A Method for Line of Sight Angle Rate Reconstruction Based on Strapdown Seeker

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Abstract—The line of sight (LOS) information which detected by strapdown seeker is coupled with missile attitude information. On the other hand, since the instantaneous field of view is wider, the noise levels of strapdown seeker are higher than those of conventional gimbals seeker. According to the relationship among coordinates which are used to describe motion of missile, analysis of the attitude decoupling algorithm was deduced in detail. Based on the proposed algorithm, the LOS reconstruction filter was designed to obtain more precise line of sight angle, where, the theory of Central Difference Kalman Filter (CDKF) which based on Central Difference (CD) method was applied to deal with the filtering problem of nonlinear system. Compared with EKF, the solving of Jacobi matrixs can be avoided and only one adjustable parameter h was introduced which leads to a reduction of the computational cost. Based on this, the algorithm was suitable for engineering application. The results of computer simulation confirmed the validity of the proposed inertial LOS reconstruction filter, which can be used to improve the precision of inertial LOS and LOS rate estimation. The method meanwhile can be used as a new method to estimate inertial LOS rate.

Keywords—Strapdown Seeker, LOS Rate Reconstruction, Central Difference Kalman filter, Fadin- MemoryFilter

I. INTRODUCTION

Proportional navigation (PN) guidance law based on line of sight (LOS) angle and its rate was applied in most modern homing seeker missile. All hardware structure of the detectors were directly fixed on the strapdown seeker. The structure of guidance system is simple and reliable by removing stabilization and tracking platform. The problems of application for some tactical missiles which was limited by space were solved. Adopting Strapdown seeker makes the integrated guidance and control design become possible. However, it also brings two serious disadvantages. Firstly, only LOS angle information relative to body coordinate system can be measured, thus the metrical LOS angles will couple with body attitude motion which finally leads to high nonlinearity. Secondly, because of the wider instantaneous field of view, more serious measurement noise especially non-Gaussian noise will be induced into the strapdown guidance system compared with the conventional system[1]. Thus, Jan K proposed using filter method[2] to obtain the target line of sight angle rate. Tingting Sun establish the mathematical model of stapdown optical image seeker[3]. Guojiang Zhang establish the second-order of the LOS angle based on the target maneuvering model[4]. Pei Wang proposed using Nonlinear Tracking-Differentiator to estimate the angle rate[5].

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LOS angle rate was obtained by using optimal filtering theory based on the model mentioned above. Since the range and relative velocity are unobservable to strapdown seekers, it is difficult to obtain a satisfactory result.

In this paper, we focus on the strapdown seeker which can only get LOS in body coordinates. Based on the relationship of LOS, LOS in body frame (BLOS) and rate gyros' information, the LOS first-order dynamic model for strapdown seeker was constructed, and a LOS reconstruction filter was designed by using the model and CDKF method.

CDKF have some advantages such as, three order precision for Gauss distributed random variable, less cost of computations compared with UKF theory. The results of simulation in MATLAB environment have showed that the method has a good performance in estimating the LOS angle. The precision of the estimation can be improved based on this method.

II. THE LOS RECONSTRUCTION METHODS

A. the relationship among the required coordinates

We take the ground coordinate as the inertial frame. The coordinate systems which needed to reconstruct LOS angle rate were showed as Fig .1. The relationships of coordinate transformation among these coordinate systems were showed as Fig .2.

The yaw angle q_{IH} and the pitch angle q_{IV} in the inertial reference frame S_I are essential to implement guidance law design. Since the optical system was fixed on the missile's body, only the yaw angle q_{BH} and pitch angle q_{BV} of LOS in body frame S_B can be obtained. Missile attitudes in inertial space are changing all the time during the process of guidance. Hence, it is essential to establish the "Mathematic Platform" to decouple the missile attitudes information from strapdown seeker's detecting information.

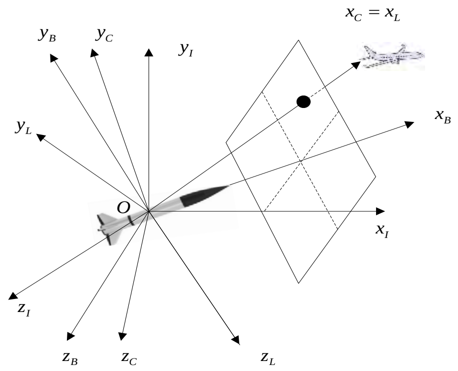

Figure 1 the relationship of coordinate systems

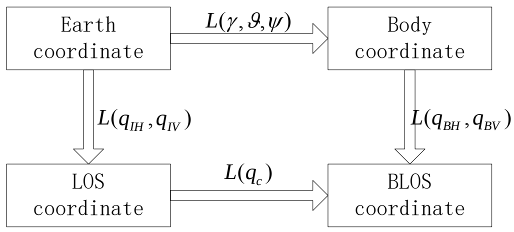

Figure 2. the relationships of coordinate transformation

B. the LOS decoupleing method

As showed in Fig .1, the target has the same position $(R_r,0,0)$ both in LOS coordinate and BLOS coordinate. According to the space position invariance principle, the target position in inertial frame S_I is

$$[x \quad y \quad z]^T = T(q_{IH},q_{IV})\cdot[R_r \quad 0 \quad 0]^T \tag{1}$$

The target position in body frame S_B is

$$[x_b \quad y_b \quad z_b]^T = T(q_{BH},q_{BV})\cdot[R_r \quad 0 \quad 0]^T \tag{2}$$

Accord to the transform relation between body frame S_B and inertial frame S_I, we can deduce that

$$[x \quad y \quad z]^T = T(\psi,\vartheta,\gamma)\cdot T(q_{BH},q_{BV})\cdot[R_r \quad 0 \quad 0]^T \tag{3}$$

The q_{IH} and q_{IV} can be obtained by the following equation

$$[q_{IH} \quad q_{IV}]^T = [-\arctan(a_3/a_1) \quad \arcsin(a_2)]^T \tag{4}$$

where,

$$\begin{cases} a_1 = \cos\vartheta\cos\psi\cos q_{BV}\cos q_{BH} - \sin\vartheta\cos\psi\cos\gamma\sin q_{BV} \\ \quad +\sin\psi\sin\gamma\sin q_{BV} - \sin\vartheta\cos\psi\sin\gamma\cos q_{BV}\sin q_{BH} \\ \quad -\sin\psi\cos\gamma\cos q_{BV}\sin q_{BH} \\ a_2 = \sin\vartheta\cos q_{BV}\cos q_{BH} + \cos\vartheta\cos\gamma\sin q_{BV} \\ \quad +\cos\vartheta\sin\gamma\cos q_{BV}\sin q_{BH} \\ a_3 = -\cos\vartheta\sin\psi\cos q_{BV}\cos q_{BH} + \sin\vartheta\sin\psi\cos\gamma\sin q_{BV} \\ \quad +\cos\psi\sin\gamma\sin q_{BV} + \sin\vartheta\sin\psi\sin\gamma\cos q_{BV}\sin q_{BH} \\ \quad -\cos\psi\cos\gamma\cos q_{BV}\sin q_{BH} \end{cases} \tag{5}$$

So the decoupling equation was established. We can reconstruct the q_{IH} and q_{IV} according to missile attitudes which was measured by strapdown seeker. The inertial LOS rate can be obtained by differential network. The method proposed above ignored the second-order dynamic characteristics of BLOS angle and missile attitudes. The extraction of the LOS angle rate dynamic performance was restricted by the seeker sampling frequency. We can reduce the LOS angle rate's lag by introducing missile attitude which was detected by rate gyro in inertial navigation system.

The inertial coordinate system conversion to LOS coordinate can be realized through the following methods. We assumption that the angle rate of LOS coordinate relative to the BLOS coordinate is $\mathbf{\Omega}_3$

$$\mathbf{\Omega}_3 = [-\dot{q}_c \quad 0 \quad 0]^T \tag{6}$$

The angle rate vector of the frame S_c relative to the frame S_B represented in coordinate frame S_B is

$$\mathbf{\Omega}_2 = [\dot{q}_{BV}\sin\dot{q}_{BH} \quad \dot{q}_{BV} \quad \dot{q}_{BV}\cos\dot{q}_{BH}]^T \tag{7}$$

where, $\dot{q}_{BH}$ and $\dot{q}_{BV}$ are the yaw and pitch angle in body frame respectively, which can be obtained by derivative network.

The angle rate vector of the frame S_B relative to the frame S_I represented in coordinate frame S_B is

$$\mathbf{\Omega}_1 = [\omega_x \quad \omega_y \quad \omega_z]^T \tag{8}$$

According to the Eq. 6-8, we can get the angle rate vector of the frame S_C relative to the frame S_I represented in coordinate frame S_I is

$$\mathbf{\Omega} = T_B^I(\mathbf{\Omega}_1 + \mathbf{\Omega}_2) + T_{LOS}^I\mathbf{\Omega}_3 \tag{9}$$

In addition, the angle rate vector of the frame S_{LOS} relative to the frame S_I represented in coordinate frame S_I can be written as follows

$$\mathbf{\Omega} = [\dot{q}_{IV}\sin q_{IH} \quad \dot{q}_{IH} \quad \dot{q}_{IV}\cos q_{IH}]^T \tag{10}$$

Thus, the relation between the rate $\dot{q}_{IV}, \dot{q}_{IH}$ and $\dot{q}_{BV}, \dot{q}_{BH}$ are established. But the rate $\dot{q}_c$ can't be obtained by strapdown seeker. So decoupling matrix can be constructed to deal with this problem. The LOS rate vector can be written as follows

$$[\dot{q}_{IH} \quad \dot{q}_{IV}]^T = \mathbf{M}_q \mathbf{\Omega} \tag{11}$$

where,

$$\mathbf{M}_q = \begin{bmatrix} -\tan q_{IV} \cos q_{IH} & 1 & \tan q_{IV} \sin q_{IH} \\ \sin q_{IH} & 0 & \cos q_{IH} \end{bmatrix} \tag{12}$$

According to Eq. 4, the observation equation can be written as

$$\begin{bmatrix} q_{BH} \\ q_{BV} \end{bmatrix} = \begin{bmatrix} -\arctan\left(b_3/b_1\right) \\ \arcsin\left(b_2\right) \end{bmatrix} \tag{13}$$

where,

$$\begin{cases} b_1 = \cos\vartheta\cos\psi\cos q_{IV}\cos q_{IH} + \sin\vartheta\sin q_{IV} \\ \quad + \cos\vartheta\sin\psi s\cos q_{IV}\sin q_{IH} \\ b_2 = -\sin\vartheta\cos\psi\cos\gamma\cos q_{IV}\cos q_{IH} + \sin\psi\sin\gamma\cos q_{IV}\cos q_{IH} \\ \quad + \cos\vartheta\cos\gamma\sin q_{IV} - \sin\vartheta\sin\psi\cos\gamma\cos q_{IV}\sin q_{IH} \\ \quad - \cos\psi\sin\gamma\cos q_{IV}\sin q_{IH} \\ b_3 = \sin\vartheta\cos\psi\sin\gamma\cos q_{IV}\cos q_{IH} + \sin\psi\cos\gamma\cos q_{IV}\cos q_{IH} \\ \quad - \cos\vartheta\sin\gamma\sin q_{IV} + \sin\vartheta\sin\psi\sin\gamma\cos q_{IV}\sin q_{IH} \\ \quad - \cos\psi\cos\gamma\cos q_{IV}\sin q_{IH} \end{cases} \tag{14}$$

III. THE DESIGN OF LOS RECONSTRUCTION FILTER BASED ON CDKF METHOD

CDKF is nonlinear filter, Compared with UKF algorithm, it has some advantages such as moderate calculation cost, high precision, less adjust parameter and good robustness.

The background noise of strapdown seeker, and high nonlinearity of decoupling information make it difficult to calibrate the system noise. A simple method is define the $\mathbf{\Omega}$ as $\mathbf{\Omega}^I_{c\mathrm{Im}}$ plus Gaussian white noise. Weighted statistical linearization method provides a way to solve the problem that the random variable will transmit its nonlinear characteristics to the next one via the nonlinear function. The method will select sever deterministic sampling points, and calculate, with these sample points as the function inputs, the value of nonlinear function. The random variable's statistical linearization can be realized by using regression techniques. The foundation of CDKF is central difference approximation, which is a method for calculating the statistics of random variable undergone a nonlinear function[6-8]. We suppose that states are Gaussian random variables, therefore, only mean and covariance are essential to be approximated.

According to the calibrations of seeker and navigation system, the statistic of BLOS noise vector $\mathbf{n}_q$, the BLOS rate noise vector $\mathbf{n}_{\omega q}$, the missile attitude noise vector $\mathbf{n}_z$, the rate gyro output noise vector $\mathbf{n}_{rg}$. The statistic of $\mathbf{\Omega}$ can be calculated by Central Difference approximation. By using CD method, the mean and the covariance of output variables can be captured precisely up to the second order, and higher order information can be partially incorporated in the mean and the covariance, which leads to even higher precision. The implementation is extremely rapid because it is unnecessary to calculate the Jacobian matrixs[9,10]. The mean $\mathbf{\Omega}^I_{C\mathrm{Im}}$ and covariance $\mathbf{R}_\mathbf{v}$, obtained by using CD, are used as input and the covariance of zero-mean process noise v respectively.

Ignored the high-order information of LOS angle, the Eq. 11 can be rewritten in discrete-time form as follows

$$\begin{aligned} \begin{bmatrix} q_{IH}(k+1) \\ q_{IV}(k+1) \end{bmatrix} &= F\left\{q_{IH}(k), q_{IV}(k), \mathbf{\Omega}^I_{C\mathrm{Im}}(k), v(k)\right\} \\ &= \begin{bmatrix} q_{IH}(k) \\ q_{IV}(k) \end{bmatrix} + T \cdot M_q(k) \cdot \left\{\mathbf{\Omega}^I_{C\mathrm{Im}}(k) + v(k)\right\} \end{aligned} \tag{15}$$

where, T is the strapdown seeker's sampling time. It is the same that Eq. 11 can be rewritten in discrete form as follows

$$\begin{aligned} \begin{bmatrix} q_{BH}(k) \\ q_{BV}(k) \end{bmatrix} &= H\left\{q_{IH}(k), q_{IV}(k), \psi(k), \vartheta(k), \gamma(k), \mathbf{n}_z(k), \mathbf{n}_q(k)\right\} \\ &= \begin{bmatrix} -\arctan\{b_3(k)/b_1(k)\} \\ \arcsin\{b_2(k)\} \end{bmatrix} + \mathbf{n}_q(k) \end{aligned} \tag{16}$$

Because of the nonlinearity in process Eq. 15, the CDKF algorithm is applied and the structure of LOS reconstruction filter is depicted in Fig .3.

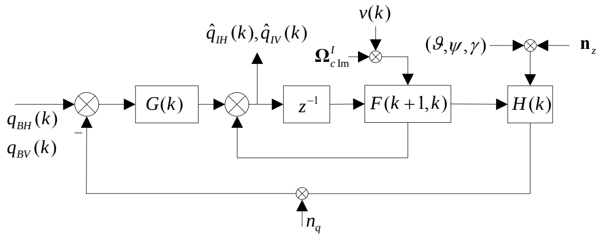

Figure. 3 the structure frame of LOS reconstruction filter

The details of LOS reconstruction filter by using CDKF algorithm are as follows

1) Initialization

Let the adjust parameter $h = \sqrt{3}$. Considering that the process noise v_k and noise n_z are not additive noises, and assumes that the process noise is not related with state in the initial estimate. The augmented state vector is selected as

$$\mathbf{x}^a_k = \left[\left(\mathbf{x}^x_k\right)^T \quad \left(\mathbf{x}^v_k\right)^T\right]^T = \left[\left(\mathbf{x}_k\right)^T \quad \left(v_k\right)^T\right]^T$$

The initial mean and covariance are

$$\hat{x}^a_0 = [\hat{q}_{IH}(0) \quad \hat{q}_{IV}(0) \quad 0 \quad 0 \quad 0]^T, \mathbf{P}^a_0 = diag\{\mathbf{P}_{\mathbf{x}0}, \mathbf{R}_\mathbf{v}\}$$

Calculate the initial input $\mathbf{\Omega}^I_{c\mathrm{Im}}(0)$ and covariance of input noise $R_v(0)$.

2) For sampling time $k = 1, 2, \ldots, \infty$, Calculate the sigma points

$$\boldsymbol{\chi}^a_{k-1} = \left[\hat{\mathbf{x}}^a_{k-1} \quad \hat{\mathbf{x}}^a_{k-1} + h\sqrt{\mathbf{P}^a_{k-1}} \quad \hat{\mathbf{x}}^a_{k-1} - h\sqrt{\mathbf{P}^a_{k-1}}\right]$$

3) Time updates

$$\boldsymbol{\chi}^x_{k|k-1} = F\left(\boldsymbol{\chi}^x_{k-1}, \mathbf{\Omega}^I_{C\mathrm{Im}}(k-1), \boldsymbol{\chi}^v_{k-1}\right)$$

$$\hat{\mathbf{x}}_k^- = \sum_{i=0}^{2L} \mathbf{w}_i^{(m)} \boldsymbol{\chi}_{i,k|k-1}^x$$

$$\mathbf{P}_{\mathbf{x}_k}^- = \sum_{i=1}^{L} [\mathbf{w}_i^{(c1)} \left(\boldsymbol{\chi}_{i,k|k-1}^x - \boldsymbol{\chi}_{i+L,k|k-1}^x\right)\left(\boldsymbol{\chi}_{i,k|k-1}^x - \boldsymbol{\chi}_{i+L,k|k-1}^x\right)^T +$$
$$\mathbf{w}_i^{(c2)} \left(\boldsymbol{\chi}_{i,k|k-1}^x + \boldsymbol{\chi}_{i+L,k|k-1}^x - 2\boldsymbol{\chi}_{0,k|k-1}^x\right) \times$$
$$\left(\boldsymbol{\chi}_{i,k|k-1}^x + \boldsymbol{\chi}_{i+L,k|k-1}^x - 2\boldsymbol{\chi}_{0,k|k-1}^x\right)^T]$$

where,

$$\begin{cases} w_0^{(m)} = (h^2 - L)/h^2 & i = 0 \\ w_i^{(m)} = \dfrac{1}{2h^2}, w_i^{(c1)} = \dfrac{1}{4h^2}, w_i^{(c2)} = \dfrac{h^2-1}{4h^4} & i = 1,2,\text{L}\ ,2L \end{cases}$$

4) Measurement updates equations

In the process of measurement updates, let the system state and measurement noise merge into one augmented state vector.

$$\hat{\mathbf{x}}_{k|k-1}^m = \left[\left(\hat{\mathbf{x}}_k^-\right)^T \quad \bar{n}^T\right]^T \quad \mathbf{P}_{k|k-1}^m = diag\{\mathbf{P}_{\mathbf{x}_k}^-, \mathbf{R}_n\}$$

$$\boldsymbol{\chi}_{k|k-1}^m = \left[\hat{\mathbf{x}}_{k|k-1}^m \quad \hat{\mathbf{x}}_{k|k-1}^m + h\sqrt{\mathbf{P}_{k|k-1}^m} \quad \hat{\mathbf{x}}_{k|k-1}^m - h\sqrt{\mathbf{P}_{k|k-1}^m}\right]$$

Thus, the measurement updates equations are

$$\boldsymbol{\gamma}_{k|k-1} = \mathbf{H}\left(\chi_{k|k-1}^x, \chi_{k|k-1}^n\right) \quad \hat{\mathbf{y}}_k^- = \sum_{i=0}^{2L} \mathbf{w}_i^{(m)} \boldsymbol{\gamma}_{i,k|k-1}$$

$$\mathbf{P}_{\hat{\mathbf{y}}_k}^- = \sum_{i=1}^{L} [\mathbf{w}_i^{(c1)} \left(\boldsymbol{\gamma}_{i,k|k-1} - \boldsymbol{\gamma}_{i+L,k|k-1}\right)\left(\boldsymbol{\gamma}_{i,k|k-1} - \boldsymbol{\gamma}_{i+L,k|k-1}\right)^T +$$
$$\mathbf{w}_i^{(c2)} \left(\boldsymbol{\gamma}_{i,k|k-1} + \boldsymbol{\gamma}_{i+L,k|k-1} - 2\boldsymbol{\gamma}_{0,k|k-1}\right) \times$$
$$\left(\boldsymbol{\gamma}_{i,k|k-1} + \boldsymbol{\gamma}_{i+L,k|k-1} - 2\boldsymbol{\gamma}_{0,k|k-1}\right)^T]$$

$$\mathbf{P}_{\mathbf{x}_k \hat{y}_k} = \sqrt{w_1^{(c1)} \mathbf{P}_{x_k}^-} \left[\boldsymbol{\gamma}_{1:L,k|k-1} - \boldsymbol{\gamma}_{L+1:2L,k|k-1}\right]$$
$$\mathbf{K}_k = \mathbf{P}_{\mathbf{x}_k \hat{\mathbf{y}}_k} \mathbf{P}_{\hat{\mathbf{y}}_k}^{-1}$$
$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{y}_k - \hat{\mathbf{y}}_k^-)$$
$$\mathbf{P}_{\mathbf{x}_k} = \mathbf{P}_{\mathbf{x}_k}^- - \mathbf{K}_k \mathbf{P}_{\hat{\mathbf{y}}} \mathbf{K}_k^T$$

5) Calculation the input $\boldsymbol{\Omega}_{c\,\mathrm{lm}}^I(0)$ and covariance of input noise $R_v(k)$ in measurement values of seeker and navigation system by the method of CD.

6) Back to 2)

Since the square root of the covariance matrix must be calculated when time updates, the method may has large computation. The square root of the covariance matrix can be calculated by Cholesky decomposition to deal with the problem of large computation[9]. Compared with the standard form, the square root form not only improves the computational efficiency, but also to ensure the covariance matrix to be a positive definite matrix. Thus, more stable numerical calculation results can be obtained.

IV. SIMULATION RESULTS

In this section, the performance of LOS reconstruction filter is validated by a simple homing missile intercept simulation. The simulation condition is depicted in Table 1. DFMF is used as low-pass filter and derivative network [10,11]. The memory length parameter of DFMF is 0.8 in simulations. Compared with the LOS calculated by solving directly and low-pass filter, the LOS estimation curves are shown as Fig .4. Good performances are obtained by using LOS reconstruction filter, and then the precision of LOS estimation is improved by applying the output of the proposed filter. Fifty Monte Carlo simulations are performed. The mean and standard deviation of estimation error of LOS reconstruction angle rate by using EKF and CDKF are showed as Table 2. We can see that the proposed method has a higher precision to enhance the performance of guidance system compared with EKF.

TABLE 1. Simulation Condition

Subsystem	Assumption
Missile Initial Position	$[0,7000,0]^{\mathrm{T}}$ (m)
Missile Velocity	306 (m/s)
Target Initial Position	$[8000,0,1000]^{\mathrm{T}}$ (m)
Target Velocity	$[27 \quad 0 \quad 20\sin(0.05\pi t)]$ (m)
PN Guidance	4
J_x	0.250956 $kg\,gm^2$
J_y, J_z	6.606011 $kg\,gm^2$
$\mathbf{n}_q$	0.0001rad
$\mathbf{n}_{\omega q}$	0.02 rad/s
$\mathbf{n}_{rg}$	0.001rad/s
$\mathbf{n}_z$	0.002rad

TABLE 2. MEAN OF RMSE

	Elevation rate(rad/s)	Azimuth rate(rad/s)
EKF	0.0066	0.0061
CDKF	0.0016	0.0019

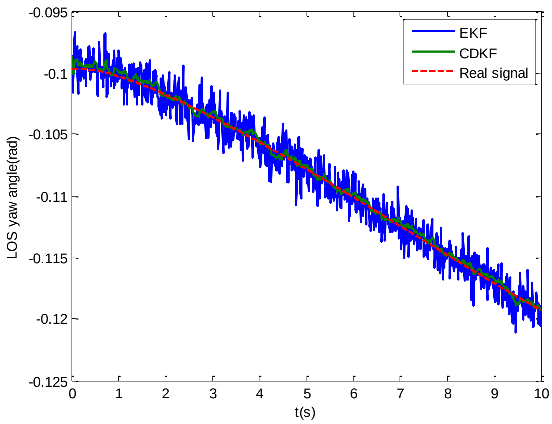

Figure. 4 the LOS pitch angle reconstruction

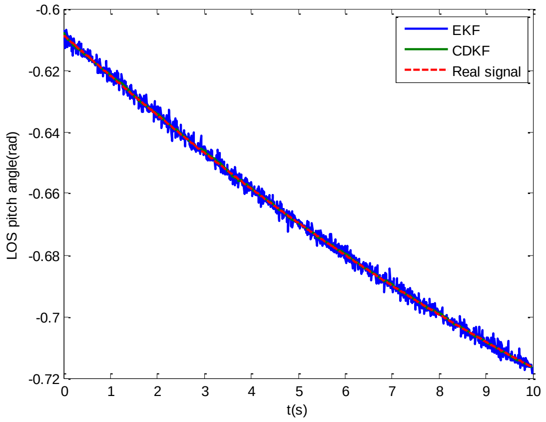

Figure. 5 the LOS yaw angle reconstruction

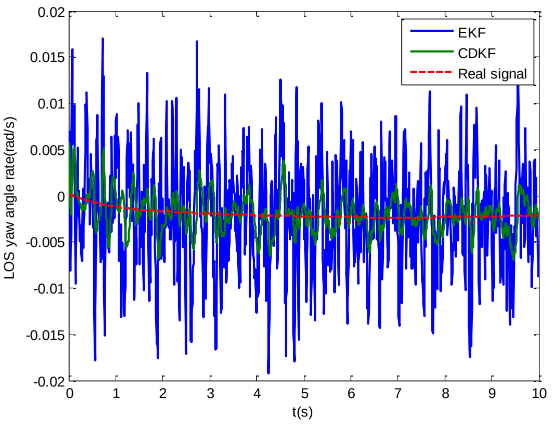

Figure. 6 the LOS pitch rate estimate

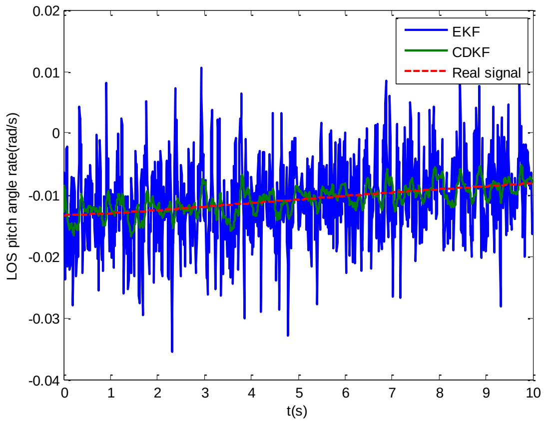

Figure.7 the LOS yaw rate estimate

V. CONCLUSIONS

By the reason of the high nonlinearity and serious measurement noise contained in the guidance system based on strapdown seeker, the CDKF is employed to estimate inertial LOS rate in this paper. Estimation results are compared with that of EKF by Monte Carlo simulations. These simulations show that the CDKF is superior to the EKF both in precision of estimation and velocity of convergence for the strapdown guidance system. Besides, calculation of Jacobian matrices can be avoided for the CDKF, which is usually a complicated process, and may suffer singularity to make the EKF infeasible.

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