

## Modelling and experimental verification of diagonal cracks in beam based on the modal strain energy

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**Abstract.** According to the linear elastic fracture mechanics and the virtual work principle, the elemental stiffness matrix of the diagonally cracked RC beam is derived, and the frequencies and modes of the diagonally cracked beam are obtained. The experiment is executed to verify the elemental stiffness matrix. It is given from the experimental result that frequencies would be lower after crack happen, and the influence of higher frequencies is more obviously than influence of the lower frequencies for crack, which is matched with the fact. So the experimental result is reliable. The built elementary stiffness matrix is used to calculate the frequencies and modes of the experimental beams. The maximum relative error of frequencies of calculating and experimental test is controlled in the 2.79%, and the MAC values between calculated mode shapes and experimental mode shapes are all higher than 0.9. The values of calculating with the elementary stiffness matrix built are matched well with the experimental values. So the stiffness matrix of RC beam element with a diagonal crack is applicable in calculation.

### Introduction

For cracks induced by fracture of concrete, the integrity of structure is destroyed and internal mechanical sections are formed which deteriorate the stress condition, which induce continuous disasters of fracture of structures. RC beam is the basic member in structures, therefore the safety of structure is determined by whether there are cracks in RC beam. Studies of crack detection of RC beam have important theoretical significance and engineering value.

Physical parameters and dynamic characters such as frequency response function and modal parameters change for cracks. The study on crack detection in beam with these change has made a great progress. Chang et al<sup>[1]</sup> use the torsional spring to simulate the crack and get the frequencies and modes. Then the locations and depths of cracks in beam are obtained with the spatial wavelet theory. Khiem et al<sup>[2]</sup> build the method to detect cracks in beam based on frequencies using nonlinear optimized technology and eigenfunction. Patil et al<sup>[3]</sup> use spring to simulate crack and separate beams with these springs. Then damage index based on frequencies and modal energy are built to identify cracks with the help of relationship between crack depth and stiffness matrix. Tian et al<sup>[4]</sup> calculate the wave function of transient flexural wave of cracked beam. Time characters of arriving times of flexural waves with different speed are obtained with wavelet translation to get the information of cracks. Kim et al<sup>[5]</sup> deduce the analytical relationship between change of modal energy and frequencies after crack occur. The models of locating cracks and crack size detection are built to identify cracks using only few frequencies. Law et al<sup>[6]</sup> obtain some formulas to detect cracks using optimization method and strain and displacement response of cracked beam in time domain.

In the former study, the research work is mainly about straight crack modelling and detection. The diagonal crack modelling and detection in RC beam has been short of studying for its complexity mechanics and more identification parameters. Based on the mechanism of diagonal crack in RC beam, the stiffness matrix of RC beam element with a diagonal crack is derived thinking about the influence of steel bar. Then the frequencies and mode shapes are obtained with the built stiffness matrix. The validity of the stiffness matrix of cracked element has been verified by experiment.

## Building of the stiffness matrix of RC beam element with diagonal crack

The stress and strain around the crack are influenced by the crack. The additional strain energy induced by crack could be obtained by integrating the stress intensity factor of the crack. In the domain of linear elastic, the displacement in the direction of the applied force can be obtained according to the Castigliano's Theory (i.e.  $\Delta = \partial U / \partial P$ ,  $U$  is the strain energy;  $P$  is the force or force couple;  $\Delta$  is the displacement in the direction of  $P$ ). The flexibility coefficient can be obtained by deriving of differentiating of the force. Then the stiffness matrix of the element with diagonal crack can be obtained according to the virtual work principle. The general strain energy of the RC beam element with a diagonal crack in fig.1 is given as:  $U = U_0 + U_1$  (Where  $U_0$  is the strain energy of the un-cracked element, and  $U_1$  is the additional strain energy induced by crack).

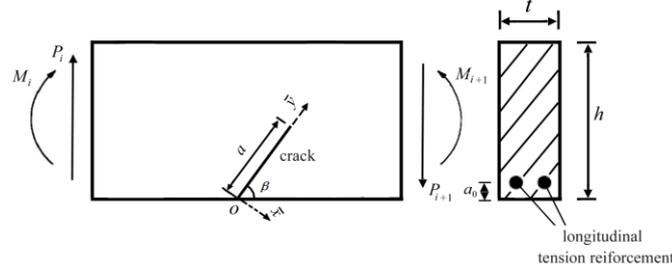


Fig.1 Model of cracked element in reinforced concrete beam

The left section of the cracked element in fig.1 is deserved fixed constraint. The right section of the cracked element is deserved moment  $M_{i+1}$  and shear force  $P_{i+1}$ . Then the strain energy of the un-cracked element is given as:

$$U_0 = \frac{1}{2} \int_0^l \frac{(M_{i+1} + P_{i+1}x)^2}{EI} dx = \left( M_{i+1}^2 l + M_{i+1} P_{i+1} l^2 + \frac{1}{3} P_{i+1}^2 l^3 \right) / 2EI \quad (1)$$

Where  $l$  is the length of the element, and  $E$  is the elastic modulus, and  $I$  is the inertia moment of section.

Under the circumstance of plane strain, the additional strain energy of the beam element induced by crack is given as:

$$U_1 = \int_A \frac{(1-\nu^2)}{EA} \left[ (K_{IM} + K_{IP})^2 + (K_{IM} + K_{IP})^2 \right] dA \quad (2)$$

In which  $A$  is the surface area of the diagonal crack. For the rectangular section in fig. 1,  $U_1$  is given as:

$$U_1 = \frac{t(1-\nu^2)}{E} \int_0^a \left[ (K_{IM} + K_{IP} - K_{IS})^2 + (K_{IM} + K_{IP} - K_{IS})^2 \right] d\alpha \quad (3)$$

Where  $\nu$  is the Poisson ratio.  $K_{IS}$  and  $K_{IS}$  are the stress intensity factor for the steel bar on crack section respectively<sup>[7]</sup>.

Under the circumstance of biaxial stress, the stress intensity factors of the diagonal crack of RC beam are given<sup>[8]</sup>. Then the expression of  $C_{mn}$  can be obtained:

$$C_{mn} = \frac{\partial^2 U_0}{\partial P_m \partial P_n} + \frac{\partial^2 U_1}{\partial P_m \partial P_n} \quad (P_1 = P_{i+1}, P_2 = M_{i+1}, m, n = 1, 2) \quad (4)$$

Then the matrix flexible coefficient  $[C]$  can be obtain from  $C_{mn}$ .

According to the condition of balance in fig.1, the following expression is obtained:

$$\begin{pmatrix} P_i & M_i & P_{i+1} & M_{i+1} \end{pmatrix}^T = [T] \begin{pmatrix} P_{i+1} & M_{i+1} \end{pmatrix}^T \quad (5)$$

The transition matrix  $[T]$  is given as followed:

$$[T] = \begin{bmatrix} -1 & -l & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}^T \quad (6)$$

Then the stiffness matrix of the cracked element can be obtained from the virtual work:

$$[K]_c = [T]^T [C]^{-1} [T] \quad (7)$$

### Experimental verification of the stiffness matrix of RC beam element with a diagonal crack

In order to verify the validity of the stiffness matrix of cracked element, the modal experiment is used with 8 RC beam. The basic parameters of concrete are tested as 28.2MPa(Compression strength of cubic specimen)、21.7MPa (Compression strength of axis) 、28.8GPa (Elastic modulus) 、0.193 (Poisson ratio) . The steel bar is made in Tangshan steel company. The diameters of the steel bar used in the experiment are all 10mm. The test result of mechanical parameters of steel bar is 320MPa (Yield strength) 、485MPa (Ultimate strength) 、28% (Extensibility  $\delta$ ) 、63% (Reduction of area  $\psi$ ) 、 $2.33 \times 10^5$  MPa(Elastic modulus). Coefficients of variation of the mechanical parameters of the steel bar are all less than 4%, so the steel bar is acceptable in the experiment.

The size of the RC beam is 1850×100×200mm. The infinite element model of the experimental beam is built in fig.2. The RC beam is divided into 9 elements uniformly. The length of the element is 200mm. Total 8 experimental RC beam are made with different crack types as shown in table.3. The A-1 is un-cracked RC beam, and 7 RC beams of series B are beams with diagonal cracks. Crack locations is expressed with element number of the beam, and the actual crack position is located in the midpoint of the element. The B-2 beam with a crack in element 2, whose depth is 112mm and angle is  $1/4\pi$ . The B-3 beam with a crack in element 4, whose depth is 67mm angle is  $1/3\pi$ . The B-4 beam with a crack in element 6, whose depth is 112mm and angle is  $2/3\pi$ . The B-5 beam with a crack in element 7, whose depth is 89mm and angle is  $3/4\pi$ . The B-6 beam with 2 cracks in element 2 and 6, whose depth is 112mm and 89mm with the angle of  $1/4\pi$  and  $2/3\pi$ . The B-7 beam with 2 cracks in element 4 and 8, whose depth is 67mm and 112mm with the angle of  $1/3\pi$  and  $3/4\pi$ . The B-8 beam with 2cracks in element 2 and 8, whose depth is 112mm and 112mm with the angle of  $1/4\pi$  and  $3/4\pi$ .

The first and second modes are obtained with equipments and modal analysis soft DASP made in china. The simplicity diagram of the connection of the equipments is shown in fig.3. The beam are divided into 9 parts uniformly. Ten nodes of the elements are used as the sample adopting points. The lump average method and the modal analysis of complex modal with single degree of freedom are used in the test. Then reports of modal analysis are obtained with the soft DASP. The modal normalized method is mass normalizing. The density of the concrete is  $2450.3\text{kg/m}^3$  calculated according to the mixture ratio of the experimental concrete. The equivalent density is  $2570.9\text{kg/m}^3$  calculated according to the ratio of reinforcement. The finite element model (as shown in fig.2) is used to calculate the modes of the un-cracked experimental beam. The modulus used in the calculating is modified according to the un-cracked experimental beam. The modified modulus of the beam is 29.23Gpa. Then the modified modulus and the stiffness matrix of the cracked element are used to calculate modes of cracked experimental beams. The calculating and experimental frequencies and modes are list in table.1.

The error is defined as followed:

$$er = |f_c - f_e| / f_c \times 100\% \quad (8)$$

where  $f_c$  is the calculating frequency, and  $f_e$  is the experimental frequency.

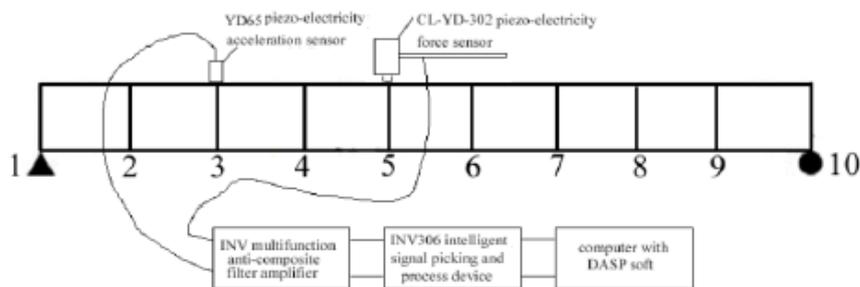


Fig.2 Connection of equipments of modal experiment

Table 1 Computational frequencies with crack element stiffness and experimental frequencies

| Beam number | First frequency    |                      |           | Second frequency   |                      |           |
|-------------|--------------------|----------------------|-----------|--------------------|----------------------|-----------|
|             | Calculated results | Experimental results | Error (%) | Calculated results | Experimental results | Error (%) |
| A-1         | 93.375             | 93.377               | 0.00      | 368.051            | 370.121              | 0.56      |
| B-2         | 92.925             | 93.807               | 0.95      | 363.413            | 368.454              | 1.39      |
| B-3         | 92.792             | 91.702               | 1.17      | 366.825            | 359.843              | 1.90      |
| B-4         | 90.165             | 91.209               | 1.16      | 362.197            | 367.316              | 1.41      |
| B-5         | 92.874             | 90.315               | 2.76      | 365.016            | 357.842              | 1.97      |
| B-6         | 91.746             | 92.971               | 1.34      | 361.214            | 367.637              | 1.78      |
| B-7         | 92.429             | 90.391               | 2.20      | 362.472            | 353.924              | 2.36      |
| B-8         | 92.561             | 91.263               | 1.40      | 359.213            | 363.621              | 1.23      |

As shown in the table.1, the experimental frequencies and calculating frequencies are matched anastomotic. The maximum of the relative error is only 2.76%, which is demonstrated that the built stiffness matrix of the cracked element can be used in the simulation with the infinite element method and have high precision. Studying of experimental frequencies In table.1, frequencies of cracked beam are all less than un-cracked beam, except for the B-2 beam, which demonstrate that cracks in RC beam would low frequencies of RC beam. This conclusion is consistent with the fact , so the experiment is successful. The calculating results with cracked element also demonstrate that cracks can low the frequencies of RC beam, which is consistent with the experiment and verify the validity of the stiffness matrix of the cracked element.

Experimental results in table.1 express that the maximum of the first frequency difference between the cracked beam and un-cracked beam is 3.062Hz of B-5 beam. The second frequency difference between B-5 beam and A-1 beam is 12.279Hz. The average of the first frequency difference of 7 cracked beams is 1.712, while the average of the second frequency difference of 7 cracked beams is 7.459. The maximum of the second frequency difference between the cracked beam and un-cracked beam is 16.197Hz of B-7 beam, while the first frequency difference between B-7 beam and A-1 beam is 2.986Hz. The experimental results express that the influence of high frequency are bigger than low frequency for cracks, which consistent with the fact. The experiment is successful.

Calculating results in table.1 express that the maximum of the first frequency difference between the cracked beam and un-cracked beam is 3.21Hz of B-4 beam. The second frequency difference between B-4 beam and A-1 beam is 5.854Hz. The average of the first frequency difference of 7 cracked beams is 1.162Hz, while the average of the second frequency difference of 7 cracked beams is 5.148Hz. The maximum of the second frequency difference between the cracked beam and un-cracked beam is 8.838Hz of B-8 beam, while the first frequency difference between B-8 beam and A-1 beam is 0.814Hz. The calculating results express that the influence of high frequency are bigger than low frequency for cracks, which consistent with the experiment. So the stiffness matrix of cracked element is fit for the calculating.

Experimental results in table.1 express that the first and second frequency of B-6 beam and B-8 beam are all lower than B-2 beam. The first and second frequencies of B-7 beam are lower than B-3 beam. The reason is that the high frequency beam has single crack and the lower frequency beam has two cracks including the single crack. All of these result show that more cracks would low the frequency of the beam, which consistent with the fact<sup>[9]</sup>. The experiment is successful. The calculating results have the same tendency with the experimental results, so the validity of the stiffness matrix of the cracked element is verified.

The modal comparison between experiment and calculating results can be executed with frequency and mode shapes. According to the mode shapes, the conclusion is given that the mode shapes are coherence and the first and second modes have been obtained correctly. The local un-flat is for measuring noise.

In order to study the correlation between the experimental and calculating mode shapes, the Modal Assurance Criterion are used as followed:

$$MAC(\phi_a, \phi_e) = \frac{|\{\phi_a\}^T \{\phi_e\}|^2}{(\{\phi_a\}^T \{\phi_a\})(\{\phi_e\}^T \{\phi_e\})} \quad (9)$$

where  $\{\phi_a\}$  and  $\{\phi_e\}$  are the calculating and experimental mode shapes. Values of  $MAC$  of experimental mode and computational mode are  $MAC_{11}$  (0.985, 0.956, 0.949, 0.951, 0.902, 0.937, 0.919, 0.942) and  $MAC_{22}$  (0.968, 0.943, 0.927, 0.942, 0.925, 0.931, 0.914, 0.948).

As we know that the values of  $MAC$  are between 0 and 1. The value of  $MAC$  is closer to 1 represent that the correlation is better. If the value of  $MAC$  is bigger than 0.9, two mode shapes are correlation. If the value of  $MAC$  is lower than 0.05, two mode shapes are not correlation. In table.5,  $MAC_{11}$  is the value of  $MAC$  between the first experimental modes and first calculating modes, and  $MAC_{22}$  is the value of  $MAC$  between the second experimental modes and second calculating modes. The value of  $MAC$  list in table.5 are all bigger than 0.9. The average of  $MAC_{11}$  of 8 experimental beams is 0.943, and the average of  $MAC_{22}$  of 8 experimental beams is 0.937, which express that the experimental and calculating modes have good correlation. Then the conclusion is given that the stiffness matrix of cracked element of RC beam is fit for the infinite element calculation.

## Conclusions

The stiffness matrix of RC beam element with a diagonal crack is derived. The experiment is used to verify the validity of the stiffness matrix of cracked element in this paper. Based on experimental results, some conclusions are given as followed:

(1) The derived stiffness matrix of RC beam element with a diagonal crack can be used in the calculation of infinite element and the calculation precision is higher.

(2) The experimental method of the beam with a diagonal crack is available and have good robustness against noise.

## References

- [1] Chang, C.C & Chen, L.W. 2005. *Detection of the location and size of cracks in the multiple cracked beam by spatial wavelet based approach*. Mechanical Systems and Signal Processing 19(1): pp. 139–155.
- [2] Khiem, N.T & Lien, T.V. 2004. *Multi-crack detection for beam by the natural frequencies*. Journal of Sound and Vibration 273(1): pp. 175–184.
- [3] Patil, D.P & Maiti, S.K.2005. *Experimental verification of a method of detection of multiple cracks in beams based on frequency measurements*. Journal of Sound and Vibration 281(1): pp.439–451.
- [4] Tian, J.Y & Li, Z. 2003. *Crack detection in beams by wavelet analysis of transient flexural waves*. Journal of Sound and Vibration 261(4): pp. 715–727.
- [5] Kim, J.T & Stubbs, N. 2003. *Crack detection in beam-type structures using frequency data*. Journal of Sound and Vibration 259(1): pp. 145–160.
- [6] Law, S.S & Lu, Z.R. 2005. *Crack identification in beam from dynamic responses*. Journal of Sound and Vibration 285(4): pp. 967–987.
- [7] Shilang Xu & Xiangrong Cai. 2010. *Experimental Study and Theoretical Models on Compressive Properties of Ultra High Toughness Cementitious Composites*, Journal of Materials in Civil Engineering (ASCE),22(10):PP. 1067-1077.
- [8] Jianying Wu and Shilang Xu. 2011 . *An augmented multicroack elastoplastic damage model for tensile cracking*,International Journal of Solids and Structures,48(18):PP. 2511-2528.

- [9] Wang, S.X & Jiang Z. 2003. *Analysis of the effect of crack on the vibration dynamic behavior and the crack identification of a cantilever beams*. Journal of vibration and shock 22(3): pp. 83–85.