Two Order Grey Markov Prediction Modeling and Its Application

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Abstract. This paper based on the Grey Theory and Markov Theory, the Two Order Grey Markov Model(GMM(2,1)) is established. The modeling process are analyzed firstly, instead of using only one theory model or only using GM(1,1) in the past, this article also through a real instance comparing GM(1, 1) ,GM (2, 1) and GMM(2,1). Experiments demonstrate that the GMM(2,1) gets the better result performance than that of the other models.

Introduction

Unknown or uncertain information is called black information. The known information is called white information. The information which contain unknown information and known information is called gray system. Since the grey model has been proposed, it has been widely employed in various fields, ranging from economics through agriculture to engineering, and demonstrated promising results. In general, people are highly interested in forecasting future tendency of some data series or event, such as investment in stock market or in the Agriculture trading. Because of the accuracy prediction can reduce the uncertain and investment risk in making decision.

GM(1,1) is the most commonly employed grey model, such as predict chaotic time series[1], predict the price of the agriculture production[2], predict the price of the stock[3-4],compute weights of the criteria[5], evaluate restoration plans for power distribution systems [6],design the evaluation[7], estimation and decision of the mechanic motion project [8] , control road tunnel ventilation[9]and so on[10]. In spite of GM(1,1) or its variants can obtain good performance on the various applications, but for acquiring the best forecasting results, an effective method, combined the Grey Theory with Markov Theory(GMM(2,1)) is proposed in our research.

Grey theory

In 1982,Professor Deng Julong founded Grey System Theory[11-13] for dealing with poor information system, usually 3-5 points, the grey model can be constructed. A brief description of the procedure of GM(1,1) is given as follows.

Step 1: for random sequence

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\} \tag{1}$$

where n is the sample size. By 1-AGO(one time Accumulated Generating Operation) $x^{(0)}$,the preprocessed series,

$$x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n)\} \tag{2}$$

where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$, for i=1,2,…,n .

By mean operation on $x^{(1)}$, the series

$$z^{(1)} = \{z^{(1)}(1), z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)\} \tag{3}$$

where $z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)$, (k= 2,3,…,n)

Step 2: obtained the grey differential equation and its whiting equation of GM(1,1,), respectively, as follows:

$$x^{(0)}(k)+az^{(1)}(k)=b,\quad k=2,3,\ldots,n \tag{4}$$

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b, \tag{5}$$

where a and b is the developing coefficient and grey input, respectively.

Step 3:Let $\theta=(a,b)^{\mathrm{T}}$ be the parameters vector. By using least square method (Hsia,1979), the parameters a and b can be obtain as

$$\hat{\theta}=(X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}Y=\begin{bmatrix} a \\ b \end{bmatrix} \tag{6}$$

where

$$X=\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(\mathrm{n}) & 1 \end{bmatrix} \quad \text{and} \quad Y=\begin{bmatrix} X^{(1)}(2) \\ X^{(1)}(3) \\ \vdots \\ X^{(1)}(\mathrm{n}) \end{bmatrix}$$

X denotes the accumulated matrix and Y represents the constant vector. The approximate relation can be obtained by substituting the θ into the differential equation.

Step 4: the solving of equation (5) can be represented as

$$\hat{X}^{(1)}(k+1)=\left[X^{(1)}(1)-\frac{b}{a}\right]e^{-ak}+\frac{b}{a} \qquad (k=1,2,3,\ldots,n) \tag{7}$$

where $\hat{X}^{(0)}(1)=X^{(0)}(1)$.

Step 5: by IAGO(inverse AGO), the recovered value $\hat{x}^{(0)}(k)$ is:

$$\hat{x}^{(0)}(k)=\hat{x}^{(1)}(k)-\hat{x}^{(1)}(k-1)=(1-e^{a})\left[x^{(0)}(1)-\frac{b}{a}\right]e^{-a(k-1)} \tag{8}$$

Given $k=1,2,3,\ldots,n$, the predictive series value is

$$\hat{x}^{(0)}=(\hat{x}^{(0)}(1),\ \hat{x}^{(0)}(2),\ \hat{x}^{(0)}(3),\ldots,\ \hat{x}^{(0)}(\mathrm{n})) \tag{9}$$

Step 6: calculate the residual error and accuracy (Deng, 1988) as:

$$e(k)=\left|\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%, \qquad k=2,3,4,\ldots,\mathrm{n} \tag{10}$$

$$\varepsilon=(1\text{-}e)\times 100\% \tag{11}$$

Above is the process of GM(1,1,) modeling, The following we will describe the procedure of GM(2,1) that is given as follows. For the sake of simplicity, we will only highlight the different place.

Step 1: for the original random sequence $x^{(0)}$, obtained $x^{(1)}$ by 1-AGO and $a^{(1)}x^{(0)}$ by 1-IAGO(one time Inverse Accumulated Generating Operation),

$$a^{(1)}x^{(0)}=\{a^{(1)}x^{(0)}(2),a^{(1)}x^{(0)}(3),a^{(1)}x^{(0)}(4),\ldots,a^{(1)}x^{(0)}(n)\} \tag{12}$$

where n is the sample size, $a^{(1)}x^{(0)}(k)=x^{(0)}(k)-x^{(0)}(k-1),\quad (k=2,3,4,\ldots,n)$. By mean operation on $x^{(1)}$, the series $z^{(1)}(k)=0.5x^{(1)}(k)+0.5x^{(1)}(k-1)$, $(k=2,3,\ldots,\mathrm{n})$ can be obtained.

Step 2: obtained the grey differential equation and its whiting equation of GM(2,1,), respectively, as follows:

$$a^{(1)}x^{(0)}(k)+a_1x^{(0)}(k)+a_2z^{(1)}(k)=b, \tag{13}$$

$$\frac{d^2x^{(1)}}{dt^2}+a_1\frac{dx^{(1)}}{dt}+a_2x^{(1)}=b, \tag{14}$$

where a_1 , a_2 and b is the developing coefficient and grey input, respectively.

Step 3:Let $\theta=(a_1,a_2,\ b)^{\mathrm{T}}$ be the parameters vector. Let

$$X=\begin{bmatrix}-x^{(0)}(2) & -z^{(1)}(2) & 1\\ -x^{(0)}(3) & -z^{(1)}(3) & 1\\ \vdots & \vdots & \vdots\\ -x^{(0)}(\mathrm{n}) & -z^{(1)}(\mathrm{n}) & 1\end{bmatrix} \quad \text{and} \quad Y=\begin{bmatrix}a^{(1)}x^{(0)}(2)\\ a^{(1)}x^{(0)}(3)\\ \vdots\\ a^{(1)}x^{(0)}(n)\end{bmatrix}$$

By using least square method (Hsia,1979), the parameters a_1 , a_2 and b can be obtain as

$$\hat{\theta}=(X^{T}X)^{-1}X^{T}Y=(a_1, a_2, b)^{T} \tag{15}$$

The approximate relation can be obtained by substituting the θ into the differential equation.

Step 4: the solving of equation (14) has three kinds of solution: let x* is the special solution of Eq.(14), $\overline{x}^{(1)}$ is the general solution on the general solution of homogeneous equation of Eq.(14).

Then the solution of Eq.(14) is $\hat{x}^{(1)}(k)=x^{*}+\overline{x}^{(1)}$,

① There are a pair of unequal real root of characteristic equation, then $\overline{x}^{(1)}(k)=c_1e^{r_1t}+c_2e^{r_2t}$

② There are a pair of equal real root of characteristic equation, then $\overline{x}^{(1)}(k)=(c_1+c_2)e^{r_1t}$

③ There are a pair of conjugate complex root , then $\overline{x}^{(1)}(k)=e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$,

where $\hat{X}^{(1)}(1)=X^{(1)}(1)=X^{(0)}(1)$. $\hat{X}^{(0)}(1)=X^{(0)}(1)$.

Markov chains theory

A Markov chain is a particular type of stochastic process, which must satisfy the Markov property, namely, the future evolution of the system of the condition probability depends only on the current states of the system and not on its history. As a forecasting method Markov process can be used to predict the future by these occurred events.

Let the state space of a Markov chain $\{X_n\}$ be S, the current state be i and the next state be j, then the transition probability is written as

$$P_{ij}=prob\{X_{k+1}=j\mid X_k=i\} \qquad (i,j\in S,\quad k=0,\ 1,\ 2,\ \ldots) \tag{16}$$

where the P_{ij} is independent of k.

The transition probability matrix is $P=(p_{ij})$, then the elements of the matrix P satisfy the following two properties:

$$(1)\quad p_{ij}\geq 0,\quad \forall\quad i,j\in S \quad ; \quad (2)\ \sum_{j\in S}p_{ij}=1,\quad \forall i\in S \tag{17}$$

Experiment results

The experiment focus on the data sets $x^{(0)}$=(79.9,79.94,79.96,80,80.29,81.14,81.02,81.57,81.93, 82.88,83.38,82.87,83.3,83.58,83.81,84.45,83.2,83.36,83.08,82.43,83.13,83.2,82.47,83.06,83.45) (Cai, 2010), which was researched use GM(1,1,). In this paper, there are three experiment results show as Tables 1-2, Table 1 list the all prediction results of GM(1, 1) ,GM (2, 1) and GMM(2,1), Table 2 shows the mean error and accuracy of all prediction models. From these two tables, it is obvious that the performance of GMM(2,1) is the best among all the prediction models.

Table 1 real data series and predicted results

	GM(1,1)			GM(2,1)			GMM(2,1)		
$x^{(0)}(k)$	$\hat{x}(k)$	error	$e(k)$	$\hat{x}^{(0)}(k)$	error	$e(k)$	$\hat{x}^{(0)}(k)$	error	$e(k)$
79.94	79.947764	-0.00776	0.000009	79.71692	0.22308	0.0027	79.91201	0.02799	0.0003501
79.96	80.05695	-0.09695	0.001213	79.89958	0.06042	0.0075	80.10151	-0.14151	0.001769
80	80.189958	-0.06197	0.000775	80.16144	-0.18996	0.0024	80.10019	-0.10019	0.0012523
80.29	80.351971	-0.06197	0.213693	81.84617	-1.55617	0.01938	80.11	0.18	0.00224187
81.14	80.549319	0.59068	0.007279	81.09011	-0.04989	0.00614	81.29813	-0.15813	0.0019488
81.02	80.789708	0.23029	0.002842	81.14321	-0.12321	0.00152	80.91675	0.10325	0.001274
81.57	81.082527	0.48747	0.005976	81.51513	0.05487	0.00672	82.07613	-0.50613	0.0062048
81.93	81.43921	0.49079	0.00599	82.01003	-0.08003	0.00977	82.0221	-0.0921	0.001124
82.88	81.873684	1.00631	0.012148	82.80782	0.07218	0.0087	83.18109	-0.30109	0.0036328
83.38	82.402918	0.97708	0.011718	83.39813	-0.01813	0.00217	83.54119	-0.16119	0.001933

Table 2 Performance of three models

Model	Mean error	Accuracy
GM(1,1)	0.050405	0.949595
GM(2,1)	0.04711	0.95289
GMM(2,1)	0.002173	0.99787

Conclusion

This paper based on the Grey Theory and Markov Theory, the Two Order Grey Markov Model(GMM(2,1)) is established. From the above analysis can be obtained the GM (1, 1) model is not satisfied to the random fluctuations larger sequence, the prediction results will be high or low, the prediction accuracy is low too. The GM (2, 1) model is modified by adding a reflecting lifting situation factors on GM(1,1) model , make the prediction results more accurate. But the GM (2, 1) model to predict the data which is related to time, the forecast result and the real value of total there is a unit of time delay. So in this paper GMM(2,1) model is established, its essence is the shock characteristics of Markov transition probability matrix was introduced into the grey GM (2, 1)model, the original GM (2, 1) index part of the model does not change, but modified the grey period. Thus the GMM(2,1) model has the characteristics of higher accuracy than GM (1, 1) and GM (2,1).

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