

Two-tuple Linguistic Multi-attribute Decision-making Based on Grey Target Theory

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Abstract—Nowadays, selection of an optimal investment program has become a challenging task for the decision makers in the Electric Power Company. Investment program selection for a company can be viewed as a complicated multi-criteria decision making (MCDM) problem which requires consideration of selection attributes. Moreover, decision makers tend to use multi-granularity linguistic term sets for expressing their assessments because of their different backgrounds and preferences, some of which may be uncertain and incomplete. Therefore, this paper studies the multi-attribute decision-making problem under two-tuple linguistic environment and proposes a new approach based on grey target theory. A new two-tuple linguistic bull's-eye is defined and the two-tuple linguistic bull's-eye related coefficient is derived consequently. Then, a new optimization model is provided by applying maximize deviation principle to determine the attributes weights. The order of alternatives is listed by comparing the two-tuple linguistic bull's-eye relative related degree. Finally, an example of power communication resources investment illustrates the applicability of the proposed method.

Keywords-grey target theory; two-tuple linguistic; multi-attribute decision-making; attribute weights; bull's-eye related coefficient

I. INTRODUCTION

The key of MADM problem lies on choosing the best alternative from a feasible set. To do this, decision maker should provide his or her assessment of each alternative on each attribute. During the course of decision making, in reality, attribute values are not precisely denoted by crisp numbers but expressed as fuzzy numbers, such as the triangular fuzzy number [1-5], the linguistic variable [6-9] etc. In such types of literature, extension principle or the symbolic method is always applied for information aggregation, where the results usually do not match any of the initial linguistic terms and an approximation process must be developed to express the result in the initial expression domain Yang(2012). To overcome this disadvantage and avoid information loss, Herrera and Martinez Yang(2012) proposed the two-tuple linguistic model, composed of a linguistic term and a real number, to represent the linguistic assessment information based

on the concept of symbolic translation. Some researchers has studied two-tuple linguistic MCDM problem. For example, Ding (2010) put forward a kind of subjective and objective attribute weight integration method based on binary semantic information processing. Zhang (2011) presented a decision making method based on an extended two-tuple linguistic information processing approach. Yi (2012) expanded the density intermediate operator, giving two-tuple linguistic information clustering methods.

Liu(2013) using the ideas and methods of grey system theory proposed gray related decision making model based on a interval two-tuple linguistic dynamic gray related group. Gu(2014) presents a risk associated with multi-criteria decision method based on binary semantic prospects. Analysis Gu (2014) gives a multi-criteria decision-making method of risk type based on two-tuple linguistic prospects correlation analysis. On another side, grey target decision, which can modeling on the uncertainty information in the decision-making process, gradually attracts the attention of scholars.

For example, Liu(2009) proposed a new multi-objective weighted grey target decision model. Song(2010) build the positive and negative bull's-eye decision model. Liu (2013) proposed a multi-objective grey target decision method based on prospect theory for interval number multiple attribute decision making problems with risk attitude. The results show that the two-tuple linguistic can realize effective decision-making information expression, and grey target decision has advantage on the decision-making process. But so far it has not yet found multi-attribute decision making studies in the combination of grey target theory with two-tuple linguistic.

This paper studies multiple attribute decision making problems of which attribute value is two-tuple linguistic, with unknown attribute weights, and proposes a two-tuple linguistic multiple-decision making method based on the grey target theory. Combining with the thought of grey target theory, the method defines the two-tuple linguistic bull's eye and its relative degree; it takes the deviations maximization as aim and gets the optimal attribute weights; it also sorts the program according to the two-

tuple linguistic bull's-eye relative degree. Finally, comparing with the existing articles, the example demonstrates the effectiveness of this method.

II. PRELIMINARIES

A. Two-tuple linguistic

Herrera and Martínez (2000) proposed the 2-tuple fuzzy linguistic representation model, which is based on the concept of symbolic translation. A 2-tuple is used to express the linguistic information, where represents the linguistic label center of the information and is a numerical value representing the value of symbolic translation.

Let $S = \{s_0, s_1, L, s_q\}$ be a finite linguistic term set with odd cardinality, where s_i represents a possible linguistic term for a linguistic variable. For example, a set of seven terms S can be expressed as follows: S

$\{s_0 = N(\text{none}), s_1 = VL(\text{very low}), s_2 = L(\text{low}), s_3 = M(\text{medium}), s_4 = H(\text{high}), s_5 = VH(\text{very high}), s_6 = P(\text{perfect})\}$. And it is required that the linguistic term set should satisfy the following characteristics:

- 1) The set is ordered: $s_i > s_j$, if and only if $i > j$.
- 2) There is a negation operator: $Neg(s_i) = s_j$ such that $j = g - i$.
- 3) Max operator: $\max(s_i, s_j) = s_i$, if and only if $i \geq j$.
- 4) Min operator: $\min(s_i, s_j) = s_i$, if and only if $i \leq j$.

Definition 1. Let θ be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S , i.e., the result of a symbolic aggregation operation. $\theta \in [0, q]$ and $q + 1$ is the cardinality of S . Let $i = \text{round}(\theta)$ and $\alpha = \theta - i$ be two values such that $i \in [0, q]$, and then α is called a symbolic translation, where $\text{round}(\theta)$ is the usual round operation.

Definition 2. Let $S = \{s_0, s_1, L, s_q\}$ be an ordered linguistic term set and $\theta \in [0, q]$ be a value supporting result of a symbolic aggregation operation, then the two-tuple that expresses the equivalent information to θ is obtained with the following function:

$$\Delta : [0, q] \rightarrow S \times [-0.5, 0.5],$$

$$\Delta(\theta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, & i = \text{round}(\theta), \\ \alpha = \theta - i, & \alpha \in [-0.5, 0.5], \end{cases} \quad (1)$$

where s_i has the closest index label to θ and α is the value of the symbolic translation.

Definition 3. Let $S = \{s_0, s_1, L, s_q\}$ be an ordered linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function such that from a 2-tuple it returns its equivalent numerical value $\theta \in [0, q]$,

$$\text{where } \begin{cases} \Delta^{-1} : S \times [-0.5, 0.5] \rightarrow [0, q], \\ \Delta^{-1}(s_i, \alpha) = i + \alpha. \end{cases} \quad (2)$$

It is obvious that the conversion of a linguistic term $s_i (s_i \in S)$ into a linguistic 2-tuple consists of adding a

value zero as symbolic translation: $\Delta(s_i) = (s_i, 0), i = 0, 1, 2, L, g$.

Definition 4. The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order. Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, with each one representing a linguistic assessment as follows:

- 1) If $k < l$, then (s_k, α_1) is smaller than (s_l, α_2) ;
- 2) If $k = l$, then
 - if $\alpha_1 = \alpha_2$, then (s_k, α_1) and (s_l, α_2) represent the same information;
 - if $\alpha_1 < \alpha_2$, then (s_k, α_1) is smaller than (s_l, α_2) ;
 - if $\alpha_1 > \alpha_2$, then (s_k, α_1) is bigger than (s_l, α_2) .

B. Grey target theory

In grey target theory through a set of model series, it finds out data the most close to the target to construct the standard model, then the grey target bull's-eye comes into being. Liu (2013) presents the definition of the bull's-eye as follows:

Definition 5. Assume that $r_j^+ = \max_i(r_{ij}^+), j = 1, 2, L, n$, then $r^+ = \{r_1^+, r_2^+, L, r_n^+\}$ is the most effective vector, called the bull's-eye. Assume that $r_j^- = \min_i(r_{ij}^-), j = 1, 2, L, n$, then $r^- = \{r_1^-, r_2^-, L, r_n^-\}$ is the worst effective vector, called the negative bull's-eye.

According to the gray target theory, closeness of each indicator to the bull's-eye (bull's-eye degrees) reflects the merits of indicators. Sorting and grading assessment of the bull's-eye determine the level of each mode.

III. TWO-TUPLE LINGUISTIC BULL'S-EYE AND TWO-TUPLE LINGUISTIC BULL'S-EYE RELATED DEGREE

A. Two-tuple linguistic bull's-eye

In the actual decision-making process, decision-making information is usually difficult to be directly quantified numerically. The use of linguistic information to describe is not only consistent with human cognitive habits, but also to avoid the loss of information in the decision-making process. In this paper, to the case of Two-tuple linguistic linguistic, we define the Two-tuple linguistic bull's-eye and the two-tuple linguistic bull's-eye related coefficient as follows:

Definition 6. Assume that $(x_j^+, \alpha_j^+) = \max_i((x_{ij}^+, \alpha_{ij}^+)), j = 1, 2, L, n$,

then $(x^+, \alpha^+) = \{(x_1^+, \alpha_1^+), (x_2^+, \alpha_2^+), L, (x_n^+, \alpha_n^+)\}$ is the most effective vector in the two-tuple linguistic grey target decision, called the two-tuple linguistic bull's-eye. Assume that

$$(x_j^-, \alpha_j^-) = \min_i((x_{ij}^-, \alpha_{ij}^-)), j = 1, 2, L, n,$$

then $(x^-, \alpha^-) = \{(x_1^-, \alpha_1^-), (x_2^-, \alpha_2^-), L, (x_n^-, \alpha_n^-)\}$ is the worst effective vector in the two-tuple linguistic grey target decision, called the negative two-tuple linguistic bull's-eye.

According to the grey target theory, the closeness to the bull's-eye of each index reflects the merits of indicators. In the view of this, we use grey correlation analysis

method to define the related coefficient of each indicator the two-tuple linguistic bull's-eye.

Definition 7. Assume that (x^+, α^+) and (x^-, α^-) ($j = 1, 2, L, n$) are the two-tuple linguistic bull's-eye, then

$$(\xi_{ij}^+, \eta_{ij}^+) = \Delta \left(\frac{\min_i \min_j D_{ij}^+ + \rho \max_i \max_j D_{ij}^+}{D_{ij}^+ + \rho \max_i \max_j D_{ij}^+} \right), \quad (3)$$

$$(\xi_{ij}^-, \eta_{ij}^-) = \Delta \left(\frac{\min_i \min_j D_{ij}^- + \rho \max_i \max_j D_{ij}^-}{D_{ij}^- + \rho \max_i \max_j D_{ij}^-} \right), \quad (4)$$

are called the two-tuple linguistic bull's-eye related coefficient, where $D_{ij}^+ = |\Delta^{-1}(x_{ij}, \alpha_{ij}) - \Delta^{-1}(x_j^+, \alpha_j^+)|$,

$D_{ij}^- = |\Delta^{-1}(x_{ij}, \alpha_{ij}) - \Delta^{-1}(x_j^-, \alpha_j^-)|$ and $i = 1, 2, L, m$, $j = 1, 2, L, n$. $\rho \in [0, 1]$ is resolution factor, generally we admit that $\rho = 0.5$.

B. Two-tuple linguistic bull's-eye (relative) related degree

Definition 8. Assume that $(\xi_{ij}^+, \eta_{ij}^+)$ and $(\xi_{ij}^-, \eta_{ij}^-)$ are the two-tuple linguistic bull's-eye related coefficient, then

$$(\xi_i^+, \eta_i^+) = \Delta \left(\sum_{j=1}^n w_j \cdot \Delta^{-1}(\xi_{ij}^+, \eta_{ij}^+) \right), \quad (5)$$

$$(\xi_i^-, \eta_i^-) = \Delta \left(\sum_{j=1}^n w_j \cdot \Delta^{-1}(\xi_{ij}^-, \eta_{ij}^-) \right), \quad (6)$$

are called the two-tuple linguistic bull's-eye relative related degree, where w_j are attribute weights.

$$\text{And } (\xi_i, \eta_i) = \Delta \left(\frac{\Delta^{-1}(\xi_i^+, \eta_i^+)}{\Delta^{-1}(\xi_i^+, \eta_i^+) + \Delta^{-1}(\xi_i^-, \eta_i^-)} \right), \quad (7)$$

is called the two-tuple linguistic bull's-eye relative related degree of program A_i ($i = 1, 2, L, m$). It is easy to find that the closer program is to the bull's-eye, the more far away is from the negative bull's-eye, the greater is (ξ_i, η_i) , then the corresponding programs is better.

IV. TWO-TUPLE LINGUISTIC MULTI-ATTRIBUTE DECISION-MAKING MODEL BASED ON GREY TARGET THEORY

A. Problem description

Considering a Multi-attribute Decision-making problem, it is supposed that there are m programs A_1, A_2, L, A_m , and n properties G_1, G_2, \dots, G_n . The linguistic evaluation of each program A_i under each attributes G_j is x_{ij} , so as to constitute a decision matrix. According to the above conditions, try to determine the best program.

B. Weighting model

In the actual decision-making process, because of the complexity of the objective things and the limitation of decision maker's cognition, decision makers are often difficult to give definite attribute weights. It appears the situation that the information on attribute weights is completely unknown. Therefore, how to reasonably

determine the attribute weights is an important issue. In fact, the reference gives the entropy maximization^[16], the deviations maximization^[17] and grey correlation method^[18] etc. for determining attribute weights. The principle of the deviations maximization consistent with the habits of most decision makers, and is easy to implement.

The principle of the deviations maximization is when there is no big difference of an attribute among all programs, then the attribute plays a small role in the decision and should be given greater weight; On the other hand, the attribute should be given less weight. This article is based on the principle of deviations maximization, establishing the optimization model as follows:

$$\begin{aligned} \max f &= \sum_{i=1}^m \sum_{j=1}^n (\Delta^{-1}(\xi_{ij}^+, \eta_{ij}^+) + \Delta^{-1}(\xi_{ij}^-, \eta_{ij}^-)) w_j \\ \text{s.t. } \sum_{j=1}^n w_j^2 &= 1, w_j > 0. \end{aligned} \quad (8)$$

Thus to construct Lagrange function:

$$L(w_j, \lambda) = \sum_{i=1}^m \sum_{j=1}^n (\Delta^{-1}(\xi_{ij}^+, \eta_{ij}^+) + \Delta^{-1}(\xi_{ij}^-, \eta_{ij}^-)) w_j + \frac{\lambda}{2} \left[\sum_{j=1}^n w_j^2 - 1 \right] \quad (9)$$

For the partial derivative and assume that:

$$\frac{\partial L}{\partial w_j} = \sum_{i=1}^m \sum_{j=1}^n (\Delta^{-1}(\xi_{ij}^+, \eta_{ij}^+) + \Delta^{-1}(\xi_{ij}^-, \eta_{ij}^-)) + \lambda w_j = 0,$$

$$\frac{\partial L}{\partial \lambda} = \sum_{j=1}^n w_j^2 - 1 = 0 \quad (10)$$

Then we get the optimal solution of attribute weights:

$$w_j = \frac{\sum_{i=1}^m (\Delta^{-1}(\xi_{ij}^+, \eta_{ij}^+) + \Delta^{-1}(\xi_{ij}^-, \eta_{ij}^-))}{\sqrt{\sum_{j=1}^n \left(\sum_{i=1}^m (\Delta^{-1}(\xi_{ij}^+, \eta_{ij}^+) + \Delta^{-1}(\xi_{ij}^-, \eta_{ij}^-)) \right)^2}}, \quad j=1, L, n. \quad (11)$$

Do the normalized processing with w_j , we can get optimal attribute weights:

$$w_j = \frac{\sum_{i=1}^m (\Delta^{-1}(\xi_{ij}^+, \eta_{ij}^+) + \Delta^{-1}(\xi_{ij}^-, \eta_{ij}^-))}{\sum_{j=1}^n \sum_{i=1}^m (\Delta^{-1}(\xi_{ij}^+, \eta_{ij}^+) + \Delta^{-1}(\xi_{ij}^-, \eta_{ij}^-))}, \quad j = 1, 2, L, n. \quad (12)$$

C. Decision-making method

For the above decision problem, the paper proposes a two-tuple linguistic multi-attribute decision making method based on grey target theory. Specific steps are as follows:

Step 1. Convert the linguistic decision matrix to two-tuple linguistic decision matrix. Using definition 1, convert the linguistic decision matrix $X = (x_{ij})_{m \times n}$ to two-tuple linguistic decision matrix $D = (x_{ij}, 0)_{m \times n}$.

Step 2. Determine the two-tuple linguistic positive and negative bull's-eye.

(1) The two-tuple linguistic positive bull's-eye :

$$(x^+, \alpha^+) = \{(x_1^+, \alpha_1^+), (x_2^+, \alpha_2^+), L, (x_n^+, \alpha_n^+)\}$$

$$(x_j^+, \alpha_j^+) = \max((x_{1j}, \alpha_{1j}), (x_{2j}, \alpha_{2j}), L, (x_{mj}, \alpha_{mj})).$$

(2) The two-tuple linguistic negative bull's-eye :

$$(x^-, \alpha^-) = \{(x_1^-, \alpha_1^-), (x_2^-, \alpha_2^-), L, (x_n^-, \alpha_n^-)\}$$

$$(x_j^-, \alpha_j^-) = \min((x_{1j}, \alpha_{1j}), (x_{2j}, \alpha_{2j}), \dots, (x_{mj}, \alpha_{mj})).$$

Step 3. Using Eqs. (3)-(4) to calculate the related coefficient $(\xi_{ij}^+, \eta_{ij}^+)$ and $(\xi_{ij}^-, \eta_{ij}^-)$ of two-tuple linguistic bull's-eye.

Step 4. Using Eq. (9) to determine optimal attribute weights w_j .

Step 5. Using Eqs. (5)-(6) to calculate the related coefficient (ξ_i^+, η_i^+) and (ξ_i^-, η_i^-) of two-tuple linguistic bull's-eye.

Step 6. Using Eq. (7) to calculate the two-tuple linguistic bull's-eye relative related degree (ξ_i, η_i) .

Step 7. Sort of programs. The greater the two-tuple linguistic bull's-eye relative related degree, the more excellent the appropriate program.

V. AN EXAMPLE

Henan Electric Power Company needs to consider a variety of large data and resources in communication resource management, such as transport networks, switching networks, business networks, data networks and access network. In order to optimize data processing and improve efficiency, grid communication resource investment program A_1 , A_2 and A_3 must be compared to pick the excellent one. A variety of resources in this communication resource system are integrated into security benefits G_1 , economic benefits G_2 , social benefits G_3 and management efficiency benefits G_4 of the four attributes of each investment program to be evaluated. After experts scoring, linguistic decision matrix is shown in Table 1.

TABLE I. LINGUISTIC DECISION MATRIX

	G_1	G_2	G_3	G_4
A_1	FZ	HZ	HC	C
A_2	HZ	YB	FZ	YB
A_3	YB	C	HZ	HC

1) Using the definition 1, convert the decision matrix into the corresponding linguistic two-tuple linguistic decision matrix.

$$D = \begin{pmatrix} (FZ, 0) & (HZ, 0) & (HC, 0) & (C, 0) \\ (HZ, 0) & (YB, 0) & (FZ, 0) & (YB, 0) \\ (YB, 0) & (C, 0) & (HZ, 0) & (HC, 0) \end{pmatrix}$$

2) Using equation (10) to determine two-tuple linguistic bull's-eye.

$$(x^+, \alpha^+) = ((FZ, 0), (HZ, 0), (FZ, 0), (YB, 0))$$

$$(x^-, \alpha^-) = ((YB, 0), (C, 0), (HC, 0), (HC, 0))$$

3) Using equation (4), (5) to calculate the related coefficient of two-tuple linguistic bull's-eye.

$$(\xi_{ij}^+, \eta_{ij}^+) = \begin{pmatrix} (HC, 0) & (HC, 0) & (FC, 0.333) & (HC, -0.286) \\ (HC, -0.286) & (HC, -0.444) & (HC, 0) & (HC, 0) \\ (FC, 0.455) & (FC, 0.455) & (HC, -0.286) & (HC, -0.444) \end{pmatrix}$$

$$(\xi_{ij}^-, \eta_{ij}^-) = \begin{pmatrix} (FC, 0.455) & (FC, 0.455) & (HC, 0) & (HC, -0.286) \\ (HC, -0.444) & (HC, -0.286) & (FC, 0.333) & (HC, -0.444) \\ (HC, 0) & (HC, 0) & (FC, 0.385) & (HC, 0) \end{pmatrix}$$

4) Using equation (10) to determine attribute weights: $W = (0.251, 0.251, 0.226, 0.272)$.

5) Using equation (6), (7) to calculate the related coefficient of two-tuple linguistic bull's-eye.

$$(\xi_1^+, \eta_1^+) = (HC, -0.228), (\xi_2^+, \eta_2^+) = (HC, -0.183), (\xi_3^+, \eta_3^+) = (HC, -0.459),$$

$$(\xi_1^-, \eta_1^-) = (HC, -0.351), (\xi_2^-, \eta_2^-) = (HC, -0.455), (\xi_3^-, \eta_3^-) = (HC, -0.139)$$

6) Using equation (8) to calculate the tuple linguistic bull's-eye relative related degree.

7) Sort the alternatives. According to the tuple linguistic sorting method, apparently there is $(\xi_2, \eta_2) > (\xi_1, \eta_1) > (\xi_3, \eta_3)$, thus the optimal program is A_2 .

It will be put forward comparative analysis of two-tuple linguistic multi-attribute decision-making method in this paper and Yang(2012) to show the characteristics of the method in this paper.

According to the method in ref. Yang(2012), it assumes that the attribute weights are respectively $W = (0.2, 0.4, 0.3, 0.1)^T$ and $W = (0.3063, 0.2700, 0.1893, 0.2344)^T$ (the weights are obtained by the method), the calculation example of each program in the evaluation of 2-tuple linguistic is shown in table 2.

TABLE II. SORTING BY THE METHOD IN YANG(2012)

	$W = (0.5, 0.2, 0.1, 0.2)^T$	$W = (0.251, 0.251, 0.226, 0.272)^T$
M_1	(HZ, -0.5)	(Z, -0.47)
M_2	(Z, 0.3)	(Z, 0.18)
M_3	(YB, -0.4)	(YB, -0.344)
Sorting	A_3 p A_2 p A_1	A_3 p A_1 p A_2

This paper put forward a two-tuple linguistic multi-attribute decision-making method based on grey target theory. Compared with the result of the method in reference[10], we found that:

(1) In reference Yang(2012), it requires that attribute weights are completely known, and does not give the weighting method in the situation that weights are completely unknown. From Table 2, the results of the weights of $W = (0.251, 0.251, 0.226, 0.272)^T$ and $W = (0.5, 0.2, 0.1, 0.2)^T$ are calculated to be opposite, it is difficult to draw a definitive conclusion. This paper presents a objective weighting method based on the principle of deviations maximization, so as to overcome the influence of subjective decision-makers. In the situation that the weight information is completely unknown, considering the comprehensive evaluation of each attribute, we get the conclusion that program A_2 is the optimal one. And this conclusion is consistent with the result (weight is) by the method in conference Yang(2012).

(2) In Yang(2012), it is only a simple weighting of each program, and the interaction between the various programs is not considered, which leads that the decision-making process will ignore the role of certain information.

Considering the interaction between the various programs, this paper presents a method based on grey target decision theory, which sorts of programs by determining the relative distance (ie. the relative bull's-eye related degree) between the bull's-eye.

VI. CONCLUSIONS

For two-tuple linguistic multi-attribute decision making problem, we propose a fuzzy multi-attribute decision making method based on grey target theory. The paper proposes the objective weighting method, which takes each attribute mutual influence on decision-making into account and leads decision-making more practical. The paper proposes concepts of two-tuple linguistic bull's-eye, two-tuple linguistic bull's-eye related coefficient and the two-tuple linguistic relative related bull's-eye degree, broadening the thinking of making-decision with the gray target decision theory. Finally, using two-tuple linguistic nature to sort can avoid information loss and other issues in the process of quantifying the value of linguistic information in previous. In the future studies, the interval valued two-tuple linguistic multi-attribute decision making problem can be further considered.

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