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Weighted Multiple Model Adaptive Control of Time-varying Systems*

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Abstract

This paper is concerned with the weighted multiple model adaptive control (WMMAC) of stochastic plant with time-varying parameters, such as slowly time-varying plant or parameter jump plant. An improved weighting algorithm is proposed for this kind of control systems. The effectiveness (sharp and correct 0-1 convergence) of the weighting algorithm and the robust stability of the resulting closed-loop WMMAC system has been verified through many simulation results.

Keywords: Stability, Convergence, Weighting algorithm, Multiple model adaptive control.

1. INTRODUCTION

As we know, weighting algorithm plays an important role in WMMAC systems. According to reference [1], the closed-loop stability of a WMMAC system mainly depends on three conditions: first, the model set includes the true model of the plant or a closet one to the plant; second, the weighting algorithm converges correctly; third, each local controller stabilizes its corresponding model. Besides it is obvious that the convergence rate of a weighting algorithm is related to the transient process of the WMMAC system.

In classical weighted multiple model adaptive estimation (WMMAE) and WMMAC [2−8], as well as robust multiple model adaptive control (RMMAC) [9−11], probabilistic weighting algorithm was adopted, which is based on multiple Kalman filters, dynamic hypothesis testing, and Bayes' theorem. Some convergence results on the probabilistic weighting algorithm have been obtained [12−15]. In [10], Fekri, Athans and Pascoal pointed out that under certain ergodicity and stationarity assumptions, one of the posterior probabilities will converge with probability 1 to unity and will 'identify' the model closest to the true plant, i.e. the one with smallest

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Baram Proximity Measure (BPM)[10,12].

However, since using Kalman filters to drive the hypothesis testing, the classical WMMAC scheme can suffer from poor performance due to either large initial state estimate error or inaccurate knowledge of the disturbance/noise statistics. Additionally, the complexity of the supervisor may hinder its application because every candidate controller requires a Kalman filter and a post posterior evaluation [16]. Thus, some substitutive methods have been proposed to improve the above-mentioned situation. In [16-19], fuzzy rules based weighting algorithms replaced the probabilistic weighting algorithm. In [1], a new weighting algorithm based on model output error, is proposed. As a result, the long-standing issue, i.e, the closed-loop stability of WMMAC, has been addressed for a class of discrete-time stochastic plant under smooth conditions. The breakthrough result also benefited from the virtual equivalent system (VES) theory [20].

This paper is intended to further improve the convergence rate of weighting algorithm, as well as to adapt the weighting algorithm to time-varying systems such as parameter jump system and slowly time-varying system.

2. DESCRIPTION OF WMMAC

Consider the following discrete-time stochastic plant with single input and single output (SISO)

$$P(k): A(k,q^{-1})y(k) = q^{-d}B(k,q^{-1})u(k) + \omega(k) \tag{2.1}$$

where

$$\begin{aligned} A(k,q^{-1}) &= 1 + a_1(k)q^{-1} + \cdots + a_{na}(k)q^{-na} \\ B(k,q^{-1}) &= b_0(k) + b_1(k)q^{-1} + \cdots + b_{nb}(k)q^{-nb} \end{aligned}$$

$$d \geqslant 1; na \geqslant 1; nb \geqslant 1$$

where $y(k)$, $u(k)$ and $\omega(k)$ are the output, input and zero-mean white noise of the system, respectively, $y(k) = 0, u(k) = 0, \omega(k) = 0$ for $k < 0$, and that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} [\omega(i)]^2 = R < \infty \tag{2.2}$$

The plant $P(k)$ can be stable or non-stable, minimum phase or non-minimum phase. Its output $y(k)$ can be rewritten as

$$y(k) = \phi^T(k-d)\theta(k) + \omega(k) \tag{2.3}$$

where

$$\begin{aligned} \phi^T(k-d) =& [y(k-1),...,y(k-na),u(k-d),..., \\ & u(k-d-nb)] \end{aligned} \tag{2.4}$$

$$\theta(k) = [-a_1(k),...,-a_{na}(k),b_0(k),b_1(k),...,b_{nb}(k)] \tag{2.5}$$

The WMMAC system consists of three components, the model set $\mathbb{M} = \{M_i, i = 1,2,...,N\}$, the 'local' controller set $\mathbb{C} = \{C_i, i = 1,2,...,N\}$, and the weighting algorithm. The i^{th} model, controller, and weight are denoted as M_i, C_i, and $p_i(k)$, respectively.

The control objective is to track the bounded reference signal $y_r(k) < \infty$.

'Local' controllers can be designed according to any existing methods, such as pole assignment, H^∞ method, mixed-μ synthesis tools, PID, etc.

Each 'local' controller C_i generates local control $u_i(k)$; the 'global' control applied to the plant is obtained by weighting of the local controls, i.e.

$$u(k) = \sum_{i=1}^{N} p_i(k)u_i(k) \tag{2.6}$$

A simple and direct weighting algorithm is proposed in [1], which depends only on the model output errors. It gets rid of the inconvenience of Kalman filters and off-line membership functions. Details will be given in the next section.

For each model $M_i \in \mathbb{M}$, its output is given by

$$y_i(k) = \phi^T(k-d)\theta_i \tag{2.7}$$

where θ_i is the parameter vector of model M_i. Further, define the output error of each model M_i, i.e.

$$e_i(k) = y(k) - y_i(k) = y(k) - \phi^T(k-d)\theta_i \tag{2.8}$$

As we will see in the next section, $e_i(k)$ is used to calculate $p_i(k)$.

3. WEIGHTING ALGORITHMS WITH CONVERGENCE ANALYSIS

First, we review a weighting algorithm put forwarded in [1]. Then, a modified weighting algorithm will be proposed in order to improve the convergence rate, as well as to be suitable for time-varying plant.

Weighting algorithm 1 [1]:

$$l_i(0) = \frac{1}{N}; p_i(0) = l_i(0) \tag{3.1}$$

$$l_i'(k) = 1 + \frac{1}{k}\sum_{p=1}^{k} e_i(p)^2 \tag{3.2}$$

$$l_{min}(k) = \min_i l_i'(k) \tag{3.3}$$

$$l_i(k) = l_i(k-1)\frac{l_{min}(k)}{l_i'(k)} \tag{3.4}$$

$$p_i(k) = \frac{l_i(k)}{\sum_{i=1}^{N} l_i(k)} \tag{3.5}$$

Based on Weighting Algorithm 1, a modified version is proposed as follows.

Weighting algorithm 2:

$$l_i(0) = \frac{1}{N}; p_i(0) = l_i(0) \tag{3.6}$$

$$l_i'(k) = 1 + \frac{1}{k}\sum_{p=1}^{k} e_i(p)^2 \tag{3.7}$$

$$l_{min}(k) = \min_i l_i'(k) \tag{3.8}$$

$$\beta_i(k) = \frac{l_{min}(k)}{l_i'(k)} \tag{3.9}$$

$$l_i(k) = \begin{cases} l_i(k-1) & \text{if } \beta_i(k) = 1 \\ l_i(k-1)[\beta_i(k)]^{ceil(\frac{2}{1-\beta_i(k)})} & \text{if } \beta_i(k) < 1 \end{cases} \tag{3.10}$$

$$p_i(k) = \frac{l_i(k)}{\sum_{i=1}^{N} l_i(k)} \tag{3.11}$$

where $ceil(x)$ is the ceiling function that generates the smallest integer not less than x, i.e.,

$$ceil(x) = \min\{n \in \mathbb{Z} | x \leqslant n\}$$

To adapt the above mentioned algorithms to slowly time-varying or parameter jump systems, there are two methods to avoid the zero lock-in condition of p_i from occurring. One method is to set lower bounds for weights $p_i, i = 1, 2, ..., N$ as in [21]; The other is to reset each weight according to (3.6) when the parameters change is detected. In the simulations of this paper, we adopted the latter method. In detail, the numerical value order of $l_i(k)$ is used to detect the parameter changes. The related programme is as follows.

$$\vdots$$

$$A = [l_1'(k) l_2'(k) l_3'(k) l_4'(k) ...];$$

$$[l_{min}(k), index(k)] = min(A);$$

calculates $l_i(k)$ according to Weighting Algorithm 2

if $l_1(k) = 0 || l_2(k) = 0 || l_3(k) = 0 || l_4(k) = 0 \cdots$

if $index(k) = index(k-1)$

resets Weighting Algorithm 2;

end

end

$$\vdots$$

4. SIMULATION RESULTS

To verify the effectiveness of the proposed weighting algorithm and the performance of the resulting closed-loop WMMAC system, a lot of simulation results have been conducted with MATLAB® version 6.5. Owing to the space limitation, the corresponding photographes are omitted here. In a word, the weighting algorithm has very good convergence performance under various situations, and the corresponding WMMAC systems have satisfactory stability and tracking performance.

5. CONCLUSIONS AND FUTURE WORK

The stability and convergence of the closed-loop WMMAC system depend only on three conditions, i.e., the stabilizing characteristic of each 'local' controller; the effective model set to cover the uncertainty of plant to be controlled; and the appropriate convergence of the weighting algorithm. Thus, WMMAC provide us with more confidence and flexibility in robust adaptive control practices.

The future research will be focused on two directions. One is to analyze the stability and convergence of WMMAC systems of non-linear time-varying plants. The other is to further relax the convergence conditions of weighting algorithms.

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