Fractal dimension of well logging curves associated with the texture of volcanic rocks

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Abstract—Fractal dimension is usually an effective approach that used to describe the nonregular geometry objects in the nature and nonlinear systems. We investigate the fractal characteristics of well logging curves acquired from Liaohe oil field, China. To calculate the fractal dimension of well logging curves, box-counting method was proposed, we study the method systematically, and calculate the fractal dimension of logging curves using neutron logging(CNL), density logging(DEN), acoustic logging(AC)、 resistivity logging(RLLD) and gamma-ray logging(GR) from 108 wells, 1100m sections with a total of 8800 logging data. Then we found the corresponding relation between fractal dimension of well logging curves and the texture of volcanic rocks. It was observed that the fractal dimension of CNL、 DEN and AC have same value, and the fractal dimension of volcanic lava is lower than the fractal dimension of pyroclastic rock. To validate the accuracy of conclusion, we take X-10 well, 2300-2580m sections for example, describe the relationship between the fractal dimension and the texture of volcanic rock in certain depth. Compare the predicting result of texture of volcanic rocks with core data, the accuracy reaches 90%, our studies demonstrated that the fractal dimension of well logging curves can be applied to predict the texture of volcanic rocks.

Keywords-Well logging curves; fractal dimension; box-counting; window technique; volcanic rocks

I.   INTRODUCTION

Fractal theory was initially proposed by B.B. Mandelbrot in 1975. Fractal is used to describe irregular figures with no characteristic scales but with self-similarity or self-affinity [1]. As a measure of the complexity of fractal, fractal dimension has developed into a powerful tool for describing features and complexity of nonlinear systems, and has been increasingly applied in various fields of science [2-4].

The Earth experienced several geological processes can be viewed as a non-linear process, has led to strong heterogeneity in rock lithology, permeability and porosity distribution. The heterogeneity provides premise and feasibility that the study of fractal theory is applied to geophysics [5, 6]. In an attempt to solve the problems in geology, many scholars investigate the fractal characteristics of well logging data, and make a great progress [7]. On the one hand, a lot of theoretical research about the fractal characteristic of well logging curves has been done, which mainly focus on self-similarity, fractal interpolation, fractal correction multi-fractal analysis and calculating method of fractal dimension [8]; On the other hand, the geological information in well logging data was quantitative interpreted by using fractal dimension of well logging curves. Furthermore, fractal dimension has a wide application in the prediction of fracture, reservoir heterogeneity, sequence stratigraphy, sedimentary facies analysis and rock pore structure description.

In this paper, we first research the box-counting method calculating fractal dimension of well logging curves. In addition, we discuss the relationship between fractal dimension of well logging curves and the texture of volcanic rocks.

II.   BOX-COUNTING DIMENSION

Fractal dimension is a useful concept in describing natural objects, which gives their degree of complexity. There are various closely related notions of fractional dimension. From the theoretical point of view, the most important are the Hausdorff dimension, the packing dimension. However, the box counting dimension is widely used in practice, may be due to the ease of implementation [9, 10].

The box-counting dimension is motivated by the square boxes. In this method, for a planar set S, we construct boxes, which are a square δ on a side and S is covered with square boxes, and the number of square boxes of a certain size is counted to see how many of them are necessary to cover S completely. When $\delta \to 0$, the total area covered by square boxes will converge to the measure of S. This can be expressed mathematically as

$$D_B = \lim_{\delta \to 0}(\log N_\delta(S) / \log(1/\delta))$$

Where $N_\delta(S)$ is the total number of boxes of size δ required to cover the S entirely. However in practice, the box-counting algorithm estimates fractal dimension by counting the number of boxes required to cover S for several box sizes, and fitting a straight line to the log-log plot of $N_\delta(S)$ versus δ. The slope of the least square best fit straight line is taken as an estimate of the box-counting dimension D_B.

Well logging curves are non-regular plane curves, which was connected with measured value for each sample points. To satisfy the needs of point-by-point

calculations, we adopted the window technique, which actually was analyzed for well logging curves from one window length.

In this paper, we take the K37 well, 3150-3151m sections with a total of 9 sample points for example (Fig .1), and reasearch the calculation procedure for box-counting dimension of well logging curves.
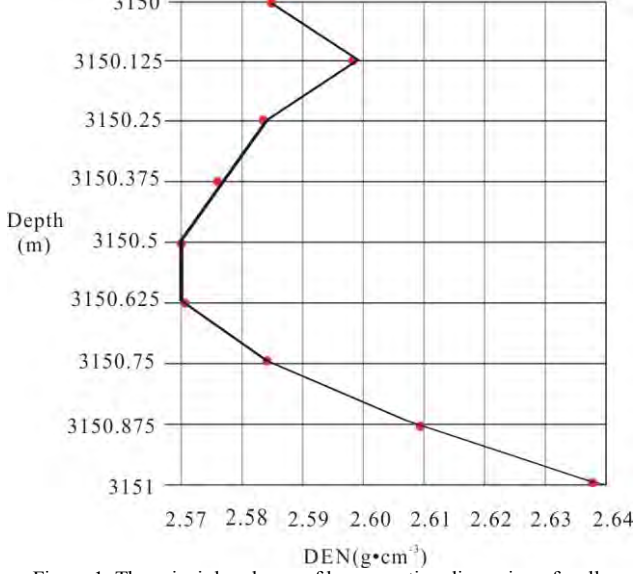

Figure 1 The principle scheme of box-counting dimension of well-logging curves

1) Where the window is 1-**m** in length, the logging curves in one window length were mapped into a square $L = 1$ on a side. Where the starting depth of well logging curve is H_{start}, and the ending depth is H_{end}, the depth of sample point is H_{log} and then the maximum value of sample point is V_{max} and the minimum value is V_{min}, the log value of each sample point is V_{log}, so the coordinates of each sample point in the plane.

$$V_x = L\left(V_{log} - V_{min}\right) / V_{max} - V_{min}$$
$$V_y = L\left(H_{log} - H_{start}\right) / V_{end} - V_{start} \quad (2.1)$$

2) We calculate the equations of sampling points of two adjacent lines. The line equation as follows can be derived from the two-point form of straight-line equation:

$$y = V_{y(i)} + \left(V_{y(i+1)} - V_{y(i)}\right)\left(x - V_{x(i)}\right) / V_{x(i+1)} - V_{x(i)}$$
$$\left(V_{x(i)} \le x \le V_{x(i+1)}\right) \quad (2.2)$$

Where the number of sampling points in one window is 9, So $i =1,2,\ldots,8$.

3) We adopt grid processing for planar graph. Suppose to the grid points in horizontally or vertically are equal to M. Then the coordinates for each split point is as follows:

$$X_{grid(i)} = iL / M$$
$$Y_{grid(i)} = iL / M \quad (i = 0,1,L,M) \quad (2.3)$$

So the planar graph is decomposed into $M \times M$ small squares that are the boxes of $\delta = L / M$ side length. The followings are the equations used to describe boxes:

$$x_i = X_{grid(i)}$$
$$y_i = Y_{grid(i)} \quad (2.4)$$

4) We solve the simultaneous equations (2.2) and (2.4), then the solution of equations are the points at which well logging curves intersects boxes, and W is called the number of intersection points. There are two kinds of situation that calculate the number of boxes: in case the intersection points do not include the boundary points, then $N_\delta = W - 1$ is called the number of boxes. In another case, the intersection points include the boundary points, then $N_\delta = W - 2$ is called the number of boxes.

5) We conduct K times grid processing for planar graph ($K = 100$), and obtain a total of 100 groups data of $(\delta_{(i)}, N_{\delta(i)})$ ($i = 1,2,L,K$).

Then we obtain the $(\lg\delta_{(i)}, \lg N_{\delta(i)})$ by logarithmic transformation of $(\delta_{(i)}, N_{\delta(i)})$. Afterwards, we do the least squares line fitting for $(\lg\delta_{(i)}, \lg N_{\delta(i)})$, the straight line equation $y = kx + b$ is derived. The result is shown in Fig .2.
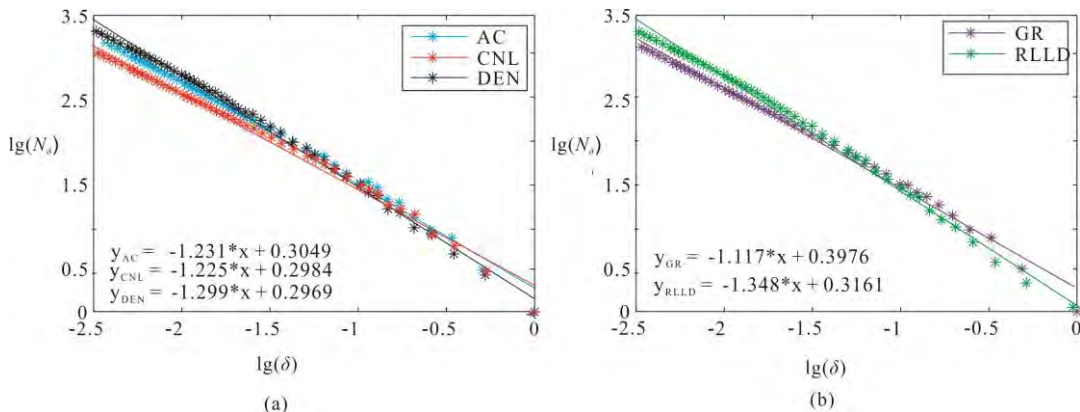

$$y_{AC} = -1.231*x + 0.3049$$
$$y_{CNL} = -1.225*x + 0.2984$$
$$y_{DEN} = -1.299*x + 0.2969$$

$$y_{GR} = -1.117*x + 0.3976$$
$$y_{RLLD} = -1.348*x + 0.3161$$

(a)   (b)

Figure 2 The diagram of calculating a box-counting dimension

III. RESULTS AND DISCUSSION

The program for calculating fractal dimension of well logging curves was implemented in Matlab2012a by using box-counting method. We sort out five logging parameters including gamma-Ray (GR), density (DEN) resistivity (RLLD), acoustic slowness (AC), and neutron porosity (CNL), which are influenced by the texture of volcanic rocks. Then we calculate the box-counting dimension of well logging curves from 108 wells, 1100m sections with a total of 8800 logging data in Liaohe basin. In this paper, the window is 1 m in length, the sampling rate in one window is 9 and the times of grid processing is 100. The box-counting dimension of logging curves of volcanic rocks is shown in Table 1.

TABLE 1 THE BOX-COUNTING DIMENSION OF LOGGING CURVES OF VOLCANIC ROCKS

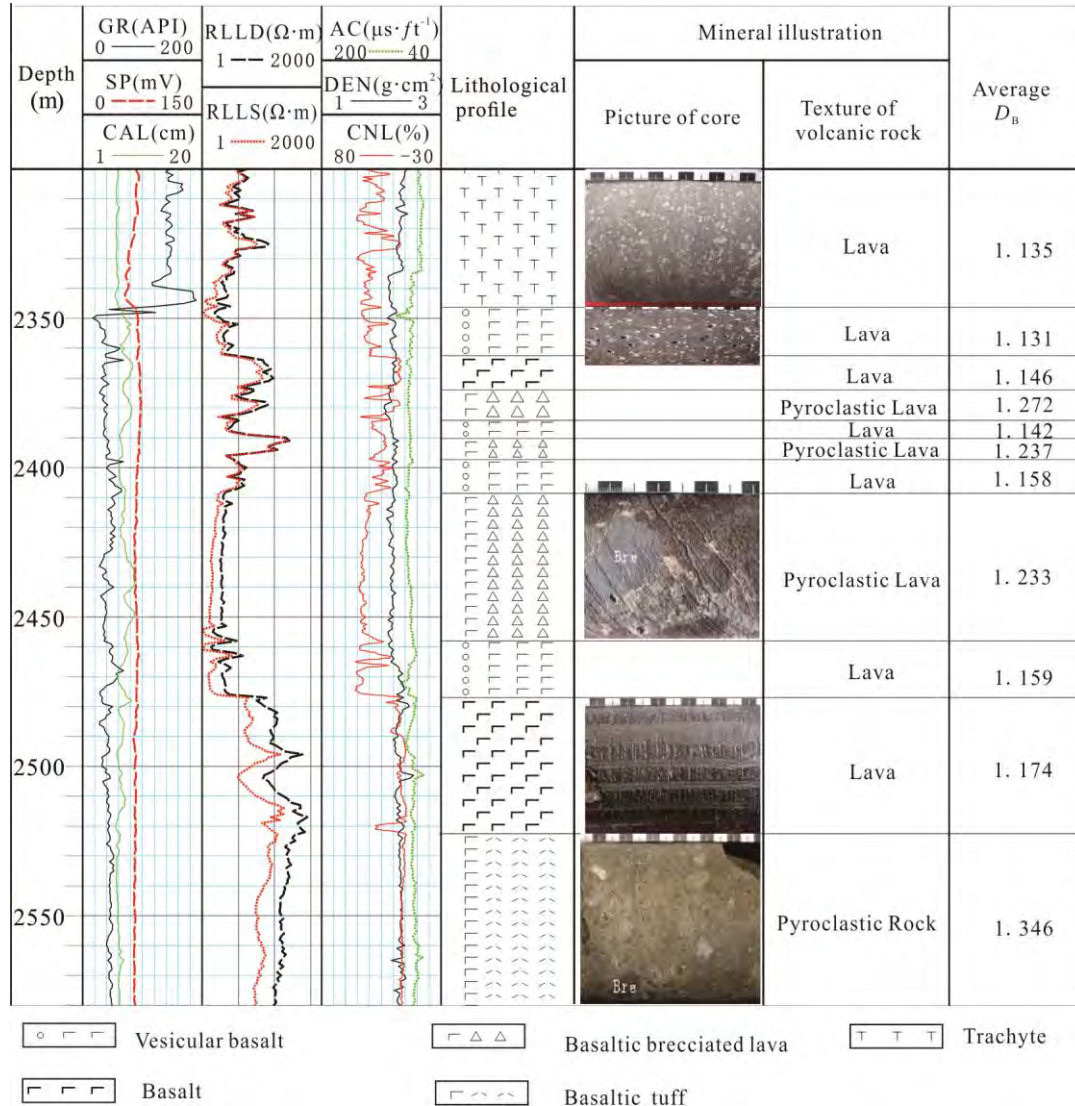| Texture of volcanic rock	Fractal dimension of well logging curves					Average dimension
	D_{RLLD}	D_{AC}	D_{CNL}	D_{DEN}	D_{GR}	D_A
Lava	1.224	1.123	1.113	1.147	1.092	1.140
Pyroclastic lava	1.307	1.265	1.226	1.292	1.118	1.253
Pyroclastic rock	1.356	1.348	1.353	1.230	1.248	1.321
Sink-pyroclastic rock	1.132	1.103	1.092	1.085	1.050	1.094
supergene rock	1.208	1.129	1.143	1.171	1.133	1.157
plutonic rock	1.211	1.133	1.161	1.168	1.123	1.159

Figure 3. Well-logging interpretation result from X-10 Well, 2300-2580m section.

For the purpose of investigate the relationship between fractal dimension and texture of volcanic rocks, the texture characteristic of volcanic rocks was analyzed based on core data of geology. Seen from Table 1, two results are as follows: 1）D_{DEN}, D_{AC} and D_{CNL} are called the fractal dimension of DEN, AC and CNL respectively, they have same value for one sort of volcanic rock. In addition, D_{RLLD} is called the fractal dimension of RLLD, which is higher than D_{AC}, D_{DEN} and D_{CNL}, and D_{GR} is called the fractal dimension of GR, which is lower than D_{AC}, D_{DEN} and D_{CNL}. 2）We research the distribution regularity of fractal dimension of volcanic rocks. The more complicate of the texture of volcanic rock, the higher is the fractal dimension. The fractal dimension of volcanic lava is lower than the fractal dimension of pyroclastic rock. Furthermore, this paper describes the application of fractal dimension to distinguish the supergene rock from sub-volcanic rock, or separate plutonic rock from volcanic lava, but the effect is not obvious.

To validate the accuracy of corresponding relation between fractal dimension and volcanic rock, we calculate the box-counting dimension for X-10 well, 2300-2580m sections, and predict the texture of volcanic rock in corresponding formation depth. Then we contrast the predicting result with the lithology of core data, the corresponding rate reaches 90%. As shown in Fig .3, the well logging curves is a fractal system with self-similar structure, which degree of complexity can be represented quantitatively by using fractal dimension. Lastly, the result indicates fractal dimension of well logging curves can be used to predict the texture of volcanic rocks.

IV. CONCLUSIONS

In this paper, our study has illustrated well logging curves a fractal system with self-similar structure, which degree of complexity can be represented quantitatively by using fractal dimension. Moreover, the D_B values of well logging curves have been calculated by applying the box-counting method. The range of box sizes chosen and the times of grid processing are crucial in determining D_B values. In addition, we have chosen logging parameters of GR, RLLD, AC, DEN and CNL of 108 wells, 1100m

sections with a total of 8800 logging data to calculate their fractal dimension. On the basis of calculating their fractal dimension, it has been concluded that fractal dimension of well logging curves can be used to predict the texture of volcanic rocks. However, we found that the effect of application of fractal dimension to distinguish the supergene rock from sub-volcano rock, or separate plutonic rock from volcanic lava is not obvious.

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