Parameter design of the two vertical arrays system for separation of Eigenrays in shallow water environment

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Abstract. This study numerical demonstrates the principle of parameter design of the two vertical source-receiver arrays system for separation of eigenrays in shallow water multipath environment. The two arrays performed transmission and reception of acoustic pulse signal repeatedly, and use a double beamforming algorithm to obtain arrival angle and launch angle of each eigenray simultaneously. Comparison of parameter design is made between the situations that the range between the two arrays are 100m and 1km. The result show that the arrival angle, launch angle and arrive time of adjacent eigenray are close to each other as the source-receiver range increase, and the selection of operation frequency, element spacing, array length of the two arrays system should make the system to have a narrower main lobe angle.

Introduction

Underwater acoustic multipath propagation phenomenon is typical and common in the shallow water environment. It means that acoustic pulse signal transmitted by a source reach the receiver through propagation along a lot of eigenrays between the source and the receiver. Each eigenray carry part of acoustic intensity and has a different travel time delay. Due to multipath propagation, the arrival signal waveform became distortion, fluctuation, de-correlation and frequency smearing, and this characteristics are negative effects to the performance of the active sonar^[1]. Fortunately, multipath propagation can be taken advantage of in underwater acoustic engineering area owing to the feature that it produce wider underwater volume influence comparing with direct propagation. As a successful application case, acoustic ocean tomography technique^[2] found by W.H.Munk invert physical ocean parameters such as sound velocity, temperature and current speed through multipath travel time perturbation. One of the most important problems in tomography implement process is distinguishing stable eigenrays, the conventional approach to identify a eigenray uniquely is finding it in the arrival signal waveform directly according to its arrival time. However, in shallow water environment, it is difficult to separate eigenrays in the time domain, because the time delay difference between adjacent eigenray decrease as the range between the source and receiver increase, the adjacent eigenray can not be separated when arrival time difference is less than the transmitted pulse length.

In shallow water environment, sound velocity can be taken as approximately constant, sound rays travel in straight lines and bounce both on the surface and bottom, so that wave front of arrival rays at the receiver can be approximated as plane with an angle obliquely relative to the vertical plane. This arrival angle is another effective feature to separate eigenrays besides arrival time and the vertical arrays are effective means to provide angle resolution ability. In order to further enhance eigenrays resolution in time and space domain, Philippe Roux^[3] present a shallow water transmission experiment between a vertical array of sources (VSA) and a vertical array of receivers (VRA), and use the double-beaming algorithm to separate and identify each eigenray. The double beamforming algorithm transform each eigenray as a three dimensional highlight spot featured by intensity, arrival time, arrival angle, and launch angle simultaneously. The use of the double beamforming algorithm

lays the foundation for shallow water remote sensing^[4,5] using travel time, launch angle and arrival angle of selected eigenrays. To obtain adequate angle resolution, it is critical to design proper arrays parameters which consist of element number, element spacing, signals frequency and s. Among the various influence factors, range between the VSA and VRA play an important role to constrict the separation ability of the two vertical arrays system. The goal of this paper is to give a detailed arrays parameter analysis of the VSA and VRA and its design principle.

Parameter design of the VSA and VRA system

The eigenrays separation system in the shallow water includes an N_s elements vertical array of sources and an N_r elements vertical array of receivers. The element spacing of the VSA and VRA are d_s and d_r respectively, and the range between them is R. In order to illuminate how the vertical array identify eigenrays by their arrival angle or launch angle, firstly, we let the individual m element of the VSA transmit a CW pulse signal q(t) centered at frequency f, all elements of the VRA receive the arrival signal simultaneously, the received data are dependent on depth and they may be transformed to arrival angle and time two dimensional domains through the classic time-delay beamforming calculation:

$$p(t,\theta_r) = \frac{1}{N_r} \sum_{n=1}^{N_r} w_n p \left[t + \tau_1(\theta_r, z_n^r), z_m^s, z_n^r \right]$$

$$(1)$$

$$u_r = \frac{1}{N_r} \sum_{n=1}^{N_r} w_n p \left[t + \tau_1(\theta_r, z_n^r), z_m^s, z_n^r \right]$$

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$$(1)$$

Fig.1 Depict of constitution of the VSA and VRA system

where w_n is weight coefficients for the VRA, θ_r is the angle of the wave front at the VRA, z_m^s and z_n^r are depths of the *m* element of the VSA and *n* element of the VRA respectively, $p(t, z_m^s, z_n^r)$ is arrival signal waveform received by the *n* element of the VRA, the time moment *t*=0 is corresponding to the origin time moment during pulse transmission process by the *m* element of the VSA. The time delay in Eq.1 is represented as:

$$\tau_1(\theta_r, z_n^r) = \left(z_n^r - z_0^r\right) \sin \theta_r / c \tag{2}$$

where z_0^r is the center depth of the VRA and *c* is the sound speed in water.

Next, we use a double beamforming algorithm to obtain arrival angle and launch angle together based on the transmission and reception repeatedly between VSA and VRA. The operation procedure of the two vertical array systems may be described as follow: Each source element of the N_s elements VSA transmit a CW pulse signal q(t) successively, and the depth dependent arrival signal is recorded on the N_r elements VRA after each transmission process. The $N_s \times N_r$ dimensional data matrix between each source and each receiver in the time domain is formed as result of the entire acquisition procedure. The data matrix takes the form of a pressure field $p(t, z_m^s, z_n^r)$ recorded at a receiver depth z_n^r ($n=1,...,N_r$) for an emission at a source depth z_m^s ($m=1,...,N_s$). The double beamforming algorithm is carried out as follow:

$$p(t,\theta_s,\theta_r) = \frac{1}{N_s N_r} \sum_{m=1}^{N_s} \sum_{n=1}^{N_r} v_m w_n p \left[t + \tau_1(\theta_r, z_n^s) + \tau_2(\theta_s, z_m^s), z_m^s, z_n^r \right]$$
(3)

where v_n is weight coefficients for the VSA, θ_s is the angle of the wave front at the VSA, and the time delay τ_2 in Eq.3 is represented as:

$$\tau_2(\theta_s, z_m^s) = \left(z_m^s - z_0^s\right) \sin \theta_s / c \tag{4}$$

where z_0^r is the center depth of the VSA. In addition, the means of variables involved with the VRA are the same with Eq.1.

The main principle of parameters design of the VSA and VRA is ensuring the two vertical arrays system to have sufficient resolution in time and space in order to separate each eigenray in the launch angle, arrival angle and time domain. The resolution requirement is dependent on eigenrays structure at specified geometry and environment parameters such as water depth *H*, sound velocity profile c(z), bottom property, sources and receiver position. Firstly, the main lobe angle $\theta_0^{[6]}$ of the VRA must be smaller than the minimum difference of arrival angle between adjacent eigenray, it means that:

$$\theta_0 = \sin^{-1}(\lambda / N_r d_r) < \Delta \beta_{\min}$$
⁽⁵⁾

where β_n (n=1,...,N.) are arrival angle of each eigenray. Secondly, the grating lobe angle^[6] of the VRA must be larger than the maximum arrival angle β_{max} of eigenray, it means that:

$$\theta_s = \sin^{-1}(\lambda/2d_r) > \beta_{\max}$$
(6)
According to Eq.5 and Eq.6 simultaneously, the frequency inequality is:

$$\frac{c}{N_r d_r \sin \Delta \beta_{\min}} < f < \frac{c}{2d_r \sin \beta_{\max}}$$
(7)

In the same way, the frequency inequality due to the VSA is:

$$\frac{c}{N_s d_s \sin \Delta \alpha_{\min}} < f < \frac{c}{2d_s \sin \alpha_{\max}}$$
(8)

where α_n (*n*=1,...,*N*.) are launch angle of each eigenray.

Indirect method for pulse propagation calculation in frequency domain

In order to study eigenrays separation ability of the two vertical arrays system at different array parameters, source and receiver positions, it is necessary to obtain the raw arrival signal in time domain. In this paper, we adopt an indirect method of pulse propagation calculation in frequency domain, and the calculation flow chart is illustrated in Fig.2. Firstly, the frequency range $[f_{\min}, f_{\max}]$ in calculation is determined according to frequency spectrum $\tilde{q}(f)$ of transmit pulse signal. Then, the acoustic deliver function in frequency domain $\tilde{p}(f, z_m^s, z_n^r)$ at a receiver depth z_n^r ($n=1,...,N_r$) for an emission at a source depth z_m^{s} is computed by KRAKEN^[7] normal mode model. Finally, the simulated arrival signal waveform $p(t, z_m^s, z_n^r)$ in time domain is given by invert Fourier transformation, that is:

$$p(t, z_m^s, z_n^r) = \int_{f_{\min}}^{f_{\max}} \widetilde{q}(f) \widetilde{p}(f, z_m^s, z_n^r) e^{-i2\pi f t} df$$

$$\tag{9}$$

Applying the FFT algorithm, the integral Eq.9 may be replaced by a discrete sum in a time window length T, that is:

$$p(t_j, z_m^s, z_n^r) = \Delta f \operatorname{Re}\left\{\sum_{l=0}^{N_f} \left[\widetilde{q}(f_l) \widetilde{p}(f_l, z_m^s, z_n^r) \right] e^{-i\frac{2\pi dj}{N_f}} \right\}$$
(10)

It is noted that the time window length T must be selected large enough that it include the whole arrival signal waveform in order to eliminate the aliasing effects in time domain. However, for the computational reasons, it is desirable to choose T as short as possible since the length of the window determines the frequency sampling spacing by $\Delta f=1/T$. Here, we choose the window length lasting from the trigger time when a source transmits a pulse signal to the time when the whole arrival signal is totally received. In Eq.10, the time and frequency are discretized as $t_j=j\Delta t$ ($j=0,1,\ldots,N_t$), $f_l=l\Delta f+f_{\min}$ ($l=0,1,\ldots,N_f$), and the total number of frequency points and time points are $N_f = (f_{\max}-f_{\min})/\Delta f+1$ and $N_t = f_s T$ respectively. The time sampling frequency is determined through $f_s>2 f_{\max}$ according to the Nyquist criterion, and the time sampling spacing is $\Delta t=1/f_s$.

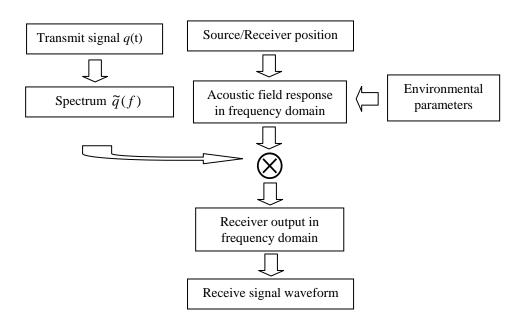


Fig 2 Flow chart of the indirect method for pulse propagation calculation in shallow water environment

Numerical simulation and analysis

The shallow water environment model shown in Fig.1 is considered, the water volume has properties: depth H=20m, density $\rho=1000$ kg/m³, sound speed c=1500m/s and the bottom medium has properties: density $\rho_1=1500$ kg/m³, compressional speed $c_1=1600$ m/s with a attenuation of $\alpha_1=0.2$ dB/ λ . Firstly, we consider the situation that the horizontal range between the VSA and VRA is 100m, and make the centers of VSA and VRA at the same depth that is $z_0{}^s=z_0{}^r=10m$. The plot of eigenray group between the two centers are shown in Fig.3(a) calculated by the BELLHOP model^[8], and the arrival angles of the first nine eigenrays are 0° , $\pm 11.3^\circ$, $\pm 21.8^\circ$, $\pm 30.9^\circ$, $\pm 38.6^\circ$ successively, then, the minimum difference of arrival angle between adjacent eigenray is $\Delta\beta_{\min}=7.7^\circ$ and the maximum arrival angle of eigenray group is $\beta_{\max}=38.6^\circ$. Next, we design the array parameters of the VRA according to Eq.7 which is a multi-parameter inequality. If we let the total length of VRA is $L_r=N_rd_r=4m$, and its element spacing equal half the wave length that is $d_r=\lambda/2$. Then, the right hand of Eq.7 found naturally, and the left hand of Eq.7 make the frequency of the received signal to satisfy the relationship that is f>2.8kHz. Here, we can choose the frequency to be f=10kHz, then, the main lobe angle is $\theta_0=2.15^\circ$ which satisfy the resolution demand. In the same way, the array parameters of the VSA can be designed according to Eq.8, here, we let the VSA has the same parameters with the VRA.

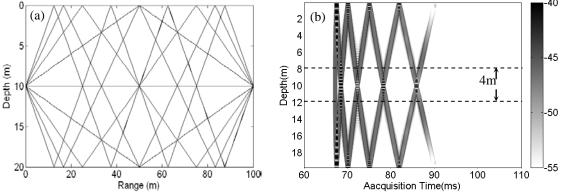


Fig 3 Multi-path propagation structure when the source-receiver range is 100m. (a) The eigenrays plot between source and receiver. (b) Depth-versus-time representation of arrival signal.

A numerical simulation of depth versus time representation of the arrival signal is shown in Fig. 3(b) using the above mentioned indirect method in frequency domain. In this numerical example, the central element of the VSA transmits a CW pulse with a 10kHz central frequency, and the arrival signal are received in the whole water depth at 100m range from the source. The arrival signal data between the two dashed lines in Fig.3(b) is correspond to the signal received by the VRA, and it can be transformed to the arrival angle versus time domain (θ_r ,t) by the conventional beamforming algorithm, the result is shown in Fig.4(a). When the additional feature of launch angle is considered, it is helpful to distinguish different eigenray in the three-dimensional domain (θ_s , θ_r ,t), however, the three-dimensional fields results should be shown in the (θ_s , θ_r) domain of each time slices for convenience. The double beamforming process was performed on the VSA and the VRA simultaneously, an example of double beamforming results at t=72ms is shown in Fig.4(b), and the two intensity spots corresponds to the two eigenrays which have arrival angle $\pm 21.8^{\circ}$ respectively.

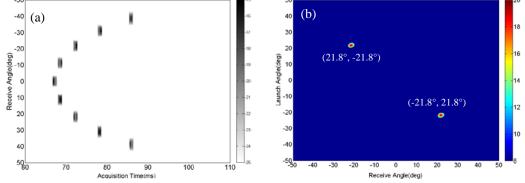


Fig.4. The beamforing results when the range between the VSA and the VRA is 100m. (a) The intensity in (θ_r, t) domain of beamforming result performed on the VRA for an emission at central element of the VSA. (b) Time slice results in (θ_s, θ_r) domain from double beamforming at *t*=72ms.

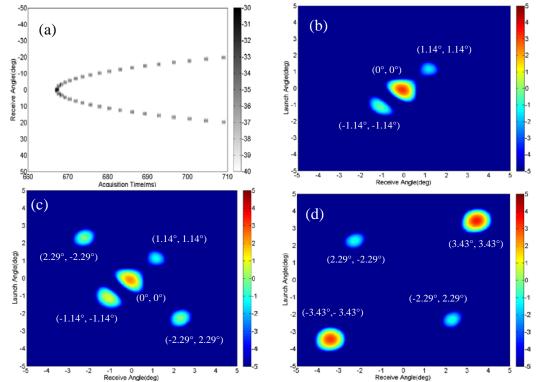


Fig.5. The beamforing results when the range between the VSA and the VRA is 1km. (a) The intensity in (θ_r, t) domain of beamforming result performed on the VRA for an emission at central element of the VSA. (b), (c) and (d) Time slice results in (θ_s, θ_r) domain from double beamforming at t=667ms, t=667.5ms and t=668ms respectively.

We note that, the difference of launch angle, arrival angle and arrive time of adjacent eigenray decrease as the source-receiver range increase in shallow water environment, so that it demands the

vertical arrays system to have a higher resolution in space and time domains at longer range. Next, we consider the situation that the range between the VSA and VRA is 1km, other simulation parameters are the same with Fig.4's calculation input. In this example, the minimum difference of arrival angle between adjacent eigenray is $\Delta\beta_{\min}=1.1^{\circ}$ and the maximum arrival angle of eigenray group is $\beta_{\text{max}}=18.8^{\circ}$ for the first 35 eigenrays. We can not follow the array parameters configuration of the situation that the range between the VSA and VRA is 100m, that configuration of vertical arrays system do not satisfy the resolution demand in this circumstances. If we still transmit the same acoustic pulse signal centered around 10kHz, according to Eq.(4), the total length of VSA or VRA should be increased in order to raise angle resolution. Here, we assume $L_r = L_s = 8$ m which is two times of that in Fig.4's calculation, as a result, the main lobe angle of the VSA and the VRA are 0.9°. On the other hand, we do not change the elements number which means that $d_s = d_r = \lambda$. Under this parameters setup, the frequency conditions Eq.(6) and Eq.(7) are fulfilled. A numerical simulation is given in the situation that the range between the VSA and the VRA is 1km. Fig.5(a) give the results of beamforming in (θ_r, t) domain performed on the VRA for an emission at central element of the VSA. Fig.5(b), (c) and (d) show the time slice results in (θ_s, θ_r) domain from double beamforming at t=667ms, t=667.5ms and t=668ms respectively, it is showed that each intensity spot in the domain (θ_s, θ_r) correspond to a eigenray, and the eigenrays which are close to each other are separated successfully under the array parameters configuration above.

Summary

In this paper, the principle of parameter design of the two vertical source and receiver arrays system for separation of eigenrays in shallow water multi-path environment is numerical demonstrated. The two arrays performed transmission and reception of acoustic pulse signal repeatedly, and a double beamforming algorithm is used to obtain arrival angle and launch angle of each eigenray simultaneously. Through parameter design comparison between the situations that the range between the two arrays are100m and 1km. it is showed that the arrival angle, launch angle and arrive time of adjacent eigenray are close to each other as the source-receiver range increase, and the selection of operation frequency, element spacing, array length of the two arrays system should make the system to have a narrower main lobe angle.

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