Volatility Forecasting：Model-Free Implied Volatility

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Abstract. Volatility in the financial market is an important variable, which in asset pricing, investment, risk management and policy-making process plays an important role. Methods for predicting volatility are mainly divided into two categories, one is the historical information method, based on the historical information to predict the future volatility; the other is the implied volatility method, calculating the expectation of the future volatility based on the market price of the option. We propose a model-free implied volatility method to measure the volatility. The model-free implied volatility does not depend on the option pricing model, and extracts information from all the option contracts. We provide empirical evidence from the S&P 500 index option that the model-free implied volatility is more accurate than GARCH model in predicting the future volatility.

Introduction

During the past thirty years, volatility forecasting is becoming more and more important in financial engineering. Many works have been found in the theoretical and practical fields. Engle [1] proposed the first ARCH model in 1982, Bollerslev [2] and Taylor [3] added the old conditional variance into the new estimation of conditional variance, which is the generalized autoregressive conditional heteroskedasticity model (GARCH). Model-free implied volatility originated from the variance swap theory. Dupire [4] and Neuberger [5], Demeterfi, Derman, Kamal and zou (DDKZ) [6,7], Britten-Jones and Neuberger [8] expanded it further. When the underlying asset price jumps exist, Britten-Jones and Neuberger did not state explicitly. Since the jump is important in financial asset prices, Jiang and Tian [9] proved the conclusions from Britten-Jones and Neuberger remain valid when the price jumps exist, thus ensuring the generalizability of this method. Jiang and Tian [10] also demonstrated that DDKZ's variance fair value and Britten-Jones and Neuberger's yields squares are the same. Besides, they came to a conclusion that the model-free implied volatility has more information content than the Black-Scholes implied volatility. In this paper, the theories of the model-free implied volatility and the time series model are used in the American S&P500 index option market. We compare model-free implied volatility method with GARCH model from the empirical aspect.

Model-free implied volatility

The squared volatility can be expressed as the integration of call option forward prices [8]:

$$E_0^F\left[\int_0^T\left(\frac{dF_t}{F_t}\right)^2\right]=2\int_0^\infty\frac{C^F(T,K)-max(0,F_0-K)}{K^2}dK \tag{1}$$

Where, F_t denotes the forward price of the underlying asset at time t. $C^F(T,K)$ denotes the forward price of the call option at time 0, and K is the strike price.

Through the prices of current option contracts, we can get the long-term expectation for the market volatility. The Chicago Board Options Exchange began to use this new approach to calculate S&P500 volatility index (VIX) in 2003.

In Eq. 1, strike prices range from 0 to infinite, and expiration dates of different options are the same. But in the real market, strike prices are discrete and finite, then Eq. 1 can be simplified to Eq. 2:

$$E_0^F\left[\int_0^T\left(\frac{dF_t}{F_t}\right)^2\right] \approx 2\sum_{i=1}^{m}\frac{C^F(T,K_i)-max(0,F_0-K_i)}{{K_i}^2}\Delta K \quad (2)$$

Where , $\Delta K=\frac{K_{max}-K_{min}}{m}$, $K_i=K_{min}+i\cdot\Delta K$, $0\le i\le m$.

The above process will bring two types of errors: truncation errors and discretization errors.

Truncation errors derive from the limited range of strike prices. Strike prices are within a certain range of underlying asset prices. Jiang and Tian [9] discovered that if cut-off points (K_{max} or K_{min}) are far from F_0, truncation errors are small. Truncation errors are negligible if $K_{min}<F_0-2\sigma F_0$ or $K_{max}>F_0+2\sigma F_0$. Otherwise, volatilities of cut-off points should be used to replace those out of the strike price interval (K_{min},K_{max}), which means volatilities outside the interval are constant.

Discretization errors derive from discrete strike prices. The interval ΔK between each strike price may vary, but will not tend to 0. When ΔK is smaller, the discrete errors is smaller. Jiang and Tian [9] found the discretization error can be ignored when $\Delta K<0.35\sigma F_0$, where σ is the realized volatility of the underlying assets within the remaining maturity of the option. If $\Delta K>0.35\sigma F_0$, the cubic spline interpolation method should be used to add the missing option prices.

Empirical Research

Since S&P500 index option is very active last ten years, we choose all the call options (10 days to the expiration date) from January 2006 to January 2014.

Before the calculation of model-free implied volatility, errors should be analyzed at first. The realized volatility σ varies each day, also does the forward price F_0. As Fig. 1 shows, the horizontal axis represents observation sequence, and the vertical axis represents the multiple: for every highest strike price K_{max} each day, $multiple=\frac{K_{max}-F_0}{\sigma F_0}$; for every lowest one , $multiple=\frac{K_{min}-F_0}{\sigma F_0}$. Most of them exceed $2\sigma F_0$, which means truncation errors are small.

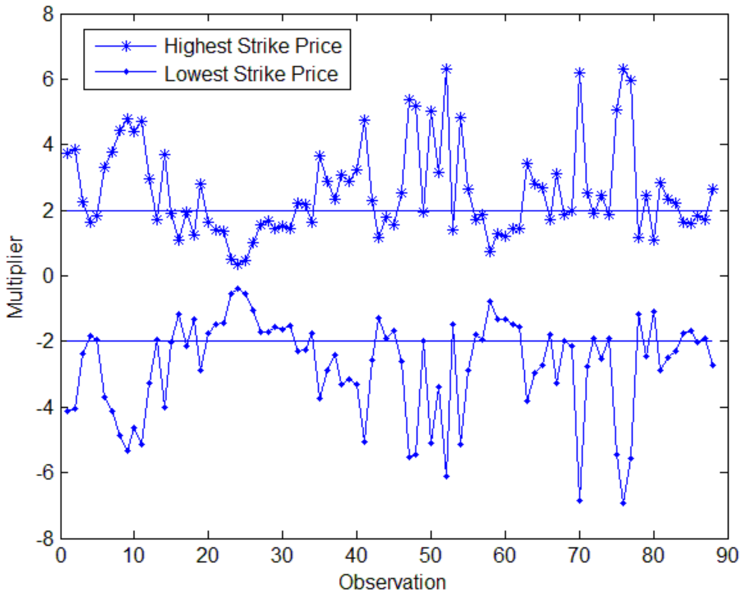

Figure. 1 Multiple for strike price (unit: σF_0)

In the sample, the strike price interval ΔK is 5 points. Fig. 2 shows that discretization errors are small in most cases. The horizontal axis represents the observation sequence, and the vertical axis represents $0.35\sigma F_0$ on each observation day. Almost all the intervals are less than $0.35\sigma F_0$.

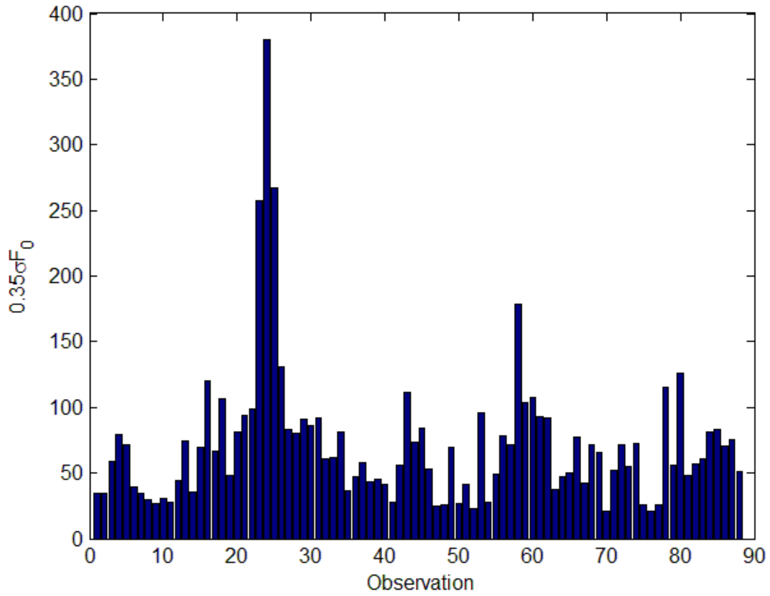

Figure. 2 Corresponding $0.35\sigma F_0$ on each observation day

Based on above analysis, discretization errors and truncation errors are relatively small for the sample, only a small part needs to use interpolation.

For a better comparation, we estimate the GARCH model using dynamic scrolling windows, and take the out of sample forecasting method [11].

According to the AIC criterion, we get the GARCH(1,1) model:

$$h_t = \alpha_0 + \alpha_1 a_{t-1}^2 + \gamma_1 h_{t-1} \tag{3}$$

Where , $0 \leq \alpha_1, \gamma_1 \leq 1$, $\alpha_1 + \gamma_1 < 1$.

We give three volatility sequences in Fig. 3. The horizontal axis represents observation days and the vertical axis represents volatility values.

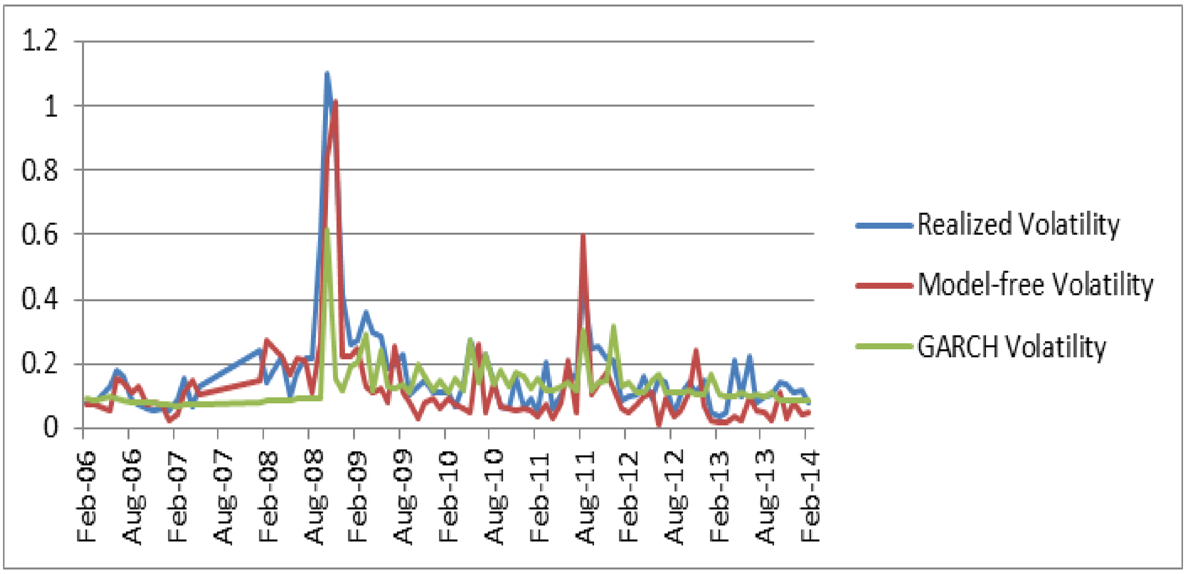

Figure. 3 Volatility sequences

From Table 1 we can see these three volatility sequences present positive skewness, and each kurtosis is great. In the description of extreme volatility values, the maximum and minimum of model-free implied volatility σ^{MF} are closer to realized volatility σ^{RE}. Compared with GARCH model, model-free implied volatility is more relevant to realized volatility.

Table 1 Descriptive statistics

	Mean	SD	Skewness	Kurtosis	Min	Max	Corrcoef with σ^{RE}
σ^{RE}	0.17	0.16	3.66	19.41	0.04	1.10	1
σ^{MF}	0.13	0.15	4.01	21.58	0.01	1.01	0.86
σ^{GAR}	0.13	0.07	3.83	23.22	0.08	0.62	0.64

Conclusions

Information in the financial markets are updatding every moment. GARCH model uses only the historical yields information. The model free implied volatility considers the latest information in the current markets from all the option contracts, including strike prices, option prices, the remaining term of options, the risk-free interest rate and underlying asset prices.

After empirical research with different models in the American S&P500 index option market, we found that, as a result of the model-free implied volatility method not relying on the option pricing model and extracting information from all the option contracts, the information content of the model-free implied volatility is more than the GARCH model volatility. The model-free implied volatility is more effective to predict the future realized volatility.

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