

# Research on the Dynamic Phase Distribution of Mixed Fluid and its Modeling

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**Abstract**—In the paper, most multiphase flow model use viscosity as an important parameter, however the viscosity of mixed fluid sometimes is unavailable or difficult to measure especially in small-scale flowing condition. In order to solve this problem, the drift-inhibition angle and expansion-inhibitions angle were deduced and used as evaluating indexes to describe the drifting trend of different ingredients among the mixed fluid. For solving above two indexes of the model, a new method was founded and gradient area stress respectively. For verifying the model, the flow process of grease in a pipe as an example was analyzed by using the model. The accordance between the analysis results and engineering phenomenon verified the validity of the model. At last, the data from experiment has proved the simulation analysis of the ingredient distribution among grease. So, the theoretical model has been very fit for practice, which provided a good theoretical basis to measure the viscosity of mixed fluid.

**Keywords**- Mixed fluid; ingredients distribution; wedge-sliding model; phase-drift

## I. INTRODUCTION

The separation and sediment process of ingredients among mixed fluid is a multi-scale and multi-factors coupling, especially when the mixed fluid as micro-particle-added lubricant is used in the porous material, its rheological behaviors present a very complex phenomenon. So it is difficult to describe accurately its inner ingredients distribution by using conventional two-phase flow model, and necessary to develop a new theoretical model for the solving of the problem.

Most of the models put the viscosity of mixed fluid as an important parameter, but there is no evidence shown that the ingredients distribution among mixed fluid at a long-term or stable state are related with the viscosity of fluid; The ingredients separation and sediment caused by non-uniform stress should belong to the problems of the long-term or stable state of mixed fluid; In addition, the viscosity data are not always available or difficult to be measured, so much as which are varying with the temperature change in general.

Early in 1962, HIGGINS, R.V. and LEIGHTON, A.J. (1962) presented a fast method to calculate thoroughly the performance of two-phase flow in reservoir rock with complex geometry. By using Stokesian Dynamics, Nott P. R. and Brady, J. F. (1994) conducted dynamic simulations of the pressure-driven flow in a channel of a non-

Brownian suspension at zero Reynolds number. Ali E. A. and Mohamed E. O. (2009) (1-6) derived a nonlinear 3D model that investigates the flow-diffusion-structure interaction occurring in mixtures. This is the result of the application of current computer technology and advanced research method in the research field of multi-phase fluids. (7-11)

In the paper, for solving above two indexes of the model, a new method was founded and gradient area stress respectively. For verifying the model, the flow process of grease in a pipe as an example was analyzed by using the model. The accordance between the analysis results and engineering phenomenon verified the validity of the model (12-16).

## II. DESIGN OF DRIFT MODEL OF HEAVIER PHASE

A barrel-ball stirring experiment was designed, Small balls with different weight are tiled on the bottom of a barrel, these balls are arranged with close-packed hexagon without stack before stirring, and the inner diameter of the barrel is nearly the outer tangential of the hexagon. Using a rod with smaller diameter than balls locating near the wall of barrel to stir and controlling the speed for avoiding stack, the result of the experiment shows that:

(1) After stirring a period of time, balls arranged in a circle shape around the rotation axis.

(2) The balls near the wall move the fastest of all balls near the axis.

(3) Regardless of the distribution of heavy balls on the bottom before, their eventual distribution places are same to near the axis, as long as the stirring time is long enough.

### 2.1 The two-phase wedge-sliding cell

Supposing the two phases in cell are both elastic-plastic bodies, and defining their inner anti-forces under pressure as:

$$F_{kA} = \lambda_k \cdot \frac{\rho_A^2 \cdot \varphi_A^2}{\rho_A \cdot \varphi_A + \rho_B \cdot \varphi_B} \cdot \frac{e^{\lambda_\delta \cdot \delta_y} - e^{-\lambda_\delta \cdot \delta_y}}{e^{\lambda_\delta \cdot \delta_y} + e^{-\lambda_\delta \cdot \delta_y}} \quad (1)$$

$$F_{kB} = \lambda_k \cdot \frac{\rho_B^2 \cdot \varphi_B^2}{\rho_A \cdot \varphi_A + \rho_B \cdot \varphi_B} \cdot \frac{e^{\lambda_\delta \cdot \delta_y} - e^{-\lambda_\delta \cdot \delta_y}}{e^{\lambda_\delta \cdot \delta_y} + e^{-\lambda_\delta \cdot \delta_y}} \quad (2)$$

The action line of the forces is perpendicular to the wedge surface.

Where,  $\delta_y$  is the yield stress of cell ( $N/mm^2$ ), which is related to the environmental temperature and

pressure and assumed to be a constant here;  $\lambda_\delta$  is the prior coefficient of power exponential and defined as,

$$\lambda_\delta = \frac{e^{(\rho_A \cdot \varphi_A + \rho_B \cdot \varphi_B)/2}}{e^{\rho_A \cdot \varphi_A} + e^{\rho_B \cdot \varphi_B}} \quad (3)$$

Clearly,  $0 < \lambda_\delta < 1$  and which is no unit.

The  $\lambda_k$  is the elastic-plastic anti-force coefficient of the cell. By the adding operation of Esq. (1) and (2) and taking  $F_{kA} + F_{kB} = \delta_y$ , after deformation,  $\lambda_k$  can be obtained as,

$$\lambda_k = \delta_y \cdot \frac{\rho_A \cdot \varphi_A + \rho_B \cdot \varphi_B}{\rho_A^2 \cdot \varphi_A^2 + \rho_B^2 \cdot \varphi_B^2} \cdot \frac{e^{\lambda_\delta \cdot \delta_y} + e^{-\lambda_\delta \cdot \delta_y}}{e^{\lambda_\delta \cdot \delta_y} - e^{-\lambda_\delta \cdot \delta_y}} \quad (4)$$

Known from the minimum entropy theory,  $F_f$  should to be the maximum static friction of two phases when flow reached its steady state, so  $F_f$  can be defined as:

$$F_f = \lambda_f \cdot (F_{kA} + F_{kB}) \quad (5)$$

The acting direction of friction is parallel to the wedge surface but opposite with the trend of movement.

The  $\lambda_f$  is the maximum static friction coefficient between two phases, which is related to the environmental temperature and pressure and assumed to be a constant here.

Substituting the Esq. (1) and (2) in (5), the following equation is obtained:

$$F_f = \lambda_f \cdot \frac{\rho_B^2 \cdot \varphi_B^2 + \rho_B^2 \cdot \varphi_B^2}{\rho_A \cdot \varphi_A + \rho_B \cdot \varphi_B} \cdot \frac{e^{\lambda_\delta \cdot \delta_y} + e^{-\lambda_\delta \cdot \delta_y}}{e^{\lambda_\delta \cdot \delta_y} - e^{-\lambda_\delta \cdot \delta_y}}$$

By one time derivation of  $C_\rho$  about  $\lambda$ , the following equation is obtained:

$$\frac{d(C_\rho)}{d\lambda} = \frac{\rho_A^2 + \rho_B^2}{\rho_B - \rho_A} - \frac{2 \cdot \rho_A^2 \cdot \rho_B^2}{\rho_B - \rho_A} \cdot \frac{1}{[(\rho_B - \rho_A) \cdot \lambda + \rho_A]^2} \quad (6)$$

### III. THE APPLICATION AND THE CHECKING OF THE MODEL

Grease is a kind of a typical multi-phases mixed fluid in which the heavier phases (such as the saponifier, densifier and additives etc.) disperse into the lighter phase (the base oil). Our studies before found that, grease flows with a velocity which has a gradient variation at the radius direction of pipe, and the front surface of the grease column bulges. the shear stress function  $\tau(r)$  at the radius direction was obtained as:

$$\tau(r) = \frac{\tau_w}{R - r^*} (r - r^*) \quad (7)$$

where,  $\tau_w$  is the shearing stress on the inner wall of pipe ( $N/mm^2$ );  $R$  is the radius and length of pipe ( $mm$ );

$r^*$  is the radius of the plug flow ( $mm$ ), and  $r^* = 2\tau_y l / \Delta p$ .

#### 3.1 The application of the model

All the parameters of  $\rho_A$ ,  $\rho_B$  and  $\Phi_A$ ,  $\Phi_B$  can be calculated respectively according to the relevant parameters of grease components. And Substituting these relevant parameters  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_4$ ,  $C_{\rho 0}$  and  $\lambda_\delta$  can be obtained respectively.

Taking the base oil as A phase, the other components as B phase, and researching cell extracted here is a micro-sector block. Measuring the value of the radial pressure of grease upon the pipe wall  $\delta_g$  ( $N/mm^2$ ) and noticing that  $H = R$  and  $x_0 = 0$ , the distribution of the radial expansion-inhibition stress can be expressed as:

$$C_h(r) = \frac{(\lambda_\delta + 1)}{R} \cdot [1 - \frac{\lambda_\delta \cdot R^{1-\lambda_\delta} \cdot r^{\lambda_\delta}}{R}] \cdot \delta_g \quad (8)$$

If the value of  $\delta_g$  is difficult to be measured such as the flow of grease in a small pipe etc., then it can be simply estimated through the equation following:

$$C_h(r) \approx \frac{(\lambda_\delta + 1)}{R} \cdot [1 - \frac{\lambda_\delta \cdot R^{1-\lambda_\delta} \cdot r^{\lambda_\delta}}{R}] \cdot \frac{1}{\cos \theta_0} \cdot [\tau(r) \cdot \sin \theta_0 - \lambda_f \cdot \lambda_k \cdot C_{\rho 0} \cdot \frac{e^{\lambda_\delta \cdot \tau_y} - e^{-\lambda_\delta \cdot \tau_y}}{e^{\lambda_\delta \cdot \tau_y} + e^{-\lambda_\delta \cdot \tau_y}}] \quad (9)$$

The internal maximum friction coefficient of grease  $\lambda_f$  is approximately taken for the maximum static friction coefficient between two phases. No consideration of the gradient variation of stress, then it is deemed that the loading forces on the grease are only  $\bar{\tau}(R)$  and  $\tau_y$ . Because the resistance force of grease are always parallel with the axial of pipe but opposite with the direction of flow, so the stress loading on the cell extracted anywhere should meet the following equation:

$$(F_{kA} + F_{kB}) + F_f = \bar{\tau}(R) - \tau_y \quad (10)$$

Substituting the Esq. (1), (2) and (5) and  $C_{\rho 0}$  in the Eq. (10), the following equation is obtained:

After deformation:

$$\lambda_f = \frac{\bar{\tau}(R) - \tau_y}{\lambda_k \cdot C_{\rho 0}} \cdot \frac{e^{\lambda_\delta \cdot \tau_y} + e^{-\lambda_\delta \cdot \tau_y}}{e^{\lambda_\delta \cdot \tau_y} - e^{-\lambda_\delta \cdot \tau_y}} - 1 \quad (11)$$

#### 3.2 The simulation analysis of the ingredient distribution among grease

the discrete values of the initial radial expansion-inhibition force  $C_h(r)$  at the beginning of grease flowing in pipe can be calculated. It is clearly known that the varying of  $C_h(r)$  is the same with the distribution of  $\tau(r)$  in the radial direction, and its maximum value  $C_h(R)$  occurs at the wall of pipe.

The discrete values of the initial drift-inhibition angle  $\theta(r)$  at the beginning of grease flowing in pipe can be calculated. Known from the figure that  $\theta(r)$  is a decreasing function about  $r$ , and its value at different  $r$  indicates the drifting trend of heavier ingredient where. It's clearly that there is a biggest trend of the heavier ingredient to separate from the base oil near the wall, and

the shear-thinning phenomenon is more apparent along the increasing direction of shear gradient.

The discrete value of  $\lambda(r)$  when the flow of grease reaches its stable state can be calculated and its curve can be drawn by the use of computer tools, shown in Fig .1. It is known from the figure that grease performs a rheological characteristic of Newtonian fluid near the wall of pipe because the fluid there has almost been a single-phase composition ( $\lambda(r=0) \approx 0$ ). There is a plug flow area occurring within  $r \in [0,3)$  and the radius of the plug flow decreases with the increase of  $\Delta P$ , and the plug flow area will disappear completely when  $\Delta P$  is higher enough

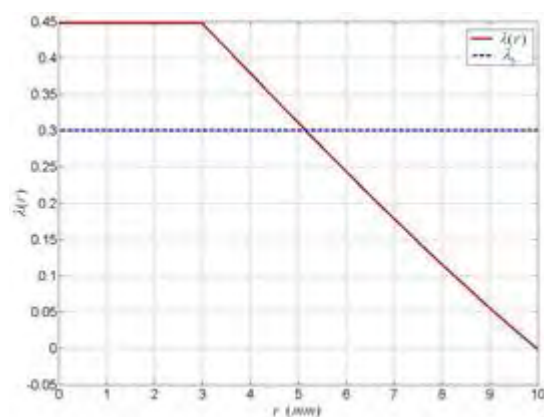


Figure 1. The distribution of  $\lambda(r)$  within grease in the steady flowing state

#### IV. CONCLUSIONS

In this study, a new method was founded and gradient area stress respectively. For verifying the model, the flow process of grease in a pipe as an example was analyzed by using the model. The accordance between the analysis results and engineering phenomenon verified the validity of the mode. The following conclusions are drawn from the experimental results:

(1) Based on the minimum entropy theory of steady-state system, a two-phase wedge-sliding model has been developed and used to describe the phenomena of the

(2) When the model is respectively applied in the conditions of a peak stress and a gradient stress, the solving methods are illustrated and meanwhile their computing steps are given in details.

(3) The phenomena of ingredient drift were studied. The simulation results show that the model can better reveal the ingredient distribution among grease when the flow reaches to its stable state.

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