVWA}_w(a_1, a_2, \cdots, a_n) = \text{LVWA}_w(a_1', a_2', \cdots, a_n').$$

Theorem 3.3 If $\mathop{\vee}_{j=1}^{n} w_j = I$, and $a_j = a$ ($j = 1, 2, \cdots, n$). Then $\text{LVWA}_w(a_1, a_2, \cdots, a_n) = a$.

Proof. Since $a_j = a$,

$$\text{LVWA}_w(a_1, a_2, \cdots, a_n) = \mathop{\wedge}_{j=1}^{n} (w_j \to a_j)$$

$$= \mathop{\wedge}_{j=1}^{n} (w_j \to a) = \mathop{\vee}_{j=1}^{n} w_j \to a = I \to a = a.$$

Theorem 3.4 Let $w = (I, I, \cdots, I)^T$. Then $\text{LVWA}_w(a_1, a_2, \cdots, a_n) = \text{Min}_i[a_i]$.

4. An approach based on the LVWA operator to multiple attribute decision making with linguistic-valued information

Consider a multiple attribute group decision making with linguistic-valued information: Assume that S is an evaluation set that is a linguistic-valued lattice implication algebra and includes both comparable and incomparable natural linguistic terms used to indicate preference information. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a discrete set of alternatives, and $U = \{u_1, u_2, \cdots, u_m\}$ be a set of attributes. Let $D = \{d_1, d_2, \cdots, d_l\}$ be a set of decision makers, and $\omega = (\omega_1, \omega_2, \cdots, \omega_l)^T$ be the weight vector of decision makers, where $\omega_k \in S$, $k = 1, 2, \cdots, l$. Suppose that $\tilde{A}^{(k)} = (a_{ij}^{(k)})_{m \times n}$ is the decision matrix, where $a_{ij}^{(k)} \in S$ is a preference value, which takes the form of linguistic value, given by the decision maker $d_k \in D$, for alternative $x_j \in X$ with respect to attributes $u_i \in U$. Group decision making problems follow a common resolution scheme composed by the following three phases:

(1) Evaluation phase: The experts are asked to give the preference values to each attribute of each alternative.

(2) Aggregation phase: It combines the individual preferences to obtain a collective preference value for each alterative.

(3) Exploitation phase: It orders the collective preference values to obtain the best alternatives.

In the following we shall utilize the LVWA operator to establish an approach to multiple attribute group decision making with linguistic-valued information.

Step 1: Experts give preference information $\tilde{a}_{ij}^{(k)}$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$, $k=1,2,\cdots,l$.

Step 2: Utilize the decision information given in matrix $\tilde{A}^{(k)}$, and the LVWA operator:

$$\tilde{a}_j^{(k)} = \text{LVWA}_w(a_{1j}^{(k)}, a_{2j}^{(k)}, \cdots, a_{mj}^{(k)}),$$
$$k=1,2,\cdots,l,\ j=1,2,\cdots,n$$

to derive the individual overall preference value $\tilde{a}_j^{(k)}$ of alternative x_j, where $w=(w_1,w_2,\cdots,w_m)^T$ is the weight vector of linguistic-valued a_i ($i=1,2,\cdots,m$), with $w_j \in S$, $j=1,2,\cdots,l$.

Step 3: Utilize the LVWA operator:

$$\tilde{a}_j = \text{LVWA}_\omega(a_j^{(1)}, a_j^{(2)}, \cdots, a_j^{(l)}),\ j=1,2,\cdots,n$$

to derive the collective overall preference value $\tilde{a}_j$ of alternative x_j, where $\omega=(\omega_1,\omega_2,\cdots,\omega_l)^T$ is the weight vector of decision makers, with $\omega_j \in S$, $j=1,2,\cdots,n$.

Step 4: Rank all the alternatives x_j, and select the optimal one(s) in according to $\tilde{a}_j$. The optimal alternative is $x_j \in X$ that $\tilde{a}_j$ is maximal. End.

5. An illustrative example

In this section, a problem of evaluating university faculty for tenure and promotion (adapted from Bryson and Mobolurin [12]) is used to illustrate the developed approach.

A practical use of the proposed approach involves the evaluation of university faculty for tenure and promotion. The attributes used at some university are u_1: teaching, u_2: research, and u_3: service, and (whose vector weights be $w=((a_7,1),(a_9,0),(a_7,1))^T$). Five faculty candidates (alternatives) x_j ($j=1,2,3,4,5$) are to be evaluated using the term set

$S=\{(a_9,1)=$ absolutely good, $(a_8,1)=$highly good, $(a_7,1)=$very good, $(a_6,1)=$quite good, $(a_5,1)=$exactly good, $(a_4,1)=$almost good, $(a_3,1)=$rather good, $(a_2,1)=$somewhat good, $(a_1,1)=$slightly good, $(a_9,0)=$absolutely poor, $(a_8,0)=$highly poor, $(a_7,0)=$very poor, $(a_6,0)=$quite poor, $(a_5,0)=$exactly poor, $(a_4,0)=$almost poor, $(a_3,0)=$rather poor, $(a_2,0)=$somewhat poor, $(a_1,0)=$slightly poor$\}$

by four decision makers d_k ($k=1,2,3,4$) (whose weight vector $\omega=((a_8,1),(a_6,0),(a_9,0),(a_5,1))^T$) under these three attributes, as listed in Tables 1-4, respectively.

Step 1: Utilize the preference information given in Table 1 and the LVWA operator (Let $w=((a_7,1),(a_9,0),(a_7,1))^T$)

$$\tilde{a}_j^{(k)} = \text{LVWA}_w(\tilde{a}_{1j}^{(k)}, \tilde{a}_{2j}^{(k)}, \tilde{a}_{3j}^{(k)}),$$

$k=1,2,3,4$, $j=1,2,3,4,5$ to derive the individual overall preference value $\tilde{a}_j^{(k)}$ of the alternative x_j:

$\tilde{a}_1^{(1)} = \text{LVWA}_w((a_9,1),(a_7,1),(a_8,0))$

$=(((a_7,1)\to(a_9,1))\wedge(((a_9,0)\to(a_7,1))\wedge((a_7,1)\to(a_8,0))$

$=(a_7,0)$

Similarly, we have

$\tilde{a}_2^{(1)}=(a_8,0)$, $\tilde{a}_3^{(1)}=(a_8,0)$, $\tilde{a}_4^{(1)}=(a_4,1)$, $\tilde{a}_5^{(1)}=(a_8,1)$,

u_i	x_1	x_2	x_3	x_4	x_5
u_1	$(a_9,1)$	$(a_7,1)$	$(a_6,1)$	$(a_9,1)$	$(a_9,1)$
u_2	$(a_7,1)$	$(a_8,1)$	$(a_9,1)$	$(a_4,0)$	$(a_8,0)$
u_3	$(a_8,0)$	$(a_9,0)$	$(a_9,0)$	$(a_7,1)$	$(a_8,1)$

Table 1: Preference information given by decision maker d_1.

u_i	x_1	x_2	x_3	x_4	x_5
u_1	$(a_7,1)$	$(a_6,0)$	$(a_8,1)$	$(a_9,0)$	$(a_9,1)$
u_2	$(a_9,1)$	$(a_8,0)$	$(a_7,1)$	$(a_7,1)$	$(a_8,1)$
u_3	$(a_6,1)$	$(a_9,0)$	$(a_9,0)$	$(a_8,1)$	$(a_7,0)$

Table 2: Preference information given by decision maker d_2.

u_i	x_1	x_2	x_3	x_4	x_5
u_1	$(a_8,1)$	$(a_6,0)$	$(a_9,1)$	$(a_8,1)$	$(a_7,1)$
u_2	$(a_9,1)$	$(a_7,1)$	$(a_8,1)$	$(a_6,1)$	$(a_8,0)$
u_3	$(a_7,1)$	$(a_8,1)$	$(a_7,1)$	$(a_9,1)$	$(a_7,0)$

Table 3: Preference information given by decision maker d_3.

u_i	x_1	x_2	x_3	x_4	x_5
u_1	$(a_7,1)$	$(a_9,1)$	$(a_9,0)$	$(a_6,0)$	$(a_6,1)$
u_2	$(a_9,0)$	$(a_9,0)$	$(a_8,1)$	$(a_8,1)$	$(a_8,0)$
u_3	$(a_8,0)$	$(a_8,1)$	$(a_9,1)$	$(a_7,1)$	$(a_6,1)$

Table 4: Preference information given by decision maker d_4.

$\tilde{a}_1^{(2)} = (a_9,1)$, $\tilde{a}_2^{(2)} = (a_8,0)$, $\tilde{a}_3^{(2)} = (a_7,1)$, $\tilde{a}_4^{(2)} = (a_7,0)$ $\tilde{a}_5^{(2)} = (a_8,0)$, $\tilde{a}_1^{(3)} = (a_9,1)$, $\tilde{a}_2^{(3)} = (a_7,0)$, $\tilde{a}_3^{(3)} = (a_8,1)$, $\tilde{a}_4^{(3)} = (a_6,1)$, $\tilde{a}_5^{(3)} = (a_8,0)$, $\tilde{a}_1^{(4)} = (a_9,0)$, $\tilde{a}_2^{(4)} = (a_9,1)$, $\tilde{a}_3^{(4)} = (a_8,0)$, $\tilde{a}_4^{(4)} = (a_8,0)$, $\tilde{a}_5^{(4)} = (a_8,1)$.

Step 2: Utilize the weight vector of decision makers, $\omega = ((a_8,1),(a_6,0),(a_9,0),(a_5,1))^T$, and the LVWA operator (let $w' = ((a_5,0),(a_7,1),(a_8,0),(a_9,1))^T$):

$$\tilde{a}_j = \text{LVWA}_\omega(\tilde{a}_j^{(1)},\tilde{a}_j^{(2)},\tilde{a}_j^{(3)},\tilde{a}_j^{(4)}) \ (j = 1,2,3,4,5)$$

to aggregate the individual overall preference values $\tilde{a}_j^{(k)}$ $(k = 1,2,3,4)$ and thus get the collective overall preference value $\tilde{a}_j$ of alternative x_j:

$$\begin{aligned}\tilde{a}_1 &= \text{LVWA}_\omega(\tilde{a}_1^{(1)},\tilde{a}_1^{(2)},\tilde{a}_1^{(3)},\tilde{a}_1^{(4)})\\ &= ((a_8,0)\to(a_8,0))\wedge((a_6,1)\to(a_9,1))\\ &\wedge((a_9,0)\to(a_9,1))\wedge((a_5,1)\to(a_9,0))\\ &= (a_8,0)\end{aligned}$$

Similarly, we have

$$\tilde{a}_2 = (a_8,0),\ \tilde{a}_3 = (a_8,0),\ \tilde{a}_4 = (a_5,0),\ \tilde{a}_5 = (a_8,1).$$

Step 3: Rank all the alternatives x_j, and select the optimal one(s) in according with $\tilde{a}_j$. The optimal alternative is $x_j \in X$ that $\tilde{a}_j$ is maximal. Thus the optimal one is x_5.

6. Conclusions

In this paper, we proposed a linguistic-valued weighted aggregation operator, which can be used in the situations where the evaluation value set is a linguistic-valued lattice implication algebra. Based on this operator, an approach to multiple attribute group decision making with linguistic-valued information is given. This approach has the following advantages:

(1) It does not require all linguistic terms to have a total order.

(2) It permits to compute with natural linguistic terms directly.

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