

A Potential Based Many-Particle Model for Pedestrian Flow

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Keywords: Speed-density relationship; Path-choice strategy; Evacuation; Lane formation

Abstract. We propose a many-particle model for pedestrian dynamics, which simply assumes that the velocity (including both magnitude and direction) of a pedestrian depends on the positions of all pedestrians. This is similar to that in the well-known social force model. However, our formulation is computationally efficient because the influences of all other pedestrians on the referred pedestrian are taken into account through the reconstruction of densities, the speed-density relationship, and the cost potential field that is also determined by densities. In the meantime, the formulation is able to simulate complex phenomena, such as the arching in front of exits, the formation of lanes in counterflow.

Introduction

In a view of molecular dynamics, pedestrians in a two dimensional walking domain Ω are taken as "many particles" in motion. The first many-particle model was proposed in [1] by establishing the motion equation of all pedestrians, with $\mathbf{x}_\alpha$ being the position, and $d\mathbf{x}_\alpha(t)/dt = \mathbf{v}_\alpha(t)$ being the velocity of the α -th pedestrian. To describe the acceleration $d\mathbf{v}_\alpha(t)/dt$, that was supposed to be governed by the Newtonian second law, the self-driven force, social force and pressure are introduced, which reflect the pedestrian's desire to the destination, the repulsion between pedestrians, and normal and share forces between two pedestrians with physical contact.

The formulation has been known as the social-force model, and has dominated for two decades with numerous subsequent works for study of pedestrian dynamics (see [2], and the references therein). Although the model (together with the improved versions) is able to reproduce typical self-organized phenomena, the social force there existing between any two pedestrians is very costly for computation. This is the case especially for overcrowded pedestrian flow. Moreover, the self-driven force directly points to a certain destination, which is not suited for a walking domain with complex geometries, e.g., for evacuation with more than one exit.

In this context, the present paper proposes a many-particle model for efficient simulation. First, we simply take the velocity $\mathbf{v}_\alpha(t)$ into account for the motion of the pedestrian. Second, the magnitude is determined by the density in position $\mathbf{x}_\alpha(t)$, $|\mathbf{v}_\alpha(t)| = V(\rho(\mathbf{x}_\alpha(t)))$ (m/s). Here, ρ ($ped./m^2$) is the density, and it is similar to that of vehicular flow that we take into account follow-the-leader behavior. Third, the direction of motion is assumed to be along an unit vector $\mathbf{e}(\mathbf{x}_\alpha(t))$, which suggests a minimized traveling time or cost from $\mathbf{x}_\alpha(t)$ to the destination Γ_0 .

Formulation

According to the forgoing discussion, our formulation can be simply written as

$$\frac{d\mathbf{x}_\alpha}{dt} = V(\rho(\mathbf{x}_\alpha(t))\mathbf{e}(\mathbf{x}_\alpha(t))), \quad \alpha = 1, \dots, N(t), \quad (1)$$

where $N(t)$ is the number of pedestrians in Ω at time t . The function $V(\cdot)$ is given by (see [3])

$$V(\rho) = v_f \exp[-19.2(\rho / \rho_{jam})^2], \quad (2)$$

where the free flow speed and jam density are taken as $v_f = 1.034 \text{ m/s}$, and $\rho_{jam} = 16 \text{ ped./m}^2$. In the following, we discuss for the density $\rho(\mathbf{x}_\alpha(t))$ in position $\mathbf{x}_\alpha(t)$ (or wherever needed) and the direction $\mathbf{e}(\mathbf{x}_\alpha(t))$ that only depend on all $N(t)$ pedestrians' positions at time t . Thus, the system becomes complete.

Reconstruction of Density. For the purpose, the walking domain is divided into square cells with the area s . Let (i, j) denote a cell as well as the scaled coordinate of the cell center. For simplicity in the discussion, let (k, l) denote a vertex or grid point, where the transformation between the sets of (i, j) and (k, l) is evident. Then, referring to Fig. 1(b) the procedure for the reconstruction is described in the following.

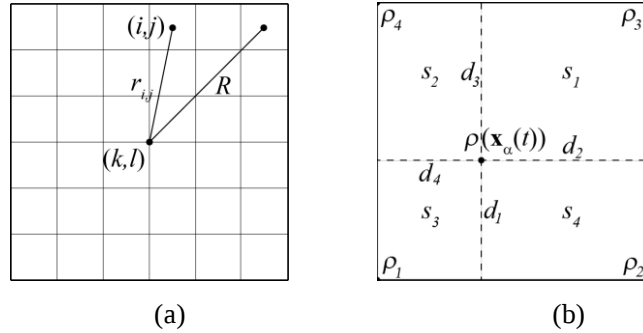


Figure 1: (a) The density at (k, l) is taken as a weighted average over 36 cells in square $A(k, l)$ through Eqs. 3-4; (b) The density (or potential) in a pedestrian's position is interpolated by the four adjacent vertex values through Eq. 5.

The density at the grid point (k, l) is taken as a weighted average over the surrounding cells:

$$\rho_{k,l} = \sum_{(i,j) \in A(k,l)} \frac{w_{i,j} n_{i,j}}{s}, \quad \sum_{(i,j) \in A(k,l)} w_{i,j} = 1, \quad (3)$$

where $A(k, l)$ is a square centered at (k, l) , with a fixed area $|A(k, l)| = |A|$; $n_{i,j}$ is the number of pedestrians whose centers are within the cell (i, j) . We note that $|A|/s$ should be an integer, and that $A(k, l)$ may contain artificial cells outside the domain, for which $n_{i,j}/s$ is replaced by the density ρ_{jam} to reflect repulsive effect on pedestrian from the wall. We set $|A| = 1 \text{ m}^2$, and $s = 1/36 \text{ m}^2$ in the simulation. The weight $w_{i,j}$ referring to the cell (i, j) is given by

$$w_{i,j} = \frac{\exp(-r_{i,j}^2 / R^2)}{\sum_{(i,j) \in A(k,l)} \exp(-r_{i,j}^2 / R^2)}, \quad (4)$$

where $r_{i,j}$ is the distance between the cell center (i, j) and the grid point (k, l) , $R = \max_{(i,j) \in A(k,l)} (r_{i,j})$.

The density in position $\mathbf{x}_\alpha(t)$ is taken as an interpolated value by using the four vertices of cell (i, j) that contains the center of the α -th pedestrian. Thus, we have

$$\rho(\mathbf{x}_\alpha(t)) = \sum_{m=1}^4 \frac{s_m}{s} \rho_m, \quad (5)$$

where ρ_m are densities at the four vertices, and s_m / s are corresponding weights (see Fig. 1(b)).

Cost Potential Field. At time t , we define a cost distribution in the walking domain Ω by

$$c(x, y, t) = \frac{1}{v(x, y, t)}, \quad (x, y) \in \Omega, \quad (6)$$

By the definition, we means that the cost for a pedestrian to move a small distance ds equals ds / v or cds , where v is the speed of motion. It is similar to the traveling of light that we assume that a steady-state flow together with a minimized cost to the destination is anticipated by all pedestrians. Therefore, the direction of motion at (x, y, t) is taken as

$$\mathbf{e}(x, y, t) = -\frac{\nabla \psi(x, y, t)}{|\nabla \psi(x, y, t)|}, \quad (7)$$

where $\psi(x, y, t)$ is called the cost potential at (x, y, t) , which satisfies the following Eikonal equation:

$$\sqrt{\psi_x^2 + \psi_y^2} = c, \quad \psi(x_0, y_0, t) = 0, \quad (8)$$

Here, $(x_0, y_0) \in \Gamma_0$, and $\psi(x, y, t)$ is the anticipated minimal cost for traveling from (x, y) to the destination Γ_0 . In this regard, the path-choice strategy is for reactive user-equilibrium. We note that similar path-choice strategy has been used in the continuum and Cellular Automata models for pedestrian dynamics (e.g. [4, 5, 6, 7]). See [6] for detailed discussion.

Given t , the numerical solution of $\psi(x, y, t)$ at all grid points (k, l) can be solved through Eq. 8 using a fast-sweeping algorithm [8], where the speed at (k, l) used in Eq. 6 is defined by $v_{k,l}(t) = V(\rho_{k,l}(t))$, and $\rho_{k,l}(t)$ is reconstructed through the discussion in Reconstruction of Density. To determine the direction of motion $\mathbf{e}(\mathbf{x}_\alpha(t))$ in Eq. 1 through Eq. 7, the potential $\psi(\mathbf{x}_\alpha(t))$ is taken as an interpolated value similarly to Eq. 5. Thus, we have

$$\begin{aligned} \psi_x(\mathbf{x}_\alpha(t)) &= \frac{1}{s} [(\psi_3 - \psi_4)d_1 + (\psi_2 - \psi_1)d_3], \\ \psi_y(\mathbf{x}_\alpha(t)) &= \frac{1}{s} [(\psi_3 - \psi_2)d_4 + (\psi_4 - \psi_1)d_2], \end{aligned}$$

where ψ_m are the potentials at the four vertices (see also Fig. 1(b)).

The forgoing discussions suggest that $\rho(\mathbf{x}_\alpha(t))$ and $\mathbf{e}(\mathbf{x}_\alpha(t))$ in Eq. 1 depend on the positions $\mathbf{x}_\beta(t)$ of all $N(t)$ pedestrians at time t . This means that the system is complete and thus can be numerically solved, given the initial and boundary conditions.

Extension to Modeling Two Pedestrian Groups. Two pedestrian groups a and b are distinguished by their destinations Γ_a and Γ_b . In this case, the two groups are separately described with Γ_0 being replaced by Γ_a and Γ_b , ρ by ρ_a and ρ_b , and so on, except that the speed-density relationship is reformulated by

$$V^c = V(\rho) \exp[-4.0(1 - \cos \theta)(\rho - \rho_c)^2], \quad (9)$$

where $c = a$ or b , $\rho = \rho_a + \rho_b$, and $\theta = \angle -\nabla \psi^a, -\nabla \psi^b >$

is the intersection angle between the directions of motion of the two groups. The function $\exp(\cdot)$ (≤ 1) reflects the interaction between the two groups, which is decreasing of θ and $\rho - \rho_c$. Eq. 9 reduces to Eq. 2 for $\theta = 0$ or $\rho - \rho_c = 0$. Initially, a prior estimate of ψ^a and ψ^b for solving θ is set to be the shortest distances from the referred coordinate to the destinations Γ_a and Γ_b , respectively. See [3,6,7] for detailed discussions.

Numerical Simulation

The system of (1) can be rewritten as

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{N}(\mathbf{X}(t)), \quad (10)$$

where $\mathbf{X}(t) = (\mathbf{x}_1(t), \dots, \mathbf{x}_{N(t)}(t))$, and $\mathbf{N}$ denotes the operation on $\mathbf{X}(t)$ that has been discussed in Section Formulation. For the discretization of Eq. 10, we adopt the first-order accurate forward Euler step, thus the scheme reads:

$$\mathbf{X}^{(n+1)} = \mathbf{X}^{(n)} + \Delta t \mathbf{N}(\mathbf{X}^{(n)}).$$

The increment $\Delta t = t^{(n+1)} - t^{(n)}$ is taken as 0.1s in the simulation.

Evacuation through Two Exits. 600 pedestrians are randomly distributed in a square hall of size $25m \times 25m$, which rightwards connects to another hall with the same size through two exits. The two exits are 3m width, and their centers are 6.5m distant from the top and bottom walls, respectively. The right side of the connected hall is open for pedestrians' evacuation, where the potential is set to be zero.

Fig. 2 shows the positions of pedestrians in the evolution. We see that the two exits had functioned as bottlenecks before $t = 10s$. As a consequence, arching effects are observed in front of the exits in the evolution from $t = 20s$ to $t = 40s$. The evacuations through two exits are expected to be simultaneous, according to all the distributions especially that at $t = 80s$. This result agrees well with the proposed optimal path-choice strategy or the user equilibrium principle, which suggests the same traveling time for those taking different paths and arriving at the last.

Lane Formation in Pedestrian Counterflow. Pedestrians enter a $10m \times 50m$ corridor from the left and the right symmetrically. From the left, 10 pedestrians enter in 3 seconds with their centers being in the line $y = 0.1m$. More precisely, the coordinates $x = 3m$, $5m$, and $7m$ are assigned to 3 pedestrians at $t = 0s$; $x = 2m$, $4m$, $6m$, and $8m$ to 4 pedestrians at $t = 1s$; and $x = 1m$, $5m$, and $9m$ to 3

pedestrians at $t = 2s$. This repeats until $t = 60s$, when there are totally 400 pedestrians in the domain, and thereafter the periodic boundary conditions are applied.

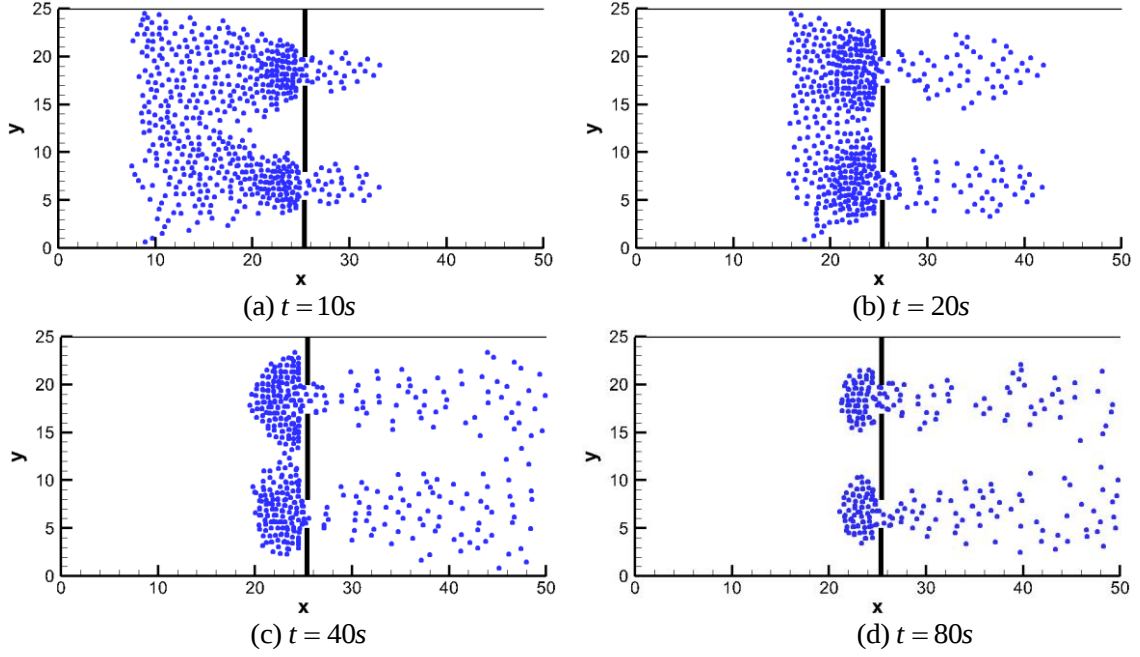


Figure 2: Evacuation of 600 pedestrians from the left square hall. They at first enter the right square through two exits. Then they get out through the right side which is set to be the destination.

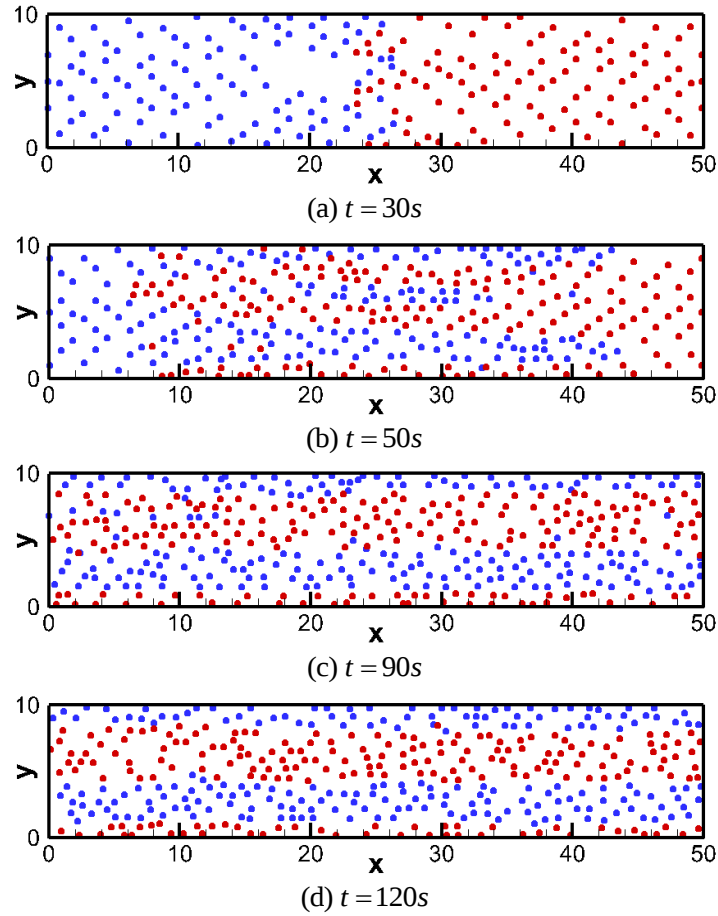


Figure 3: Pedestrian counterflow in a corridor, with observation of segregation of two pedestrian groups and the formation of lanes.

Fig. 3 shows three scenarios in the evolution. We should have observed blocking near the center because all associated with the two pedestrian groups are symmetric to the central line $x = 25m$.

However, we see the segregation of two pedestrian flows after they meet at the central line (Figs. 3(a)-(b)).

As a consequence, we observe the formation of lanes (Figs. 3(c)-(d)). Such are typically symmetry-breaking phenomena in physics. Mathematically, the expected symmetric solution (blocking) is unstable in that the noise or numerical error (as small perturbation) leads to asymmetric results which indicate relatively stable flows (Figs. 3(c)-(d)).

The segregation is ascribed to Eq. 9, which suggests that pedestrians prefer following a person ahead in the same group to mingling with the other group, so as to minimize their cost. Similar phenomena were reproduced by using the continuum model in [6] and the Cellular Automate model in [7].

Summary

Based upon the reconstruction of densities in all cells, the proposed many-particle model assumes that the speed of a pedestrian is determined by a speed-density relationship, and that the direction of motion is parallel to the negative gradient of cost potential in his/her position. Despite its being elegant in formulation, the model is efficient and able to reproduce such phenomena as evacuation in a complex walking facility, and formation of lanes in counterflow. In particular, the path-choice strategy can be associated with the user equilibrium principle that has been widely applied in transportation assignment.

Acknowledgment

This study was jointly supported by grants from the National Natural Science Foundation of China (11072141, 11272199), the National Basic Research Program of China (2012CB725404), the Shanghai Program for Innovative Research Team in Universities, a National Research Foundation of Korea grant funded by the Korean government (MEST) (NRF-2010-0029446), and the Training Program Foundation for University Young Teachers by Shanghai Municipal Education Commission.

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