

Multilevel Stochastic Dynamic Process Models and Possible Applications in Global Financial Market Analysis and Surveillance

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Abstract

This paper advances Multilevel Stochastic Dynamic Process (MSDP) models as a new framework for modeling nonstationary and nonlinear time series and complex systems, and discuss possible applications of MSDP models in global financial market analysis and surveillance. Under the heterogeneous market hypothesis, different types of market participants react to the same information differently, characterized by their time horizons or dealing frequencies. Consequently financial prices exhibit multilevel trends, cycles, and seasonality, which provide the very basis for MSDP models. Both the discrete- and continuous-time MSDP models are constructed: Multilevel Structural Time Series (MSTS) with Unobserved Components (UC) and Multilevel Stochastic Differential Equations (MSDE).

Keywords: stochastic dynamic process models, structural time series models, stochastic differential equations, multilevel trends, cycles, seasonality, swing, momentum, financial market analysis, prediction and surveillance.

1. Introduction

The theory of efficient markets reached the height of its dominance in academic finance around the 1970s. Faith in this theory was eroded by a succession of discoveries of anomalies and of evidence of excess volatility of returns, found by multidisciplinary finance researchers, largely inspired by the steadfast beliefs and first-hand empirical knowledge and wisdom of professional analysts and traders since at least 1880s throughout the entire 20th century. This includes the blossoming of research, since the 1990s, on econophysics, behavior finance, computational finance, and intelligent finance (Pan et al 2006). Evidence was found of predictability at long horizons and at shorter ones.

Although it is believed now, though still variably, by many mathematical and computational finance researchers that pockets of predictability with

probability can emerge, disappear and re-emerge in the financial markets in a stochastic and dynamical manner, not much work is reported in the literature for modeling the prices directly using stochastic and dynamical process models. Most of the stochastic differential equations in finance have concentrated on modeling price returns and volatility for the purposes of option pricing and risk management. Also, not much work on discrete-time time series models for market prices have been reported. Structural Time Series (STS) models with Unobserved Components (UC) including time-variable and state-dependent trends, cycles, seasonality and random walking noise have, however, been developed substantially since the 1980s in the environmental modeling community (Young 1974–2000) and in the macro-econometric modeling community (Harvey 1984–2005), after the advent of the state space model of the Kalman filter in the 1960s in control engineering. Nevertheless, both the stochastic differential equations and structural time series models developed so far are virtually all confined to providing a single-level model on a single time scale. In particular, although single-level stochastic differential equations can simulate realistic price time series, it is not known how to use them in predicting future prices. On the other hand, no structural time series models are available for modeling multilevel trends and swings in the real price time series.

The key perspective underlying our approach, models and methodology is the multiple characteristic levels of time scale, which corresponds essentially to the so-called *multiple time frames* referred to by professional analysts and traders. The concept of multilevel analysis includes at least two parts: the time resolution of the market price time series data, and the time horizon of modeling, prediction, investment and trading of different market participants. In its most general sense, the time scale should be continuous, so there is a scale space for any parametric model of market. However, in practice, real financial time series exhibit clear characteristic levels of time scale, each with well-defined geometrical structure of price dynamics. Much of the mainstream academic finance theories have concentrated on modeling market price

returns and volatilities, while very little has been done on modeling the prices directly for the purpose of predicting the absolute market direction and potential, as well as multiple time frame projections.

The *Multilevel Stochastic Dynamic Process (MSDP)* models to be advanced in this paper are nested process models on multiple scale levels of time for modeling the same stochastic dynamic process, level by level, and the set of stochastic dynamic process (SDP) models for each single level may share the same or similar kernel structure with time-varying state-dependent parameters. A complete set of MSDP models is characterized by two key properties: (1) the process on a given level is fully determined by the set of SDP models of that level, which is defined on top of the process of its previous higher (coarser) level; (2) the fractal dimension of the process on a given level is higher than that of its previous higher level. There are fundamental reasons for using MSDP models in financial time series, as observed by professional analysts and traders, and empirically formulated in the Dow theory of multilevel trends (Murphy 1999), Elliott waves (Prechter 1980–2004), Gann cycles and geometry (Gann and Allery 1951), as well as later evolutions of technical analysis throughout the 20th century. A primary reason is due to the heterogeneity of investors with different time horizons and other qualities, so they react to the same information differently. These kinds of considerations have been reformulated in the Heterogeneous Market Hypothesis (HMH) (Gencay et al, 2001), Fractal Market Hypothesis (FMH) and Swingtum Market Hypothesis (SMH) (Pan, 2004). In particular, SMH took a step further after HMH and FMH, stating that fractal market prices exhibit robust stochastic dynamic process patterns in the scale space of time and price, which can be described by multilevel trends, swings and momentums.

2. Multilevel Stochastic Dynamic Process (MSDP) Models

Consider a stochastic dynamic process function $x(t)$ defined in continuous time t . In general, a time series of $x(t)$ is available $(x(t_0), x(t_1), \dots, x(t_N))$, with $t_0 < t_1 < \dots < t_N$. $x(t)$ is assumed to be nonstationary and nonlinear, and multifractal. Also assume there are n characteristic levels of time scale $(s_1 > s_2 > \dots > s_n)$ such that the time on the l -th level of time scale corresponds to t/s_l . For example, a dyadic scale series is defined by $s_l = 2^{n-l}$. On the 0 -th level, $x_0(t)$ provides the

coarsest approximation to $x(t)$. On the n -th level, $l = n, s = 1$, it refers to the original time t and function $x(t)$. On the l -th level, the function $x(t)$ is approximated by $x_l(t)$ which is supposed to be determined by an m -th order differential equations

$$F_l(x_l(t), x_l^{(1)}, x_l^{(2)}, \dots, x_l^{(m)}, x_{l-1}, \theta_l, \xi_l) = 0 \quad (1)$$

where θ_l is a vector of parameters defining F_l , and ξ_l is a vector of noise terms on this l -th level.

Note that the essence of this set of MSDP models is to approximate the original function $x(t)$ through a hierarchical series of time scale levels, level by level. Each l -th level function $x_l(t)$ is determined on top of its immediate previous $(l-1)$ -th level function $x_{l-1}(t)$, and $x_l(t)$ is a finer approximation than $x_{l-1}(t)$. On each single level, the model is stochastic if the noise ξ_l is time-variable; and the model is dynamic if the parameters θ_l are also time-variable, so $x_l(t)$ is said to be state-dependent. According to the nature of time t , there are two forms of MSDP models: discrete-time structural time series models, and continuous-time stochastic differential equations. Actually, each single-level model F_l can be a set of equations.

3. Multilevel Structural Time Series (MSTS) Models with Unobserved Components (UC)

A multilevel structural time series (MSTS) model has a discrete-time form of the general MSDP model of (1), but the function $x_l(t)$ on each l -th level of time scale now has a general form of univariate unobserved component model

$$x_l(t) = x_{l-1}(t) + C_l(t) + S_l(t) + V_l(\mathcal{G}(t)) + \xi_l(t) + \zeta_l(t) \quad (2)$$

where $x_{l-1}(t)$ is the function on the last coarser level, which in fact serves as the trend or low-frequency component to $x_l(t)$; $C_l(t)$ is a sustained cyclical or quasi-cyclical component with period different from that of any seasonality in the data; $S_l(t)$ is a seasonal component corresponding to a particular kind of cycles with physical time periodicity, so $S_l(t)$ is also called physical cycles while $C_l(t)$ may be better called dynamical cycles; $V_l(\mathcal{G}(t))$ captures the

influence of a vector of exogenous variables $\mathcal{G}_l(t)$; $\xi_l(t)$ is a stochastic perturbation; i.e., colored noise which can be modeled as an autoregressive (AR) or autoregressive moving average (ARMA) process; Finally $\zeta_l(t)$ is white noise, normally defined as an independent identically-distributed process following a Gaussian distribution; i.e., $\zeta_l(t) \sim N(0, \sigma_l^2)$. Note that the key difference of this structural time series (STS) model as defined by (2) as the discrete-time kernel to a MSTS model from the standard STS model with UC in time series literature and in macro-econometric models is that the kernel (2) is a hierarchically nested time series with the trend $x_{l-1}(t)$ on the current level being the function of the last coarser level, so this trend component needs not to be determined on this level. Therefore, this kernel of (2) is in fact easier to solve from the data than the standard STS model.

The most important unobserved components in (2) are the cyclical term $C_l(t)$ and the seasonal term $S_l(t)$. Both terms can be modelled in a similar manner to dynamic harmonic regression (DHR) (Young, 2000):

$$C_l(t) = \sum_{i=1}^{n_c} \{ \alpha_{l,i}(t) \cos(\omega_{l,i}(t)) + \beta_{l,i}(t) \sin(\omega_{l,i}(t)) \} \quad (3)$$

$$S_l(t) = \sum_{j=1}^{n_s} \{ \phi_{l,j}(t) \cos(\varpi_{l,j}(t)) + \varphi_{l,j}(t) \sin(\varpi_{l,j}(t)) \} \quad (4)$$

Note that $\alpha_{l,i}(t), \beta_{l,i}(t)$, $i = 1, 2, \dots, n_c$, and $\phi_{l,j}(t), \varphi_{l,j}(t)$, $j = 1, 2, \dots, n_s$ are stochastic time-variable parameters (TVP). $\omega_{l,i}(t)$, $i = 1, 2, \dots, n_c$, and $\varpi_{l,j}(t)$, $j = 1, 2, \dots, n_s$, are the fundamental and harmonic frequencies associated with the cyclicity and the seasonality in the $x_l(t)$. In general, dynamic cycle frequencies $\omega_{l,i}(t)$ may also be TVP, but the seasonal frequencies $\varpi_{l,j}(t)$ should be constant but pertinent to the given level of time scale.

Each of the stochastic time-variable parameters $\alpha_{l,i}(t)$, $\beta_{l,i}(t)$, $\phi_{l,j}(t)$, $\varphi_{l,j}(t)$, and $\omega_{l,i}(t)$ is defined by its respective vector of two stochastic state variable $\chi(t) = (u(t), v(t))'$, where $u(t)$ and $v(t)$ are, respectively, the changing level (a time series term) and the slope of the associated TVP.

The stochastic evolution of each $\chi(t)$ is assumed to be describable by a Generalized Random Walk (GRW) process of the form (Jakeman and Young, 1979, 1984; Young et al, 1989)

$$\chi(t) = G\chi(t-1) + H\eta(t) \quad (5)$$

$$\text{where } G = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}, H = \begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix}$$

This general model of GRW comprises as special cases: scalar or smoothed random walk, local linear or damped trend, according to different parameter setting.

4. Multilevel Stochastic Differential Equations (MSDE)

On each given l -th level, the function $x_l(t)$ fluctuates about $x_{l-1}(t)$. Here, if $x_l(t)$ is the market price on the current level, $x_{l-1}(t)$ can be considered as a fundamental value in the equilibrium market model. But in our model, $x_{l-1}(t)$ refers to the dominant trend for the l -th level. The change of $x_l(t)$ can be naturally modeled as containing three parts: a stochastic drift of itself which can be understood as the persistency on the current level, an adjustment by the dominant trend $x_{l-1}(t)$ which is negatively related to the deviation of $x_l(t)$ from $x_{l-1}(t)$, and a random noise. We provide a basic model in the form of a set of stochastic differential equations as an advancement by taking advantages from classical equilibrium models such as Watson and Getz (1981) and augmented micro-market structure models by Caginalp and Balenovich (1994, 2003), Bouchaud and Cont (1998) and Ozaki et al. (2001).

An augmentation of the classical theory of price adjustment stipulates that relative price change occurs in order to restore a balance between supply S and demand D , which in turn depends on price $x_l(t)$ and price derivative $x_l'(t)$, i.e.

$$\frac{d}{dt} \log x_l = \frac{1}{x_l} \frac{dx_l}{dt} = F \left[\frac{D(x_l(t), x_l'(t))}{S(x_l(t), x_l'(t))} \right] \quad (6)$$

where D/S denotes the imbalance between supply and demand. We require function $F(D/S)$ satisfy two properties: $F(1) = 0$ and $F(D/S) = -F(S/D)$. The logarithm provides a valid form. Let

$$\phi(t) = F(D(t)/S(t)) = \log(D(t)/S(t)) \quad (7)$$

Let λ be a liquidity parameter. A set of useful stochastic differential equations can be defined as

$$\frac{d\phi}{dt} = \alpha_1 + \beta_1\phi + \kappa \log \frac{x_l}{x_{l-1}} + \gamma_1 \frac{d\zeta_1}{dt} \quad (8)$$

$$\frac{d\lambda}{dt} = \alpha_2 + \beta_2\lambda + \gamma_2 \frac{d\zeta_2}{dt} \quad (9)$$

$$\frac{1}{x_l} \frac{dx_l}{dt} = e^\lambda \phi(t) + \gamma_3 e^{\lambda/2} \frac{d\zeta_3}{dt} \quad (10)$$

where $\phi(t)$ and $\lambda(t)$ are unobserved time-variable state parameters, $\zeta_1(t)$, $\zeta_2(t)$ and $\zeta_3(t)$ are three Brownian motion processes. For the ease of reading, in this set of equations, the level subscript l is omitted for all the coefficients and parameters except the functions $x_l(t)$ from $x_{l-1}(t)$. Clearly, this set of SDE's provides a kernel for a multilevel SDE model (MSDE).

5. Discovery, Detection and Projection of High-Level Dynamics Patterns

The structural signals—multilevel trends, cycles and seasonality—extracted from the raw historical time series and real-time data streams already provide directly useable information for market analysis and trading decision-making. They also serve as a reliable and robust basis for discovery, detection and projection of high-level dynamics patterns. The space of these dynamic patterns is large and complicated; here we mention three of the most significant patterns:

- (1) Turbulence in foreign exchange markets;
- (2) Convergence of multilevel trends and harmonic coincidence of multilevel cycles and seasonality;
- (3) Log-periodic power laws for modeling financial bubbles and anti-bubbles.

6. Sample Evidence to the Feasibility and Significance of MSDP Models

Pan (2004) provides a first presentation of the multilevel modeling of financial time series in terms of multilevel dynamical and physical cycles and phases, and demonstrates a real system for predicting Australian All Ordinary Index (AORD) using the multilevel phases extracted from AORD and several key US stock indices. This system has achieved a consistent performance of predicting the daily direction of AORD High, Low and Close returns with respective correctness of about (80%, 80%, 70%). The multilevel phases used in this system correspond to an approximation of the signals as described in Section 3. For detection and projection of long-term dynamical

patterns as described in Section 5, the work of Sornette (1996-2006), Zhou and Sornette (2003-2005), and many of their publications during 1996-2006 (www.ess.ucla.edu/faculty/sornette/) provides a great deal of successful applications. In particular, there are already some econophysical models which implement the idea of MSDP models. In particular, reference should be pointed for instance to the multifractal random walk model which emerged as the continuous causal self-consistent limit of a class of cascade models (two books: Sornette 2000-2004, 2003). An important remark, given by Didier Sornette (after reading a draft of this paper), which is probably the most difficult technical point, is how to make all the dynamics of the different levels compatible. In some cases, this can be done by renormalization group methods (Sornette and Zhou 2003; Zhou and Sornette 2003; Israeli and Goldenfeld 2005).

There is a vast professional literature on multiple time frame technical analysis, including the Dow theory of multilevel trends, Elliott wave principles, Gann cycles and geometry, as well as most recent work on multi-time frame trading methods as collected in Kaufman (2005). Recent academic revisits to multiple time frame technical and quantitative analysis can be found in Ilinski (2001), Gencay et al. (2001), and Voit (2003).

References

A more detailed version of this paper is available as a Technical Report IIFP-TR-2005-1, from the Research Publications of www.iifp.net. All other references not listed below can be found from there.

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