Research on Fixed-gain Range Tracking Loop

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Abstract. The range tracking loop is an important part of measuring Radar. Based on the mathematic model, the analysis between αβ-tracker and αβγ-tracker is studied. The effect of loop parameters is analyzed. The formula of αβγ-tracker's equivalent noise bandwidth (ENBW) is derived. The simulation results provide the theoretical foundation for engineering designs.

Introduction

The Range tracking tracker offers continuous data to Radar. It has great influence on Rader performance. Algorithms of range tracking tracker are now studied widely and deeply. The kalman tracker is studied in [1]～[4], and the implement of αβ-tracker is introduced in [5]～[7]. The αβ-tracker can't eliminate the steady- state error caused by target's acceleration, and the error caused by the change of acceleration is accumulating with the tracking-time. The αβγ-tracker which is studied in [10]～[12] can eliminate the steady- state error caused by target's acceleration and reduces the acceleration changing error.

In this paper, the mathematic model of αβγ-tracker which is a type of Fixed-gain trackers is built. The expression of equivalent noise bandwidth（ENBW） is derived. The loop parameters' influences on tracking performance are analyzed between αβγ-tracker and αβ-tracker. It supplies the theoretical designs for engineering implements.

Range tracking loop

The αβγ-tracker

The range-tracking model in Radar system can be described as $x=[R\,\dot{R}\,\ddot{R}]^T$,where the R、$\dot{R}$、$\ddot{R}$ are target's range, velocity and acceleration[8][9][10].

The αβγ-tracker provides the information of target's range, velocity and smoothing predicted acceleration. It contains two loops: the predicting loop and the smoothing loop.

Smoothing loop expression:

$$\begin{cases} R_s(kT)=R_p(kT)+\alpha\left(R_m(kT)-R_p(kT)\right) \\ R'_s(kT)=R'_s(KT-T)+TR''_s(KT-T)+\dfrac{\beta}{T}\left(R_m(kT)-R_p(kT)\right) \\ R''_s(kT)=R''_p(kT)+\dfrac{2\gamma}{T^2}\left(R_m(kT)-R_p(kT)\right) \end{cases} \quad (1)$$

Predicting loop expression:

$$\begin{cases} R_p(kT)=R_s(kT-T)+TR'_s(kT-T)+T^2R''_s(kT-T) \\ R'_p(kT)=R'_s(kT-T)+TR'_s(kT-T) \\ R''_p(kT)=R''_s(kT-T) \end{cases} \quad (2)$$

Where the "p"、 "s" and "m" represent predicted, smoothing and real-time data. The "T" is tracker's sampling period. R(kT) 、R′(kT) and R″(kT) represent the information of target's range,

velocity and acceleration in k-time sampling period. The implementation of αβγ-tracker is illustrated in Fig 1.

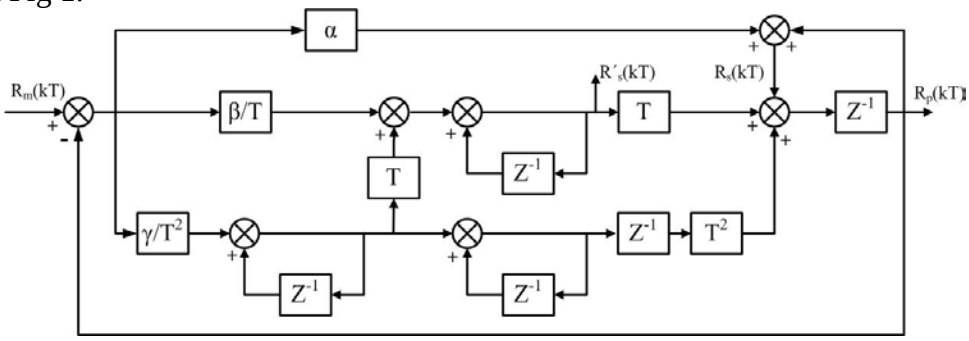

Fig 1 αβγ Range Tracking Tracker

The filtering parameters of range, ,velocity and acceleration sub-loop are α,β and γ, the expression of closed loop transfer function is as below:

$$H(z)=\frac{(\alpha+\beta+\gamma)z^2+(-\beta-2\alpha+\gamma)z+a}{z^3+(-3+\alpha+\beta+\gamma)z^2+(3-\beta-2\alpha+\gamma)z-(1-\alpha)} \quad (3)$$

The characteristic equation:

$$f(z)=z^3+(-3+\alpha+\beta+\gamma)z^2+(3-\beta-2\alpha+\gamma)z-(1-\alpha) \quad (4)$$

According to Routh principle, the stability condition of αβγ-tracker is:

$$\begin{cases}4-2\alpha-\beta>0\\2\alpha-\gamma>0\\\beta,\gamma>0\end{cases} \quad (5)$$

The αβ-tracker

The αβ-tracker can be consider as the special case that γ=0 in αβγ-tracker[12], its closed loop transfer function is

$$H(z)=\frac{(\alpha+\beta)z-a}{z^2+(-2+\alpha+\beta)z+(1-\alpha)} \quad (6)$$

The loop stability condition: $\begin{cases}\alpha>-\beta\\\beta>0\end{cases}$ (7)

The ENBW

Considering the input and system noise influence, choosing adaptive ENBW which is confirmed by loop parameters is very important. The tracking error and response characteristic of the tracker can be changed by selecting the matched ENBW. The system will have same self-adapting capability .[5][6][7]

The ENBW equation is:

$$B_n=\frac{1}{2\pi}\int_{-\infty}^{\infty}|H(\omega)|^2\,d\omega=\frac{1}{2j\pi T}\oint_{\text{单位圆}}H(Z)H(-Z)Z^{-1}dZ \quad (8)$$

Where H(w) is system's closed loop frequency characteristic, and $Z=\frac{1+w}{1-w}$. The αβγ-tracker's B_n becomes, in succession

$$B_n=\frac{1}{2\pi T}\int_{-j\infty}^{j\infty}\frac{H(w)}{1+w}\frac{H(-w)}{1-w}dw \quad (9)$$

And $w=ju$, the equation becomes:

$$B_n=\frac{1}{\pi T(4-2\alpha-\beta)^2}\int_0^{+\infty}\frac{(2\alpha+\beta)^2u^4+(\beta^2+\gamma^2-4\alpha\gamma)u^2+\gamma^2}{u^6+\frac{(4\alpha^2+4\alpha\beta+2\beta^2-4\alpha\gamma+\gamma^2-8\beta)}{(4-2\alpha-\beta)^2}u^4+\frac{\beta^2-4\alpha\gamma+2\gamma^2}{(4-2\alpha-\beta)^2}u^2+\gamma^2}du \quad (10)$$

Where

$$K_0=(2\alpha+\beta)^2, K_1=\beta^2+\gamma^2-4\alpha\gamma, K_2=\gamma^2$$

$$K_3=\frac{(4\alpha^2+4\alpha\beta+2\beta^2-4\alpha\gamma+\gamma^2-8\beta)}{(4-2\alpha-\beta)^2}, K_4=\frac{\beta^2-4\alpha\gamma+2\gamma^2}{(4-2\alpha-\beta)^2}, K_5=(4-2\alpha-\beta)^2$$

The relation for B_n is now

$$B_n=\frac{1}{\pi TK_5}\int_0^{+\infty}\frac{K_0u^4+K_1u^2+K_2}{u^6+K_3u^4+K_4u^2+K_2}du=\frac{1}{2TK_5}\bullet\left[\frac{A}{u^2+B}+\frac{C}{u^2+D}+\frac{E}{u^2+F}\right] \quad (11)$$

where

$$\begin{cases}BDF=K_2\\ BF+DF+BD=K_4\\ B+D+F=K_3\\ A+C+E=K_0\\ AF+DA+BC+FC+EB+ED=K_1\\ AFD+CBF+EBD=K_2\end{cases}\Rightarrow\begin{cases}A=\frac{B^2K_0-BK_1+K_2}{(B-D)(B-F)}\\ C=\frac{D^2K_0-DK_1+K_2}{(B-D)(F-D)}\\ E=\frac{F^2K_0-FK_1+K_2}{(D-F)(B-F)}\end{cases} \quad (12)$$

The B_n is the solution of equations:

$$\begin{cases}B_n=\frac{1}{2TK_5}\bullet\frac{K_0\sqrt{K_2}b+K_1\sqrt{K_2}+K_2a}{c}\\ b^2=K_4+2\alpha\sqrt{K_2}\\ a^2=K_3+2b\\ ab=\sqrt{K_2}+c\\ a、b、c>0\end{cases} \quad (13)$$

The ENBW of αβ-tracker is as below: [5][12]

$$B_n=\frac{(2\beta+\alpha\beta+2\alpha^2)}{2T(4\alpha-2\alpha^2-\alpha\beta)} \quad (14)$$

Analysis of tracking loop performance

Loop parameters selecting

On the premise of loops stabilizing, the ENBW is confirmed by selecting loop parameters. Two types of parameters selecting are introduced in this paper:

Type I parameters selecting method is to get minimum tracking mean square deviation: $\alpha=\xi$, $\beta=\frac{\alpha^2}{(2-\alpha)}$ in αβ-tracker, $\alpha=\xi$, $2\beta=\alpha(\alpha+\beta+\gamma)$, $\beta^2=4\alpha\gamma$ in αβγ-tracker. [9][12]

Type II parameters selecting method is critical damping: $\alpha=1-\xi^2$, $\beta=(1-\xi)^2$ (ξ is smoothing factor, when $\xi\to1$, deep smoothing) αβ-tracker, $\alpha=1-\xi^3$, $\beta=1.5(1-\xi)^2(1+\xi)$, $\gamma=\frac{(1-\xi)^3}{2}$ in αβγ-tracker. [9][10]

The relations between B_nT and ξ for αβγ-tracker and αβ-tracker are illustrated as follows,: Fig 2 is type I parameters selecting method, and Fig 3 is type II parameters selecting method.

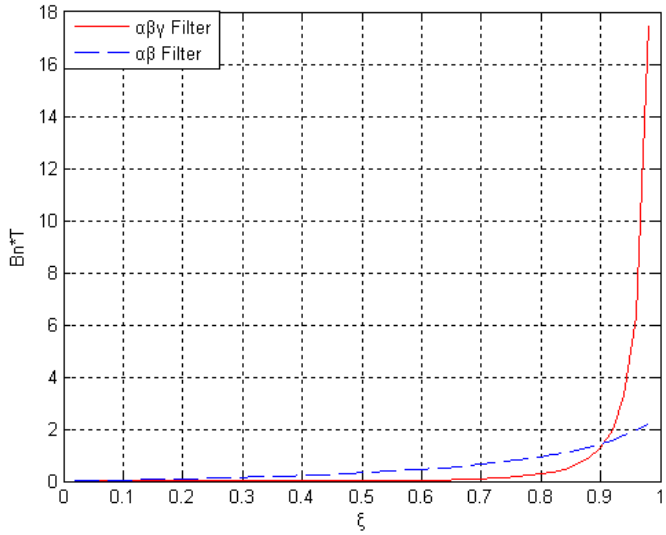

Fig 2 Relation of Bn•T and ξ(Type I parameters selecting method)

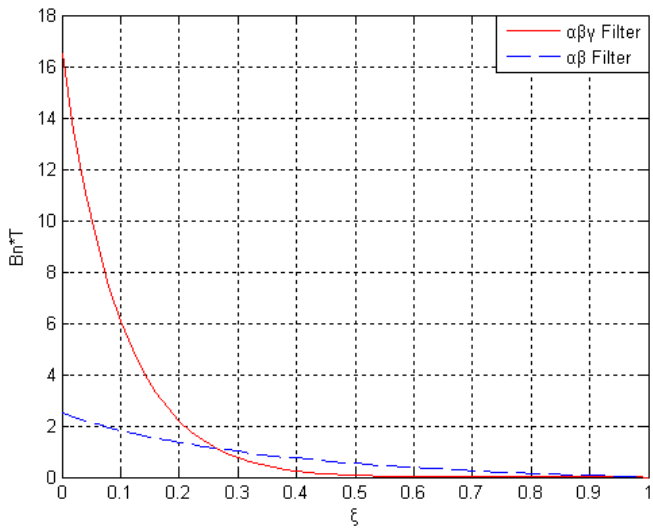

Fig 3 Relation of Bn•T and ξ(Type II parameters selecting method)

Tracking errors

Assuming the target's acceleration is A, and the acceleration changing rate is B. The steady state error of αβ-tracker caused by acceleration is $e=\lim_{z\to 1}\frac{z-1}{z}\cdot\frac{A}{1+H(z)}=\frac{A}{\beta F_r^2}$, $F_r=1/T$. The steady state error of αβγ-tracker caused by acceleration changing is $e=\lim_{z\to 1}\frac{z-1}{z}\cdot\frac{B}{1+H(z)}=\frac{B}{\gamma F_r^3}$.

The tracking error caused by thermal noise is $\sigma=\frac{\tau\bullet c}{2k\sqrt{(S/N)_0(F_r/B_n)}}$

Where τ is Radar impulse width, c is light velocity, $(S/N)_0$ is Signal-to-Noise performance, and k is ranging normalized slope coefficient (k = 1.4)

As the ENBW becoming larger, the thermal noise error raising, and the steady state error reducing. Therefore, the ENBW selection is the key to tracking performance.

Simulation analysis

Assuming Radar's parameters are as follows:

$F_r=300Hz$, $\tau=1us$, $(S/N)_0=12dB$, and $A=1000m/s^2$, $B=40m/s^3$, and the Gauss noise

is added in tracking process.

ENBW Selection

The thermal noise error is definite, when the ENBW is established. Fig 4 illustrates the relations between thermal noise error and ENBW. Assuming the $\sigma \leq 7$m, the ENBW should be under 40Hz.

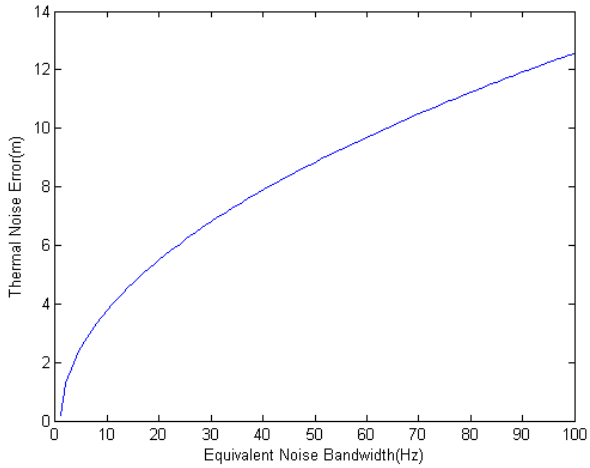

Fig 4 Relations of ENBW and Thermal Noise Error

The tracker parameters selections of two types are in table 1 and table 2.

Table 1: Tracker Parameters Selection (Type I parameters selecting method)

ENBW	αβ-tracker parameters selection	αβγ-tracker parameters selection
2Hz	α=0.02，β=0.0002（ξ=0.02）	α=0.45，β=0.14，γ=0.01（ξ=0.45）
10Hz	α=0.09，β=0.0042（ξ=0.09）	α=0.61，β=0.28，γ=0.18（ξ=0.61）
20Hz	α=0.16，β=0.0139（ξ=0.16）	α=0.68，β=0.38，γ=0.23（ξ=0.68）

Table 2: Tracker Parameters Selection (Type II parameters selecting method)

ENBW	αβ-tracker parameters selection	αβγ-tracker parameters selection
2Hz	α=0.02，β=0.0001（ξ=0.99）	α=0.71，β=0.29，γ=0.020（ξ=0.66）
10Hz	α=0.098，β=0.0025（ξ=0.95）	α=0.83，β=0.47，γ=0.046（ξ=0.55）
20Hz	α=0.19，β=0.01（ξ=0.9）	α=0.87，β=0.54，γ=0.059（ξ=0.51）

Tracking performance analysis

1) The αβγ-tracker and αβ-tracker

Fig 5 shows two types of trackers' steady state errors by type II parameters selecting method (ENBW =10 Hz).

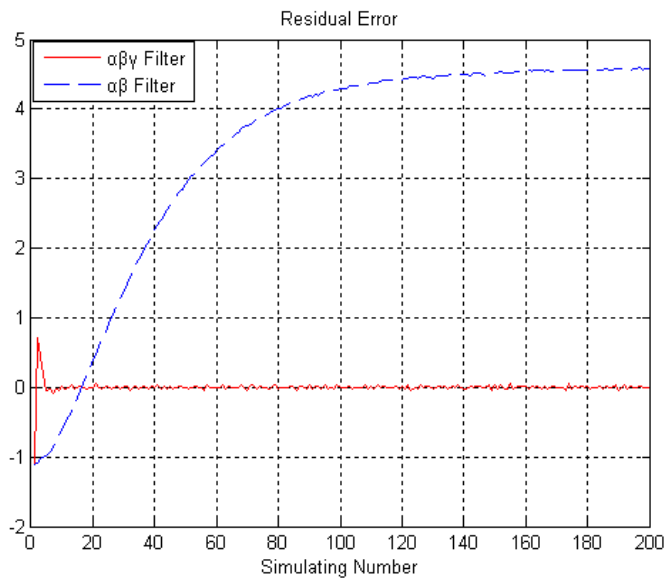

Fig 5 Performances of Tracking Loops

From Fig 5, the tracking error of αβ-tracker is cumulating with the simulating steps, the steady state error caused by acceleration is $\frac{A}{\beta F_r^2} = \frac{1000}{0.0025*300^2} = 4.44\text{m}$. The αβγ-tracker's tracking error which is depended on noise intensity converges fast.

So , the tracking error of αβγ-tracker is much less than αβ-tracker in the same ENBW condition. Comparing with αβ-tracker, the αβγ-tracker 's ENBW selecting range is much more flexible. In this simulating sample, we can select the αβγ-tracker 's ENBW=2Hz., and in the same ENBW the initial tracking error of αβ-tracker is 111m, which is overranged.

2) Tracking performance of αβγ-tracker

In type I parameters selecting method, the oscillating range and amplitude are becoming wider and larger as the ENBW getting smaller. The tracking will lost when the ENBW=4Hz. Fig 6 illustrates the tracking position residual error in the condition of ENBW=4Hz and 40Hz.

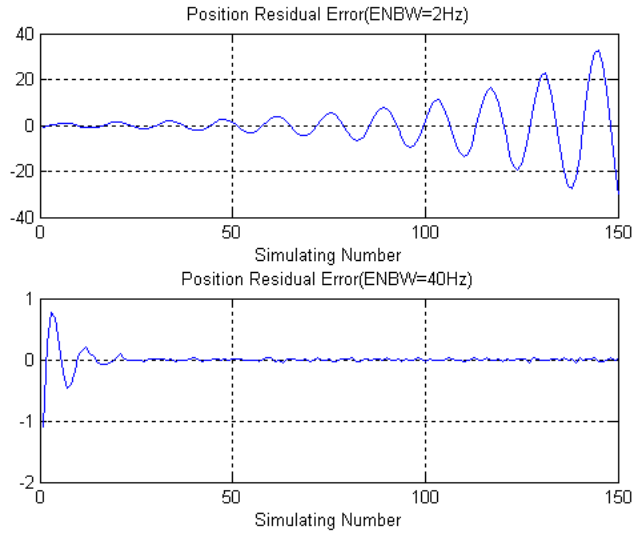

Fig 6 Tracking Loop Performance (Type I parameters selecting method)

In type II parameters selecting method, with the ENBW becoming larger, the tracking error converges faster, and the overshoot gets larger. The tracking is steady in low ENBW condition. Fig 7 illustrates the tracking position residual error in the condition of ENBW=4Hz and 40Hz.

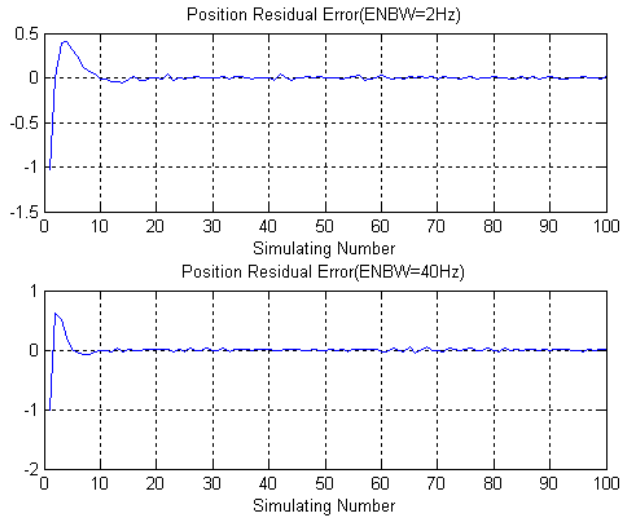

Fig 7 Tracking Loop Performance (Type II parameters selecting method)

By analyzing simulation results, the type II parameters selecting method has the better tracking performance than type I parameters selecting method for αβγ-tracker.

Conclusion

In this paper, the relationship between ENBW and loop parameters selection is analyzed. By tracking process simulation, we analyze the tracking performance of αβ-tracker and αβγ-tracker in two different loop parameters selecting method. The results can be utilized in engineering design of Radar range tracking.

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