Identifying Steady State Relationships in a Multidirectional System: A Multivariate Cointegration Analysis

W.I.C.S. Gunasinghe

Department of Economics, University of Ruhuna, Sri Lanka

Abstract

This paper aims at identifying steady state relations (cointegrated relationships) in a system that contains multidirectional nexuses for a hypothetical economic structure. The study discusses various econometric tools needed to identify long run cointegration vectors and employs these tools to derive three such relationships. This study will be useful for a researcher to identify steady state relationships within a system of multidirectional relationships among financial markets or economic variables.

Keywords: Multidirectional relationships, cointegrated vectors, econometric tools

1. Introduction.

The identification of long run relationships in multidirectional frameworks has a considerable history (Hendry and Mizen 1993: Johensen and Jusilius 1990, 1992, 1994: Boswijk 1995: Boswijk and Doornike 2003: Hendry and Juselius 2000, 2001: Pesaran *et al* 2000; Nachega 2001: Gunasinghe 2005, 2006). Although the current study does not consider a specific economic theory to derive long-run steady state relations, techniques it discusses are useful for any researcher who considers identifying long-run relations in a multidirectional system.

The paper is organized as follows. Section 2 explains the hypothetical economic structure of multidirectional relationships. Section 3 discusses the model. Section 4 discusses the mechanism of the estimation of long run steady state relationships and finally presents concluding remarks.

2. The Hypothetical Economic Structure of Multidirectional Relationships.

The dependent variable of the main relationship is $\mathbf{Y_t}$. $\mathbf{X_{st}}$ stands for explanatory variables. The identification structure given below can be used not only to remove the possible multicollinearity problem of the model but also to capture the indirect effects from $X_{1t}\ X_{3t}, X_{4t}$ *and* X_{5t} variables on Y_t as one can incorporate these long-run relationships into a vector error correction framework.

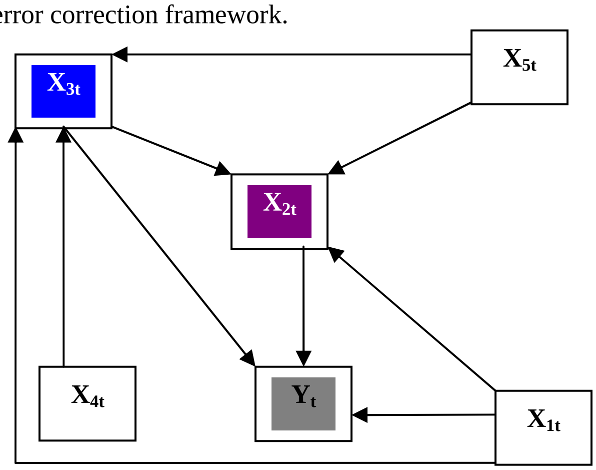

Figure 1: The Hypothetical Economic Structure of Multidirectional Relationships

According to the figure 1 above, the following long-run (steady state) relationships are defined.

$$Y_t = f(X_{1t}, X_{2t}, X_{3t}) \quad ...(2.1)$$

$$X_{2t} = f(X_{1t}, X_{3t}, X_{5t}) \quad ...(2.2)$$

$$X_{3t} = f(X_{1t}, X_{4t}, X_{5t}) \quad ...(2.3)$$

3. The Model.

Modern co-integration estimation can be applied only for the variables that are integrated of order (**I(1)**). Therefore, the paper assumes that each variable concerned in the study follows a unit root (**I(1))** process. The concept of cointegration means the stationarity of linear combination(s) of non-stationary variables (Johensen and Jusilius 1990, 1992, 1994: Boswijk 1995: Boswijk and Doornike 2003: Hendry and Juselius 2000, 2001: Pesaran *et al* 2000; Gunasinghe 2005, 2006).

Then,

$$\beta_i^{/} Z_t = (H_i \varphi_i)^{/} Z_t = (H_1\varphi_1 + H_2\varphi_2 + H_3\varphi_3)^{/} Z_t$$

$$\beta_i^{/} Z_t = [(h_1 + \tilde{H}_1 \tilde{\varphi}_1) + (h_2 + \tilde{H}_2 \tilde{\varphi}_2) + (h_3 + \tilde{H}_3 \tilde{\varphi}_3)]^{/} Z_t$$

Where β_i indicates cointegrated vectors and "i" stands for three cointegrated vectors (**i =1, 2, 3**).

$Z_t = (Y_t, X_{1t}, X_{2t}, X_{3t}, X_{4t}, X_{5t})^{/}$. H_i and $\tilde{H}_i$ are ***6 X 4*** and ***6 X 3*** design matrices respectively. φ_i and $\tilde{\varphi}_i$ are ***4 X 1*** and ***3 X 1*** hyper-parameter (free parameters to be estimated) vectors with typical elements φ_{if} and $\tilde{\varphi}_{if}$ respectively (where f denotes the number to which the parameter places) and $\mathbf{h_i}$ is a vector in $sp(H_i)$ defining the chosen normalization, and $sp(h_i, \tilde{H}_i) = sp(H_i)$ (Patterson 2000: Gunasinghe 2006).

The restriction matrices (R_is) given below are employed to derive three linearly independent row vectors (cointegrated vectors) from the multidirectional relationships. Therefore, $\mathbf{R_i}$**s** are:

$\mathbf{R_1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ **;** $\mathbf{R_2} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ and $\mathbf{R_3} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$**.** When the $\mathbf{R_i}$**s** define exclusive restrictions, the hyper- parameters φ are easily related back to the nonzero elements of β_i. For each $\mathbf{R_i}$ it is essential to define $H_i \equiv R_{i\perp}$, where H_i is orthogonal to R_i so that $R_i H_i = 0$. R_i s here are of dimension 2 X 6 row vectors and H_i s are (before normalization) 6 X 4 design matrices. These design matrices are used to determine the free parameters in each cointegration vector.

$$\beta_i^{/} Z_t = (H_i \varphi_i)^{/} Z_t = (H_1\varphi_1 + H_2\varphi_2 + H_3\varphi_3\}^{/} Z_t$$

$$\beta_i^{/} Z_t = [(h_1 + \tilde{H}_1 \tilde{\varphi}_1) + (h_2 + \tilde{H}_2 \tilde{\varphi}_2) + (h_3 + \tilde{H}_3 \tilde{\varphi}_3)]^{/} Z_t$$

$$\beta_i^{/} Z_t = \left\{\left[\begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0&0&0\\1&0&0\\0&1&0\\0&0&1\\0&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}\varphi_{12}\\\varphi_{13}\\\varphi_{14}\end{pmatrix}\right] + \left[\begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0&0&0\\1&0&0\\0&0&0\\0&1&0\\0&0&0\\0&0&1\end{pmatrix}\begin{pmatrix}\varphi_{22}\\\varphi_{24}\\\varphi_{26}\end{pmatrix}\right] + \left[\begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix} + \begin{pmatrix}0&0&0\\1&0&0\\0&0&0\\0&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}\varphi_{32}\\\varphi_{35}\\\varphi_{36}\end{pmatrix}\right]\right\}^{/} Z_t$$

Hence,

$$\beta_1^{/} Z_t = Y_t - \varphi_{12} X_{1t} + \varphi_{13} X_{2t} + \varphi_{14} X_{3t} \ldots (3.1)$$

$$\beta_2^{/} Z_t = X_{2t} - \varphi_{22} X_{1t} + \varphi_{24} X_{3t} + \varphi_{26} X_{5t} \ldots (3.2)$$

$$\beta_3^{/} Z_t = X_{3t} - \varphi_{32} X_{1t} + \varphi_{35} X_{4t} + \varphi_{36} X_{5t} \ldots (3.3)$$

4. The Mechanism of the Estimation of Long-Run Steady State Relationships.

The estimation of these hypothetical relationships needs fulfilling further two tests. First, it is necessary to define the "order of VAR". Second, selecting an "appropriate case", which stands for the pattern of possible common trends among the variables in the function, for modeling cointegration is needed. The former is normally selected on a likely hood ratio (LR) as well as adjusted LR test statistics and AIC information criteria. The latter is also selected based on a LR test. Accordingly, co-integrated VAR model consistent with both hypothetical relationships (i.e. theory consistent steady state relations) and dynamic interactions of variables can be formulated as follows.

$$\Delta Z_t = \Pi Z_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Z_{t-j} + a_0 + \varepsilon_t$$

Then,

$$\Delta Z_t = \left(\alpha_1\varphi_1^{/}H_1^{/} + \alpha_2\varphi_2^{/}H_2^{/} + \alpha_3\varphi_3^{/}H_3^{/}\right)Z_{t-1} + \sum_{j=1}^{p-1}\Gamma_j \Delta Z_{t-j} + a_0 + \varepsilon_t$$

Where $Z_t = (Y_t, X_{1t}, X_{2t}, X_{3t}, X_{4t}, X_{5t})^{/}$ *and* Δ denots diferenced oprator. a_0 is *6 X1* vector of unknown coefficients. $\Gamma_j = -\sum_{j=i+1}^{p-1} \phi_j$ *for i = 1, 2…....p-1*

(ϕ denotes parameters of the VAR model comprising 6 X 6 matrices). $\varepsilon_t \sim IN_P(0, \Omega_\varepsilon)$. If Π is of reduced rank ($r < p$), it can be written as $\Pi = \alpha\beta^{/}$ and α *and* $\beta^{/}$ indicate *(p x r)* error correction and *(r x p)* long run cointegrated matrices respectively. The rank of Π determines the number of cointegration vectors (r) and it is carried out using λ_{trace} and $\lambda_{\max}$ test statistics. P is the number of variables in the relevant cointegrated vector. Having concluded that the model contains 3 cointegrated vectors (rank (r) of Π is 3), the next step is to define restriction matrices (**$\mathbf{R_i}$ i = 1,2,3**) so that the long-run relationships defined in section 2 can be identified. In Microfit Version 04 (1997), the researcher should indicate "1" for the normalizing variable, "0" for exclusion variables and "B_s" for free parameters to be estimated in each cointegration vector.

5. Concluding Remarks.

The paper aims at identifying steady state relationships (cointegrated relationships) in a system that contains multidirectional nexuses for a hypothetical economic structure. Although the current study does not consider a specific economic theory to derive long-run steady state relations, it simply discusses various econometric tools that are needed to derive long-run relationships in a multidirectional system. Additionally, the study further reveals that the identification structure helps not only to remove the possible multicollinearity problem of the model but also to capture the indirect effects from other explanatory variables ($X_{1t}\ X_{3t}, X_{4t}\ and\ X_{5t}$) on the dependent variable (Y_t) as one can easily incorporate these long-run relationships into a vector error correction mechanism.

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Tel :(+94)0412227014 Fax: (+94)0412227014
E-mail: gunasinghe@econ.ruh.ac.lk

References

[1] H.P. Boswijk, "Identifiability of Cointegrated Systems," *Tinbergen Institute Discussion Paper # 95-78*, Faculty of Economics and Econometrics, Department of Quantitative Economics, Universiteit van Amsterdam, Netherlands,1995.

[2] H.P. Boswijk and J.A Doornike, "Identifying, Estimating and Testing Restricted Cointegrated Systems: An Overview," *Tinbergen Institute Discussion Papers*, Faculty of Economics and Econometrics, Department of Quantitative Economics, Universiteit van Amsterdam, Netherlands, 2003.

[3] W.I.C.S Gunasinghe, "The Demand for Money in Sri Lanka: A Vector Error Correction Approach," in *Proc. of the 9th International Conference on Sri Lankan Studies; Sri Lanka at Crossroads; Continuity and Change*, University of Ruhuna, December 2003, 2006 (forthcoming).

[4] _________________ "Does excess money supply generate inflation in the short run? Money in the utility function approach," *Sri Lanka Economic Journal*, Vol 06, pp. 41-62, 2005.

[5] D.F. Hendry and K Juselius, "Explaining Co-integration Analysis: part I," *Energy Journal*, Vol 21, pp. 1-42, 2000.

[6] D.F. Hendry and K Juselius, "Explaining Co-integration Analysis: part II," *Energy Journal*, Vol 21, pp. 1-45, 2001.

[7] S Johansen and K Juselius, "Maximum Likelihood Estimation and Inference on Cointegration with Applications to the Demand for Money," *Oxford Bulletin of Economics and Statistics*, Vol. 52, pp. 169-210, 1990.

[8] S Johansen and K Juselius, "Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and the UIP for the UK," *Journal of Econometrics*, Vol. 53, pp. 211-244, 1992.

[9] S Johansen and K Juselius, "Identification of the Long-Run and the Short-Run Structure: An Application to the IS/LM Model," *Journal of Econometrics*, Vol. 63, pp. 7-36, 1994.

[10] Microfit *Version 04*, 1997.

[11] J.C. Nachega, "A Co-integration Analysis of Broad Money Demand in Cameroon," *IMF Staff Papers* No-WP/01/26, 2001.

[12] K Patterson, "An Introduction to Applied Econometrics: A Time series Approach," Replika Press Pvt. Ltd, 2000.

[13] M.H. Pesaran, Y. Shin and R.J. Smith, "Structural Analysis of Vector Error Correction Models with Exogenous I(1) Variables," *Journal of Econometrics*, Vol 97, pp. 293-343, 2000