Price Formation in Genetic Learning Models of Investor Sentiment

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Abstract

This paper studies the possibilities that genetic algorithm describes investor sentiment, and time series properties of estimated models. For these purposes, first we identify the conditions for describing investor sentiment by altering parameters of genetic algorithm. Then the auto-correlations and the BDS statistics are conducted after generating sample paths. Our results show that some Monte-Carlo simulations seem to lead to dynamics reported in previous studies.

Keywords: Multiagent Model, Genetic Algorithm, Investor Sentiment, Monte-Carlo Method

1 Introduction

Genetic algorithm (from now on GA) has been often used as a learning method of agents in agent-based computational economic models [1, 2, 7, 9]. According to Arifovic, and Arifovic and Gençay, there are mainly four advantages of using GA; (1) it does not need highly computational abilities, (2) it can represent the heterogeneity of agents' belief, (3) whether a decision rule can survive or not depends on its performance, and (4) GA is able to mimic the behavior of subjects observed in experimental economics [1, 2].

On the other hand, the recent development of behavioral economics has enabled to propose descriptive models with respect to the behaviors of speculators. Besides, some models are proposed from the experimental economic points of view [6, 12], or by applying some evidences of behavioral finance for agent-based computational economic studies [4, 8].

In this study, therefore, the aims of this paper are to show the conditions requisite for GA to represent investor sentiment developed by Barberis et al. (A model of investor sentiment: hereafter MIS) [3], and to explore the similarities and differences between the time series properties of estimated models and results in previous studies.

The rest of this paper is organized as follows. The next section introduces a summary of MIS, and shows some conditions requisite for GA to describe MIS. Then, in section 3, we explored the time series properties of generated sample paths using deducted variables of MIS. And finally, section 4 concludes this paper.

2 Description of Investor Sentiment by Genetic Algorithms

Barberis et al. have developed their model of investor sentiment shown in Table 1 and 2 in order to describe market participants' "conservatism" [5] and "representativeness heuristic" [11]. The constitutions of MIS are twofold; First, a market is either in a stable state or in an unstable one. If the market is in a stable condition, the probability π_H that the price movement will be the same as the previous one is over 0.5. While if the market is in an unstable state, the probability π_L is under 0.5 (Table 1). The parameter λ_1 is the probability of transition from unstable condition to stable one, while λ_2 is the one from stable condition to unstable one (Table 2). Moreover, Barberis et al. postulate that the sum of λ_1 and λ_2 is less than unity and that λ_1 is smaller than λ_2. Second, the price movement in the economy is either $+1$ or -1. Therefore q_t, the probability that the market is unstable, is renewed by equation (1) in case that the price movement is the different from the previous one, or by (2) otherwise;

$$q_{t+1} = \frac{((1-\lambda_1)q_t + \lambda_2(1-q_t))(1-\pi_L)}{\text{denominator 1}} \tag{1}$$

where denominator $1 = ((1-\lambda_1)q_t + \lambda_2(1-q_t))(1-\pi_L) + (\lambda_1 q_t + (1-\lambda_2)(1-q_t))(1-\pi_H)$, and

$$q_{t+1} = \frac{((1-\lambda_1)q_t + \lambda_2(1-q_t))\pi_L}{\text{denominator 2}} \tag{2}$$

where denominator $2 = ((1-\lambda_1)q_t + \lambda_2(1-q_t))\pi_L + (\lambda_1 q_t + (1-\lambda_2)(1-q_t))\pi_H$.

In order to define conditions requisite for GA to describe MIS, we fed two kinds of price movements ($- + - - - + ++$ and $+-$) to agents. Since the former series has at least the minimum information with respect to diversity, this is used to calculate

Table 1: Transition probability of MIS

a. Unstable condition

	$\Delta S_{t+1} = +1$	$\Delta_{S_{t+1}} = -1$
$\Delta S_t = +1$	π_L (< 0.5)	$1 - \pi_L$
$\Delta S_t = -1$	$1 - \pi_L$	π_L

b. Stable condition

	$\Delta S_{t+1} = +1$	$\Delta_{S_{t+1}} = -1$
$\Delta S_t = +1$	π_H (> 0.5)	$1 - \pi_H$
$\Delta S_t = -1$	$1 - \pi_H$	π_H

ΔS_t: Price movement at time t

Table 2: An abstract of investor's recognition

$t \backslash t+1$	Stable cond.	Unstable cond.
Stable cond.	$1 - \lambda_2$	λ_2
Unstable cond.	λ_1	$1 - \lambda_1$

the variables of MIS. While the latter series is to estimate possible range of the q_t.

An agent has five variables, each of which is a binary bit with the following meanings:

- *judge*

  If this variable is 1 then an agent considers the market to be stable. On the other hand, if that is 0, she does the market to be unstable.

- $stable_+$, $stable_-$, $unstable_+$, $unstable_-$

  The variable $stable_+$ is used if the previous movement is +1 and an agent considers the market to be stable. If the variable is +1, she forecasts the next change is +1, while if 0 then −1. The same is true to other variables.

First, an agent judges the market condition by her *judge*, then she makes a prediction for the next movement, based on that value and on the previous change. Therefore, the roles of the agents in our model is to tell us the possibilities to represent a model of investor sentiment through their learnings.

The simulation was ran by altering parameters of genetic algorithm, i.e. crossover (0.6 or 0.8), mutation (0.01 or 0.05), learning frequency (LF) (every period or every 19-period), time horizon, i.e. how long the sum of the fitness values is needed in selecting parents[1], and fitness calculation[2]. Hence, this simulation has totally 72 kinds of results and we obtained the following conditions; First, the agents needed to know a market condition for their learnings. Second, the information used when the agents selected their parents must be up-to-date.

[1]There are three types: (ha) The corresponding fitness value. (hb) Sum of the fitness values in the last 19 periods. (hc) Total fitness values.

[2]There are also three types: (fa) An agent receives +1 if she predicts the price change precisely. (fb) She receives +1 if she predicts the price movement and, at the same time, judges the market condition properly. (fc) She receives +1 if she judges only the market condition properly. While she receives +3 if her expectation is also right about the price movement.

3 Price Formations

This section attempts to show the relations between the parameters of GA and time series properties by generating sample paths using estimated variables in the previous section and by applying them for several time series analyses.

3.1 Generation of sample paths

Since this simulation required only the variables of MIS, no agent existed or no parameter of genetic algorithm except learning frequencies was used.

The price movement ΔS_t was determined using a coefficient $\alpha = 2.0$ by equation (3) when the previous movement was +1;

$$\Delta S_t = \begin{cases} \alpha(2p_u - 1), & \text{if } rnd() < p_u \\ -\alpha(2p_u - 1)/2, & \text{otherwise} \end{cases} \quad (3)$$

where $p_u = q_t \pi_L + (1 - q_t)\pi_H$ is the probability of price-up, and by (4) when the previous change is −1;

$$\Delta S_t = \begin{cases} \alpha(2p_d - 1)/2, & \text{if } rnd() < p_d \\ -\alpha(2p_d - 1), & \text{otherwise,} \end{cases} \quad (4)$$

where $p_d = q_t(1-\pi_L)+(1-q_t)(1-\pi_H)$ and $rnd() \in (0,\ 1)$ are the probability of price-up and uniform random number respectively.

Other setups are as follows:

Number of sample paths 100

Total periods 20000

Initial q_t 0.5

Variables of MIS estimated ones in the previous section

Renewal of q_t in accordance with equations (1) and (2)

Renewal frequency of q_t the same as learning frequency (see section 2)

Initial price $S_0 = 100$, $\Delta S_t = S_t - S_{t-1} = 1$ or -1 $(t = -1,\ 0)$

ΔS_ts at $t = -1,\ 0$ were determined arbitrarily.

Table 3: Estimated models of investor sentiment

	λ_1	λ_2	π_L	π_H	Crossover	Mutation	LF	Fitness
(a)	4.14×10^{-3}	7.14×10^{-3}	0.451	0.510	0.8	0.01	1	fb
(b)	4.52×10^{-3}	6.16×10^{-3}	0.468	0.517	0.6	0.01	1	fb
(c)	4.38×10^{-2}	5.11×10^{-2}	0.438	0.511	0.8	0.05	1	fb
(d)	4.03×10^{-3}	4.75×10^{-3}	0.462	0.523	0.8	0.01	19	fb
(e)	5.09×10^{-3}	12.63×10^{-3}	0.406	0.509	0.8	0.01	1	fc

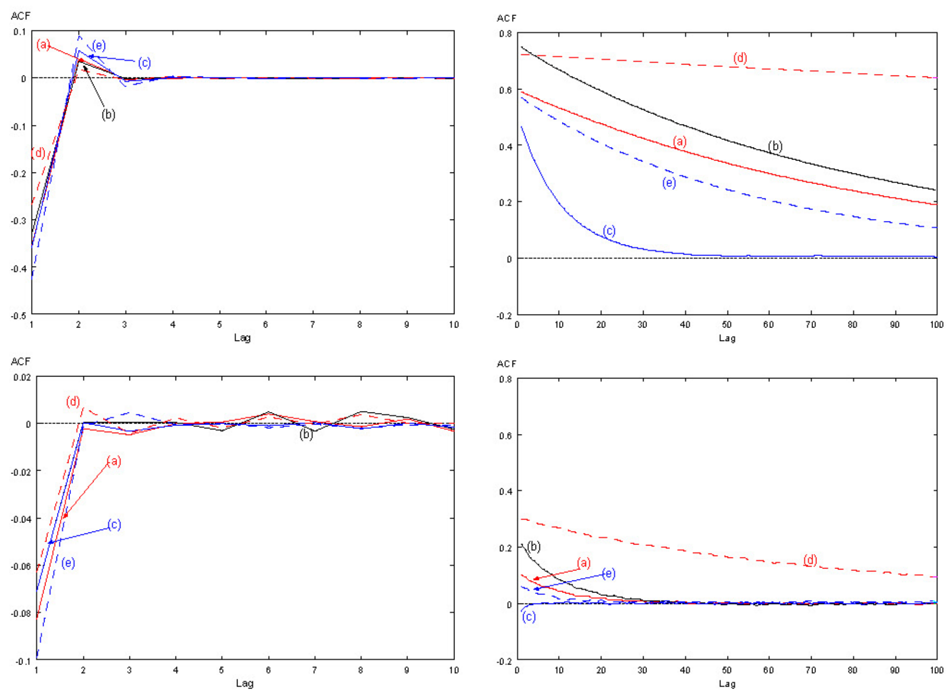

Figure 1: Auto-correlation functions (top left: 1-term return, top right: 1-term absolute return, bottom left: 8-term return, bottom right: 8-term absolute return)

3.2 Time Series Properties

This part of the section reports the results using the five parameter sets of 16 kinds of estimated models of investor sentiment (Table 3). To conduct analyses, two kinds of time series, 1-term return and 8-term return, were employed.

Figure 1 illustrates the auto-correlation functions for return series and absolute return series. The first-order values of 1-term normal return series take significantly negative in most cases and long-term auto-correlations are found for absolute return series. These are relatively similar to the findings in [10]. On the other hand, when it comes to 8-term return compared to [13], both the auto-correlation functions do not have longer-memory except parameter (d). However, these values result from just the poor learning chances.

Next, Table 4 shows the BDS statistics which tests the null hypothesis of i.i.d. process. For both the two segments, the statistics rejects the null hypothesis except some cases, but for the 1-term return series, the values are much larger than those in [2]. Combining the findings and the previous analyses, it seems that the time series properties of 1-term return series are similar not only to those of daily data but also to those of high-frequency data. On the other hand, the reason why the 8-term return series were not the same as those in [13] is the way of price formation. The possible reasons could be considered; First, while the setups in our model are to determine the price movement, those in [13] are to form the price. Besides, it is because our model is not restricted by the condition that the prices created in the Ising spin model range just from 0 to 1.

Finally, consider the relations between parameters of GA and the statistical properties. First, the differences of crossover, mutation and learning frequencies seem to determine whether a return series has memory or not. Besides, a parameter set with higher mutation made the 8-term return series

Table 4: BDS statistics

	1-term return				8-term return			
	$\epsilon = 0.25\sigma$		$\epsilon = 0.75\sigma$		$\epsilon = 0.25\sigma$		$\epsilon = 0.75\sigma$	
	$m = 2$	$m = 3$	$m = 2$	$m = 3$	$m = 2$	$m = 3$	$m = 2$	$m = 3$
(a)	698.00	1365.82	483.76	568.75	7.25	11.35	6.22	8.95
(b)	554.37	925.83	107.45	153.38	12.24	21.00	15.16	24.48
(c)	1158.62	1914.89	500.85	498.91	0.46	0.22	−1.22	−1.33
(d)	672.18	1721.63	254.18	377.62	24.52	49.63	18.66	27.89
(e)	1192.64	2237.21	679.47	782.35	5.26	7.45	4.39	6.17

σ is the standard deviation of the return series, and ϵ is the distance parameter.

not rule out the lack of persistence. Second, actual market participants may adjust their opinions judging from the differences of learning frequencies. In other words, no matter how a volatility clustering might be seen, the generated sample paths with the parameter set (d) were not similar to actual data.

4 Conclusion

This paper attempts to show whether genetic algorithm can represent a descriptive model of investor sentiment and what distinctions such an agent-based model has. For these purposes, we combined investor sentiment with genetic learning in an agent-based computational economic model, and conducted time series analyses using sample paths generated. Both the time series statistics reveal that a proper setup could lead to dynamics reported in the early studies no matter how the setups are different. Especially, investors are likely to react a piece of new information and adjust their views to the market.

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