A bubble population model in the analog ship wake

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Keywords: Bubble population model, ship wake, attenuation

Abstract. When modeling sound propagation through the uppermost layers of the ocean, the presence of bubble clouds cannot be ignored. Their existence can convert a range-independent sound propagation problem into a range-dependent one. Based on available test data and within the framework of previous work, a model of bubble population in the analog ship wake is developed. The model parameters are calculated and the model error is analyzed. This model, which coincide the test data, is then can be used to help modeling ship wake bubble distribution.

Introduction

Microbubbles have important affection in the realm of oceanography, ultrasonics and hydroacoustics. When sound propagates through the upper layers of the ocean, the presence of microbubbles cannot be ignored. Their range can be more than 10 meters underwater and the void fraction can reach 10^{-3} [1-3]. For bubble is a strong sound scatter and its scattering cross section is hundreds times of its geometric cross section, the large number of microbubbles change the acoustic impedance characteristic near-sea and influent the sound propagation.

The effects of microbubbles on acoustic propagation in water have been investigated since the 1940s. Meanwhile, the research of the bubble population model in near-sea and in the ship wake also got comprehensive attention.

In 1933, the natural frequency of a bubble was derived by M. Minnaert [4]. It showed that the sound generated by gas bubbles in liquids is associated with simple volume pulsations of the bubble without change of shape. The bubble behaves as a simple damped oscillating system with one degree of freedom. Because of the complex environment, Charles Devin, Marshall V. Hall and Michael A. Ainsile explored the resonance frequency of the bubbles in water and the damping coefficient [5-7].

In 1977, Medwin presented the bubble population model attenuating as power index between 2~6 [8]. Taken the influence of wind speed and depth into consideration, Hall proposed the near-sea population density spectrum of wind-generated bubbles in 1989 [6]. It is a kind of comprehensive bubble spectrum for a range-independent or uniform bubble layer based on published ocean measurements and previous parameterizations. Since then, Novarini developed a model for the range and depth dependence of the bubbly environment in 1998 [9], which generated a possible realization of the bubbly environment and then used to calculate the frequency-dependent change in the sound speed and attenuation induced by the presence of the bubbles plumes. The bubble population model in the ship wake has been built by Rapids [10] for the use of the characteristics of ship wake based on measurements made by Trevorrow [11].

Although some of the bubble population model in the ocean have been present in recently years, it is still rare to see the information on the bubble distribution in the ship wake.

In this paper, a simple model is set forth to parameterize the bubble population of analog ship wake. The model aims to provide acousticians with a method to estimate the rang- and depth-dependent sound speed and attenuation induced by the bubbles close to the surface. The bubble distribution is parameterized. In the development of this model, we resort to Novarini's model as the basic framework and couple with the data from analog ship wake test result. Thus, the developed model can assess the implications of surface-generated bubbles on sound propagation in the ocean.

Bubble population model

As presented above, several near-sea bubble population models had been presented. In this section four typical kinds of them will be investigated.

Medwin and Breit's model

Medwin and Breit generalized the near-sea bubble density population model by acoustical measurements in 1982, which expressed as

$$N(a) = 7.8\times10^{8}\left(\frac{a}{1\mu m}\right)^{-2.7}, \quad 30\mu m \le a \le 270\mu m \tag{1}$$

Marshall V. Hall's model

Marshall V. Hall proposed for the population density spectrum of wind-generated bubbles in the sea based on measurements made by Johnson and Cooke and by Thorpe in 1989, which expressed as

$$N(a) = N_0 G(a,z) U(w) Y(z,w), \quad 16\mu m \le a \le 1000\mu m \tag{2}$$

where N_0 is the value of N at peak, $N_0=1.6\times10^{10}\,m^{-4}$.

$G(a, z)$ is proposed as a fit to the Johnson and Cooke's data is given by

$$G(a,z) = \begin{cases} (a/a_1)^2, a < a_1 \\ 1, a_1 \le a \le a_2 \\ (a_2/a)^p, a_2 < a \end{cases}$$

in which the break points $a_1=(34+1.24z)$ μm, $a_2=1.6a_1$ μm, p is the slope of the curve, which takes the form $p=4.37+(z/2.55)^2$, where z is the depth in meter.

U is the wind-generated function. Considering the various results obtained for the wind speed dependence of population density spectrum level, it is concluded that

$$U(w) = (w/13)^3$$

in which w is the wind speed. This form is a reasonable description of $U(w)$ in the near-surface bubble layer.

Y is the depth dependent function. At bubble radius in the neighborhood of 50μm, it can be represented to a good approximation by

$$Y(z,w) = \exp(-z/L(w))$$

where $L(w)$ is the depth constant of bubble layer, a bilinear curve that fits well is

$$L(w) = \begin{cases} 0.4, w \le 7.5m/s \\ 0.4 + 0.115(w-7.5), w > 7.5m/s \end{cases}$$

Novarini's model

Novarini proposed a new kind of bubble population model in 1998.

$$N(a,z,w) = N_0 G(a) U(w) Z(z) \tag{3}$$

This model generates a possible realization of the bubbly environment, and is used to calculate the frequency-dependent change in the sound speed and attenuation induced by the presence of the bubbles plumes.

For the bubble density spectrum of the β-plume, set $N_{\beta0}=2.0\times10^{7}\mu m^{-1}m^{-3}$, at the surface and for a wind speed of 15m/s.

Table 1 Estimated properties of the bubble formations[12]

	β-plume	γ-plume
Horizontal area (m^2)	8~50	100~500
Vertical scale (m)	0.8	0.42~0.75
Time scale (s)	3.5~4.3	100~1000
Void fraction	10^{-4}~10^{-3}	10^{-7}~10^{-6}
n ($\mu m^{-1}m^{-3}$)	10^5~10^6	10^2~10^4

For bubbles larger than about 60μm, the spectral sharp function can be written as,

$$G_\beta(a)=\begin{cases}0, a<a_{\min}\\ (a/a_1)^3, a_{\min}\le a<a_1\\ 1, a_1\le a<a_2\\ (a/a_2)^{-4}, a_2\le a<a_3\\ (a_3/a_2)^{-4}(a/a_3)^{-2.6}, a_3\le a<a_{\max}\end{cases}$$

in which a_{min}=10μm, a_1=15μm, a_2=20μm, a_3=(54.4+1.984z) μm, a_{max}=1000μm.

Follow Hall, the wind-generated function takes the form by,

$$U(w)=(w/13)^3$$

The depth dependent function controls the vertical extent of the plume axis. For β-plume, a uniform bubble distribution in depth is been assumed, hence

$$Z_\beta(z)=\begin{cases}1, z\le z_{\beta\max}\\ 0, z>z_{\beta\max}\end{cases}$$

in which $z_{\beta max}$ is the maximum penetration depth. For β-plume, $z_{\beta max}$ =$1.23\times10^{-2}w^2$.

For the bubble density spectrum of the γ-plume, set $N_{\gamma 0}$=$6.0\times10^5$$\mu m^{-1}m^{-3}$.

Novarini adopt Hall's parameterization of Johnson and Cooke's data as the slope for the γ-plume, therefore, the spectral sharp function is given by

$$G_\gamma(a)=\begin{cases}0, a<a_{\min}\\ (a/a_1)^3, a_{\min}\le a<a_1\\ 1, a_1\le a<a_2\\ (a/a_2)^{-4}, a_2\le a<a_3\\ (a_3/a_2)^{-4}(a/a_3)^{-p}, a_3\le a<a_{\max}\end{cases}$$

in which a_{min}=10μm, a_1=15μm, a_2=20μm, a_3=(54.4+1.984z) μm, a_{max}=1000μm, p=4.37+$(z/2.55)^2$.

The depth dependent function has been assumed to be of the form,

$$Z_\gamma(z)=\exp(-z/d_\gamma)$$

in which d_γ=0.6(w-5)+3.5.

Here gives the simulation of these four kinds of bubble population model.

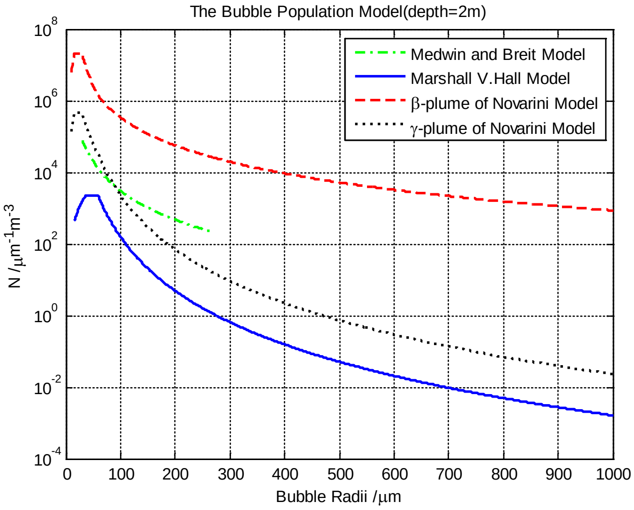

Fig.1 Four kinds of bubbles population model simulation

Bubble density spectrum population

If take the influence of resonance bubble into account only, the population density spectrum of bubbles can be written as

$$n(z,a)\approx\frac{4.6\times10^{-6}f^{3}\alpha(f)}{1+0.1z} \tag{4}$$

where $\alpha(f)$ is the acoustic attenuation coefficient(dB/m), z is the depth of bubble. Hence, the bubble density spectrum can be obtained by Eq.4 at resonance bubble.

The acoustic attenuation coefficient propagate in the bubble layer is the function of density spectrum and frequency, which takes the form

$$\alpha(f)=\frac{20}{\ln 10}\frac{c_0}{f}\int_0^{\infty}\frac{a\delta n(z,a)\,da}{\left(f_r^2/f^2-1\right)^2+\delta^2} \tag{5}$$

where f_r is the resonance frequency of bubbles, δ is the damping coefficient of bubbles, characterized as the sum of contributions from thermal diffusion, acoustic radiation, and the liquid viscosity.

The damping coefficient of a free bubble of radius a, for a given frequency ω and depth, is given by [6]

$$\delta=\left(\frac{a_r}{a}\right)^2\frac{\mathrm{Im}(B)}{\mathrm{Re}(B)}+\frac{\omega a}{c_0}+\delta_v \tag{6}$$

where the first term is the component due to thermal diffusion in the gas bubble, the second term is the component due to acoustic radiation, and the third term is due to liquid viscosity.

The bulk modulus is given by

$$B=\gamma P_0\left(1-3i(\gamma-1)\Big/\left(2\phi^2a^2\right)\left\{(1+i)\phi a\coth\left[(1+i)\phi a\right]-1\right\}\right)^{-1} \tag{7}$$

where γ is the specific heat ratio of the gas in the bubble, γ=1.4, P_0 is the hydrostatic pressure at the bubble, and

$$\phi=(\pi f/D)^{1/2} \tag{8}$$

in which D is the thermal diffusivity of the gas, $D=K_{gas}/(\rho_{gas}(a)C_p)$, K_{gas} is the thermal conductivity and C_p is the specific heat capacity at constant pressure. For air at atmospheric pressure, $D\approx1.84\times10^{-5}\mathrm{m}^2\mathrm{s}^{-1}$. c_0 is the speed of sound in liquid, a_r is the resonance radius, for a frequency f and a given depth, satisfies the equation

$$\mathrm{Re}\left[B\left(\phi a_r\right)\right]-\rho\left(2\pi f a_r\right)^2/3=0 \tag{9}$$

where ρ is the density of the surrounding medium.

The term δv can be expressed as [13]

$$\delta_v=\frac{4\mu}{\rho\omega a^2} \tag{10}$$

in which μ represents the coefficient of molecular viscosity of seawater, at 10 °C, $\mu \approx 1.4\times10^{-3}$ kg/ms [14].

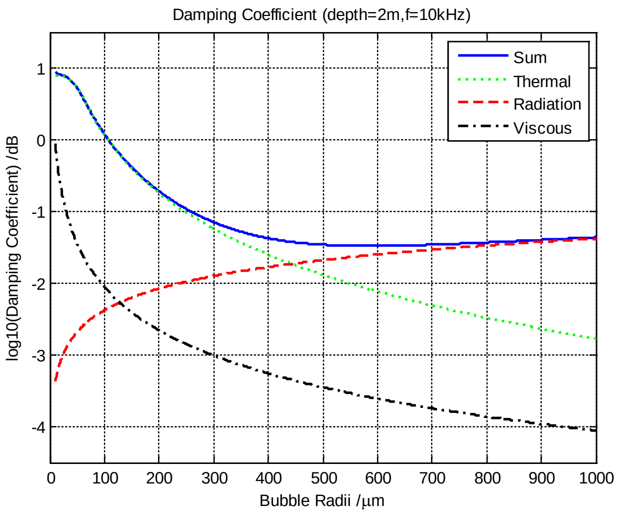

Fig.2 The damping coefficient of bubbles

Fig.2 shows the damping coefficient of bubbles varies with bubble radius at 10 kHz and depth of 2 m. The real (blue) line represents the sum of the damping coefficient, the dotted (green) line represents the thermal diffusion damping coefficient, the dashed (red) line represents the radiation damping coefficient and the dashed-dotted (black) line represents the viscous damping coefficient respectively. When a<350μm, the thermal diffusion is the main constituent of the damping coefficient, and the radiation becomes the main constituent as a>350μm. The damping coefficient reduced and then increased slowly as the bubble radius increased. It must be pointed that the bubble radius is the function of incident frequency and depth as the main constituent of the damping coefficient changed.

Developed model

According to the test data of attenuation coefficient of the analog ship wake and the bubble population models mentioned above, a simple model that meets this test data and predicts realizations of a bubble population is been developed. The analog ship wake produced by the interaction of towed ammunition and water.

Table 2 Test data of $\alpha(f)$ (depth=8m)

f (kHz)	4	5	6	7	8	9	10
$\alpha(f)$ (dB/m)	0.21	0.26	0.29	0.29	0.29	0.37	0.36

Based on the test phenomenon and the data from Table 1, we choose the model of γ–plume as the basis framework of our model. According to Eq.4 and Table 2, the model parameters has been calculated, which are: N_0=9.87×10^4, p=-3.45. Here we get the spectral sharp function,

$$G(a)=\begin{cases}0, a<a_{\min}\\(a/a_1)^3, a_{\min}\le a<a_1\\1, a_1\le a<a_2\\(a/a_2)^{-4}, a_2\le a<a_3\\(a_3/a_2)^{-4}(a/a_3)^{-3.45}, a_3\le a<a_{\max}\end{cases}$$

and the depth-dependent function $Z(z)$ and wind-generated function $U(w)$ are as same as the parameters in the γ-plume model.

The model is present in Fig.3. In the figure, the real (blue) line represents the developed bubble population model, and the dashed-dotted (red) line represents the values calculated by Eq.4 and Table 2. By compare this two curves we can get the conclusion that the developed model meets the test values well.

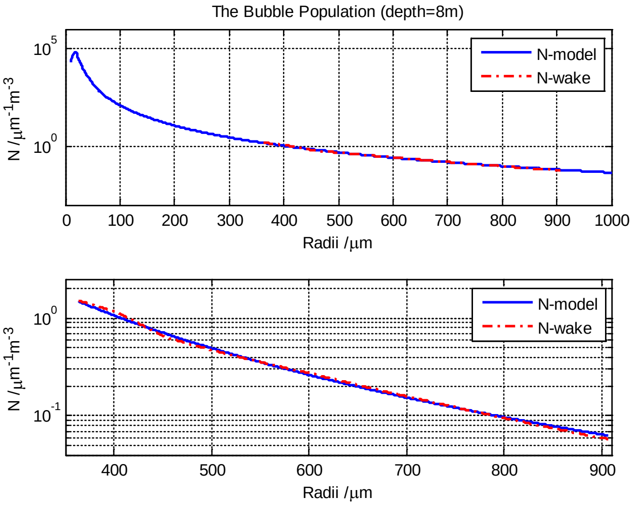

Fig.3 The bubble population model

Fig.4 shows the error of bubble population between the developed model and the test values, and the details are given in Table 3. From Fig.4 and Table 3 we can get the point that the error between model and the test result is less than 9.6%, and the mean error is 3.9%, which means this developed model can represent the bubble population distribution in analog ship wake in the depth of 8 m.

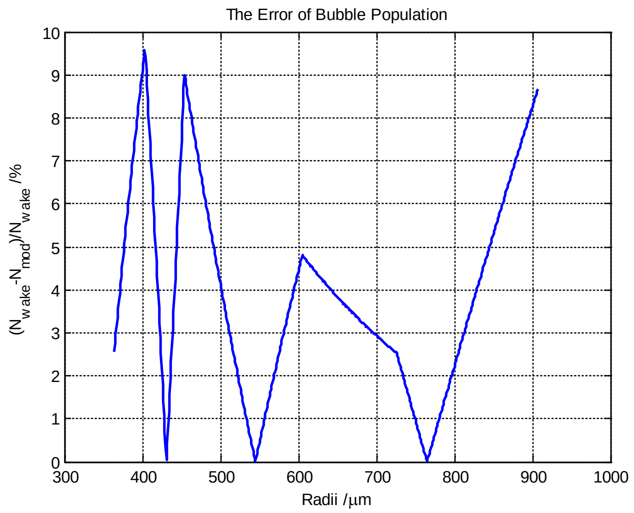

Fig.4 The error of bubble population

Table 3 Error result

	average	mean square deviation	max @ location
Error (%)	3.9	2.3	9.58@404μm

Summary

In this paper, the bubble population model and its estimate method had been described briefly. Based on the analog ship wake test data, and with the use of the previous near-sea bubble population models, we developed a simple model that predicts realizations of a range- and depth-dependent bubble population. The result of analysis shows that the developed bubble population model is quite in agreement with the bubble population from analog ship wake test data. This bubble population model we developed can be use to help modeling ship wake bubble distribution.

Reference

[1] E. Lamarre, W. K. Melville: Sound-speed measurements near the ocean surface. Journal of the Acoustical of American, Vol. 96(1994), p. 3605-3616

[2] B. K. Choi, S. W. Yoon: Acoustic bubble counting technique using sound speed extracted from sound attenuation. Journal of Oceanic Engineering, VOL. 26(2001), p. 125-130.

[3] V. Leroy, A. Strybulevych, J. H. Page and M. G. Scanlon: Sound velocity and attenuation in bubbly gels measured by transmission experiments. Journal of the Acoustical of American, Vol. 123(2008), p. 1931-1940

[4] M. Minnaert: On musical air-bubbles and the sounds and the sounds of running water. Philos. Mag. 16(1933), p. 235-248

[5] C. Devin: Survey of thermal, radiation, and viscous damping of pulsating air bubbles in water. Journal of the Acoustical of American, Vol. 31(1959), p. 1654-1667

[6] M. V. Hall: A comprehensive model of wind-generated bubbles in the ocean and predictions of the effects on sound propagation at frequencies up to 40kHz. Journal of the Acoustical of American, Vol. 86(1989), p. 1103-1117

[7] M. A. Ainsile, T. G. Leighton: Review of scattering and extinction cross-section, damping factors, and resonance frequencies of a spherical gas bubble. Journal of the Acoustical of American, Vol. 130(2011), p. 3184-3208

[8] H. Medwin: Acoustic determinations of bubble-size spectra, Journal of the Acoustical of American, Vol. 62(1977), p. 1041-1044

[9] J. C. Novarini, R. S. Keiffer, and G. V. Norton: A model for variations in the range and depth dependence of the sound speed and attenuation induced by bubble clouds under wind-driven seas. IEEE J. Oceanic Eng., Vol. 23(1998), p. 423-438

[10] B. R. Rapids, R. L. Culver: An acoustic ship wake for propagation studies. Applied Research Laboratory Technical Memorandum, File No. 00-068, (2000)

[11] M. V. Trevorrow, S. Vagle and D. M. Farmer: Acoustic measurements of microbubbles within ship wakes. Journal of the Acoustical of American, Vol. 95(1994), p. 1922-1930

[12] E. C. Monahan: Natural Physical Sources of Underwater Sound. B. R. Kerman, Ed. Boston, MA: Kluwer, p. 503, (1993)

[13] K. W. Commander, A. Prosperetti: Linear pressure waves in bubbly liquids: Comparison between theory and experiments. Journal of the Acoustical of American, Vol. 85(1989), p. 732-746

[14] S. Pond, G. L. Pickard: Introductory Dynamic Oceanography. Pergamon, Oxford, England, p. 229, (1978)