A study on water-mass-to-collector-area ratio for the flat-plate solar water heating system used in Yunnan Province

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Keywords: Solar water heating system; MAR; Flat-plate solar collector; Optimal tilt angle

Abstract. The model for calculating the water-mass-to-collector-area ratio (the ratio is abbreviated as MAR) was developed based on the useful energy output model of a flat-plate solar collector. The MARs were calculated by using the typical meteorological data of seven cities in Yunnan Province. The final water temperature of the water tank is assumed to be 60℃ during the calculation. The ranges of MARs have been given for seasonal or year-round running of the flat-plate solar water heating system. For convenient engineering applications, the quadratic fit equations between MARs and tilt angles have been obtained with high correlations (R^2>0.999). Further discussions found that the optimal tilt angles and the corresponding MARs for spring, summer, autumn, winter or year-round operation of the solar water heating system are respectively in 10 ~ 15°, 0 ~ 0°, 32 ~ 38°, 45 ~ 50°, 24 ~ 30°, 50 ~ 60 kg/m^2, 44 ~ 51 kg/m^2, 43 ~ 56 kg/m^2, 44 ~ 58 kg/m^2 and 46 ~ 52kg/m^2.

Introduction

Matching problem of the mass of water in the tank and collector area is one of the key problems of solar hot water system. To insure the high-efficiency operation of the solar water heating system and meet the water temperature requirement of the consumers, the different water-mass-to-collector-area-ratios (the ratio is abbreviated as MAR) have been recommended in different climatic regions at home and abroad. The values of the MAR in America [1] and Greece [2] are 75 kg/m^2. The value of MAR in Malaysia [3] is 50-70 kg/m^2 and in China [4] is less than 100 kg/m^2. 60-100kg/m^2 are recommended in Kunming for the flat-plate solar water heating system while the daily thermal efficiency and the end water temperature in water tank are more than 0.45 and 45℃ [5].

However, the range of the above MARs is large and is difficult to determine the values of the MAR of the solar water heating system used in different climatic regions. To solve this problem, the calculation model of the MAR of the solar water heating system has been built based the useful energy output model of the flat-plate solar collector. Previous studies have verified that the relative error of the calculated and measured values is within 10% [6]. The MARs were calculated by using the typical meteorological data of seven cities in Yunnan Province. During the discussions, the final water temperature in the tank is 60 ℃ and the azimuth angle of the solar collector is 0°. For ease of engineering applications, the relationships between the MARs and the tilt angles have been obtained for spring, summer, autumn, winter or year-round operation of the solar water heating system. In addition, the optimal tilt angles and the corresponding MARs have been given for the optimization design of the flat-plate solar hot water system.

Mathematical model

Energy output model of the flat-plate solar collector. The hourly total solar radiation I_β on the unit inclined plane can be expressed as [7]:

$$I_\beta = K_b I_b \cos\theta + \frac{1}{3} K_d I_d (2+\cos\beta) + \frac{1}{2}\rho_g K_g I_h (1-\cos\beta) \quad (1)$$

where I_b, I_d and I_h (I_h=I_b+I_d) are respectively hourly beam, diffuse and total solar radiation on the horizontal plane. β is the tilt angle and ρ_g is the reflectivity of ground. The incidence angle modifiers K_b, K_d and K_g [8] are given by K_b=1+b_0(1/cosθ-1), K_d=1+b_0(1/cosθ_d-1) and K_g=1+b_0(1/cosθ_g-1). For a single glass cover, b_0 = 0.1. The equivalent incident angles (θ, θ_d and θ_g) of the beam, diffuse and ground reflection radiation are calculated by cosθ=sinδsin(λ-β)+cosδcos(λ-β)cosω, θ_d =59.68–0.1388β + 0.001497β^2 and θ_g =90 – 0.5788β + 0.002693β^2. δ, λ and ω are the solar declination angle, the geographic latitude and the hour angle.

The energy output (Q_u) of a flat-plate solar collector under steady-state conditions can valued as

$$Q_u = A_c \int_{t_1}^{t_2} \left[(\tau\alpha) I_\beta - U_{Lf} \left(T_{abs} - T_{air} \right) \right] dt \tag{2}$$

where t_1 and t_2 are the beginning time and the end time. Ac is the collector area, (τα) is the transmission- absorption product, T_{abs} and T_{air} are the heat-absorption plate temperature and ambient air temperature. The heat loss coefficient (U_{Lf}) of the solar collector can be described as U_{Lf} = U_{top}+U_{bot}+ U_{edg}. U_{top} is the top heat loss coefficient [9], U_{bot} and U_{edg} are the bottom and edge heat loss coefficients [10].

Water-mass-to-collector-area ratio (MAR). The heat consumption (Q_h) of the hot water is determined by the following formula

$$Q_h = MC_p \left(T_{hot} - T_w \right) \tag{3}$$

where M is the water mass in the tank, C_p is the specific heat of water, T_{hot} is water temperature for use and T_w is the water temperature of the city water. When Q_u = Q_h, the energy output of the solar water heating system can meet the heat load for hot water supply. Then, the MAR can be indicated as

$$\frac{M}{A_c} = \frac{\int_{t_1}^{t_2} \left[(\tau\alpha) I_\beta - U_{Lf} \left(T_{abs} - T_{air} \right) \right] dt}{C_p \left(T_{hot} - T_w \right)} \tag{4}$$

Due to the lack of water temperature data, the water temperature of the city water can be estimated by the empirical formulas [11,12]:

$$T_w = \frac{1}{2} \left(\left(2.82 + 0.82 T_{air} \right) \frac{\left(1 + R_h^2 \right)^{0.435}}{\left(1 + 0.31 V^2 \right)^{0.056}} + 4.717 e^{0.041 T_{air}} \frac{\left(1 + R_h^2 \right)^{0.781}}{\left(1 + 0.325 V^2 \right)^{0.0325}} \right) \tag{5}$$

where R_h is the relative humidity and V is the wind velocity.

Calculations and discussions of MAR

The MARs have been calculated based on the typical meteorological data of seven cities (Mengzi, Lincang, Tengchong, Kunming, Chuxiong, Lijiang and Deqin) in Yunnan Province. During the simulation, the ransmission-absorption product (τα) is 0.81, the albedo of ground (ρ_g) is 0.2 and the specific heat of water (C_p) is 4.187 kJ/(kg·K).

Table 1 Ranges of MARs for seasonal or year-round running of flat-plate solar hot water system

City	Latitude(^{o})	Spring	Summer	Autumn	Winter	Year-round
Mengzi	23.38	28.6 ~ 55.5	22.3 ~ 50.2	34.0 ~ 48.6	43.1 ~ 55.1	32.1 ~ 50.8
Lincang	23.88	28.9 ~ 56.0	22.5 ~ 48.9	37.0 ~ 52.7	44.4 ~ 58.0	33.6 ~ 52.3
Tengchong	25.02	28.1 ~ 52.1	21.8 ~ 44.8	40.0 ~ 56.2	42.9 ~ 57.5	34.1 ~ 51.1
Kunming	25.02	31.4 ~ 59.2	21.8 ~ 46.8	30.8 ~ 43.1	41.6 ~ 54.6	31.9 ~ 49.5
Chuxiong	25.03	28.0 ~ 51.4	21.4 ~ 44.1	33.3 ~ 45.9	40.6 ~ 53.2	31.3 ~ 47.3
Lijiang	26.87	28.1 ~ 53.1	21.6 ~ 47.9	35.1 ~ 47.7	41.1 ~ 57.9	33.0 ~ 49.9
Deqin	28.48	28.0 ~ 49.6	24.0 ~ 48.2	35.0 ~ 47.7	32.8 ~ 44.2	30.9 ~ 46.1

For the tilt angle variation from 0° to 90°, the ranges of MARs for seasonal or year-round operation of solar water heating system are given in Table 1. The data in Table 1 show that the ranges of the

MARs for spring, summer, autumn, winter and year-round are respectively in 28~59kg/m^2, 21~50 kg/m^2,30~56kg/m^2, 32~58kg/m^2 and 30~52kg/m^2. The ranges are lightly large and difficult to determine the effective value of MARs for the design of the solar hot water system. For convenient engineering applications, the relationships between the MARs and tilt angles have been analyzed by using the nonlinear quadratic fit method (i.e. Equation (6)). Regression coefficients (a, b and c) and correlation coefficients (R^2) of equation (6) are shown in Table 2.

$$MAR = a + b\beta + c\beta^2 \tag{6}$$

Table 2 Regression coefficients and correlation coefficients of equation (6)

		Mengzi	Lincang	Tengchong	Kunming	Chuxiong	Lijiang	Deqin
Spring	a	55.0871	55.5406	51.5238	58.1038	50.7145	52.0176	48.7252
	b	0.0803	0.0848	0.0934	0.1367	0.0980	0.1288	0.1064
	c	-0.0042	-0.0043	-0.0040	-0.0049	-0.0039	-0.0044	-0.0038
	R^2	0.9991	0.9991	0.9991	0.9991	0.9991	0.9991	0.9991
Summer	a	50.6336	49.2748	45.1026	47.1577	44.3682	48.2826	48.4826
	b	-0.0617	-0.0515	-0.0361	-0.0433	-0.0384	-0.0307	-0.0162
	c	-0.0029	-0.0028	-0.0025	-0.0027	-0.0025	-0.0030	-0.0029
	R^2	0.9990	0.9990	0.9990	0.9990	0.9990	0.9990	0.9991
Autumn	a	43.7942	47.0452	49.1120	38.7895	40.5900	40.5190	41.4646
	b	0.2893	0.3341	0.3956	0.2581	0.3023	0.3675	0.3350
	c	-0.0045	-0.0050	-0.0055	-0.0039	-0.0043	-0.0048	-0.0045
	R^2	0.9994	0.9994	0.9995	0.9994	0.9995	0.9996	0.9996
Winter	a	42.9993	44.2734	42.7244	41.4876	40.4962	41.2198	32.6374
	b	0.5363	0.58945	0.6086	0.5569	0.5406	0.6620	0.4623
	c	-0.0059	-0.0063	-0.0063	-0.0059	-0.0058	-0.0066	-0.0046
	R^2	0.9998	0.9998	0.9997	0.9998	0.9998	0.9997	0.9997
Year-round	a	48.1299	49.0306	47.1137	46.3853	44.0426	45.5133	42.8304
	b	0.2110	0.2393	0.2655	0.2272	0.2255	0.2817	0.2218
	c	-0.0044	-0.0046	-0.0046	-0.0043	-0.0041	-0.0047	-0.0040
	R^2	0.9992	0.9992	0.9993	0.9992	0.9993	0.9993	0.9993

The data in Table 2 show that the MARs and tilt angle β are highly correlated and their correlation coefficients are more than 0.999. Using equation (6) and the coefficients in Table 2, we can quickly determine the MARs for seasonal or year-round running of the solar water heating system in Yunnan Province.

Table 3 Optimal tilt angle and corresponding MARs of the flat-plate solar water heating system

City	Spring		Summer		Autumn		Winter		Year-round	
	β_{opt} (^{o})	MAR_{opt} (kg/m^2)	β_{opt} (^{o})	MAR_{opt} (kg/m^2)	β_{opt} (^{o})	MAR_{opt} (kg/m^2)	β_{opt} (^{o})	MAR_{opt} (kg/m^2)	β_{opt} (^{o})	MAR_{opt} (kg/m^2)
Mengzi	9.56	55.47	0.00	50.63	32.14	48.44	45.45	55.19	23.98	50.66
Lincang	9.86	55.96	0.00	49.27	33.41	52.63	46.78	58.06	26.01	52.14
Tengchong	11.68	52.07	0.00	45.10	35.96	56.23	48.30	57.42	28.86	50.94
Kunming	13.95	59.06	0.00	47.16	33.09	43.06	47.19	54.63	26.42	49.39
Chuxiong	12.56	51.33	0.00	44.37	35.15	45.90	46.60	53.09	27.50	47.14
Lijiang	14.64	52.96	0.00	48.28	38.28	47.55	50.15	57.82	29.97	49.73
Deqin	14.00	49.47	0.00	48.48	37.22	47.70	50.25	44.25	27.73	45.91

According to the extreme value conditions $d(MAR)/d\beta = b + 2c\beta = 0$ and equation (6), we can obtain the optimal tilt angle (β_{opt}) and corresponding MAR_{opt}. And the vales of the β_{opt} and MAR_{opt}

are listed in Table 3. The calculated values of the optimal tilt angle in summer are negative and the tilt angle in this paper is 0-90°. Therefore, the optimal tilt angles in summer are recorded as 0° in Table 3. The data in Table 3 show that the optimal tilt angles and the corresponding MAR_{opt} for spring, summer, autumn, winter or year-round operation of the solar water heating system are respectively in 10~15°, 0~0°, 32~38°, 45~50°, 24~30°, 50~60 kg/m^2, 44~51 kg/m^2, 43~56 kg/m^2, 44~58 kg/m^2 and 46~52kg/m^2.

Results and conclusions

The calculation model of the water-mass-to-collector-area ratio (MAR) was built based on the useful energy output model of a flat-plate solar collector.

The ranges of the MARs for seasonal or year-round running of flat-plate solar water heating system have been calculated based on the typical meteorological data of seven cities in Yunnan Province.

For convenient engineering application, the relationships between the MARs and tilt angle have been conducted by using quadratic fit method. The results show that the MARs and tilt angles are highly correlated and their correlation coefficients are more than 0.999.

Further discussions show that the optimal tilt angles and the corresponding MAR_{opt} for spring, summer, autumn, winter or year-round operation of the solar water heating system are respectively in 10~15°, 0~0°, 32~38°, 45~50°, 24~30°, 50~60 kg/m^2, 44~51 kg/m^2, 43~56 kg/m^2, 44~58 kg/m^2 and 46~52kg/m^2.

Acknowledgements

The work was financially supported by the Applied Basic Research Project of the Yunnan Province (Grant No. 2013FZ111), the Project for Innovative Research Team (Grant No. TD201301), the Education Reform Project (Grant No. JGXM2012013) and the Superior Quality Course ‘Calorifics’ (Grant No. YZKC201408) of the Qujing Normal University.

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