

# An Adaptive Kalman Filtering Algorithm based on Doppler Frequency

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**Abstract.** Working in a passive mode, the result of instability, slow convergence and low convergence precision were easy to appear when the underwater target is located by using Kalman filtering algorithm, so an adaptive Kalman filtering algorithm based on Doppler frequency was proposed. The algorithm estimated the statistical characteristics of the system process noise and measurement noise in real-time, dynamically compensate error caused by linearizing observation model, and reduce the bad impact by the observation error. Through the simulation, experiments show that the algorithm performs better in aspects of convergence precision and stability.

## Introduction

Tracking and locating the target are always the hot spot in underwater warfare system [1]. In only bearing target motion analysis algorithm, we often use rectangular coordinate system to establish the target state equation and measurement equation, and the Kalman filtering algorithm [2] to estimate the target parameter information. Due to the nonlinearity of the observation equation and the measurement equation, the equations should be converted into linear equations, which is the extended Kalman filtering idea [3]. But the statistical characteristic of state noise and measurement noise are needed to be accurate, which is difficult to satisfy in practice. Therefore, the researchers put forward some algorithms, such as adaptive Kalman filtering algorithm based on Sage-Husa [4], the fading factor, the memory factor [5], the information [6], multiple model method [7] and so on. But only bearing target motion analysis algorithm requires the submarine to maneuver itself at least once, which brings certain restrictions to observability and concealment. Because the observation equation is nonlinear, so we often adopt the extended Kalman filtering algorithm [8]. To solve above problems, an adaptive Kalman filtering algorithm based on Doppler frequency is proposed in this paper. The algorithm estimates the system noise and observation noise in real-time, compensating dynamically the error of linearization and observation to reduce the impact of error. The result which is simulated shows that the algorithm can be better to eliminate the linear error and observation error as well as to improve the stability of the filter and the precision of the convergence.

## The mathematical model of the Kalman filter

In this paper, we assume that the target and observation platform both move in the same horizontal plane, and keep a constant velocity in a straight line.

**The transfer equation of the target motion.** Continuous dynamic model of the system,

$$X(k) = AX(k-1) + W. \quad (1)$$

**The observation equations of bearing and Doppler frequency.** The observation model is,

$$Z(k) = (Z_1(k) \quad Z_2(k))^T = (F(k) + V_1 \quad \bar{f}(k) + V_2)^T. \quad (2)$$

where  $F(k) = \arctan^{-1} \frac{x_m(k) - x_w(k)}{y_m(k) - y_w(k)}$  is the target azimuth,

$\bar{f}(k) = f_0 \left(1 - \frac{v_{mx}(k) - v_{wx}(k)}{c} \sin Z_1(t) - \frac{v_{my}(k) - v_{wy}(k)}{c} \cos Z_1(t)\right)$  is the Doppler frequency.

**The iterative equation of the extended Kalman filter.** The observation model is nonlinear, so the observation matrix needs to be linearized.

$$H(k) = \begin{pmatrix} \frac{\partial F(k)}{\partial X(k)} & \frac{\partial \bar{f}(k)}{\partial X(k)} \end{pmatrix}^T. \quad (3)$$

So the extended Kalman filtering process goes as follows:

1) One-step predictive state,

$$X(k|k-1) = AX(k-1|k-1). \quad (4)$$

2) One-step predictive state error covariance matrix,

$$P(k|k-1) = AP(k-1|k-1)A^T + Q. \quad (5)$$

3) The estimation of Kalman filtering gain matrix,

$$G(k) = P(k|k-1)H^T(k)(H(k)P(k|k-1)H^T(k) + R)^{-1}. \quad (6)$$

4) The information error matrix of observation,

$$e(k) = \begin{pmatrix} Z_1(k) & Z_2(k) \end{pmatrix}^T - \begin{pmatrix} F(k|k-1) & \bar{f}(k|k-1) \end{pmatrix}^T. \quad (7)$$

5) Update state forecast,

$$X(k|k) = X(k|k-1) + G(k)e(k). \quad (8)$$

6) Update error covariance matrix,

$$P(k|k) = (E - G(k)H(k))P(k|k-1). \quad (9)$$

### The Adaptive Statistical Properties of Noise

According to the literature [9], the extended Kalman filtering algorithm which assumes the noise invariance will bring in greater error, resulting in the poor effect of the target state analysis. To solve above problems, the mean and variance of the state noise and observation noise are adaptively estimated real-time in the system.

The covariance matrix of process noise and measurement noise play an important role in Kalman filtering algorithm. Firstly, discuss about the covariance matrix of process noise. According to literature [13], in common sense, if the covariance matrix of process noise is too small, state estimation will generate deviation, but if the covariance matrix of process noise is too large, state estimation will generate oscillation. The main idea is to do the statistics of the difference of the state step prediction with further prediction in certain time, and to reverse back to determine the covariance matrix of process noise based on the statistical law. When deviation is appeared from the statistical state estimation, increase the value. When oscillation is happened the statistical state estimation, reduce the value. The covariance matrix of process noise goes as follows,

$$Q(k+1) = \begin{cases} (1+d_k)Q(k), & \text{if } \Delta X(k) \text{ is biased} \\ (1-d_k)Q(k), & \text{elseif } \Delta X(k) \text{ is oscillated.} \\ Q(k), & \text{else other} \end{cases} \quad (10)$$

Where  $d_k = (1-b)/(1-b^{k+1})$ ,  $0 < b < 1$  and  $b$  is the fading factor.

Next, discuss the covariance matrix of observation noise. The adaptive method of measurement noise covariance matrix is determined by the information which is obtained by Eq.7 in the iterative process. If large observation error exists in the system, estimated filtering state will exceed actual

state. In this case, the covariance matrix of observation noise needs to be adjusted adaptively to ensure accuracy and stability of filtering. In the adaptive process, a coefficient matrix of the measurement noise covariance matrix is needed to make sure that the estimated state of filtering matches its actual state. Therefore, the coefficient matrix added to filtering algorithm can be expressed as,

$$e(k)e^T(k) = H(k)P(k|k-1)H^T(k) + S(k)R(k). \quad (11)$$

$$S(k) = (e(k)e^T(k) - H(k)P(k|k-1)H^T(k))R^{-1}(k). \quad (12)$$

In Kalman filtering algorithm, the coefficient matrix is a unit matrix, which means that estimated state has matched its actual state. Because of measurement noise and the error of calculating, the coefficient matrix computed from Eq.12 may be non-diagonal matrix, or its diagonal elements are negative, or its diagonal elements are less than 1, which is impossible in the actual dynamics. To avoid this situation, the assignment of the coefficient matrix requires the following rules:

$$S^* = \text{diag}(s_1^*, s_2^*, \dots, s_n^*). \quad (13)$$

$$G(k) = P(k|k-1)H^T(k)(H(k)P(k|k-1)H^T(k) + S^*(k)R(k))^{-1}. \quad (14)$$

$$R(k+1) = S^*(k)R(k). \quad (15)$$

### The Experimental Simulation and Analysis

In order to verify the effectiveness of adaptive Kalman filtering algorithm based on Doppler frequency, respectively take adaptive Kalman filtering algorithm and extended kalman filtering algorithm to estimate when fixing the underwater target tracking in the simulation system.

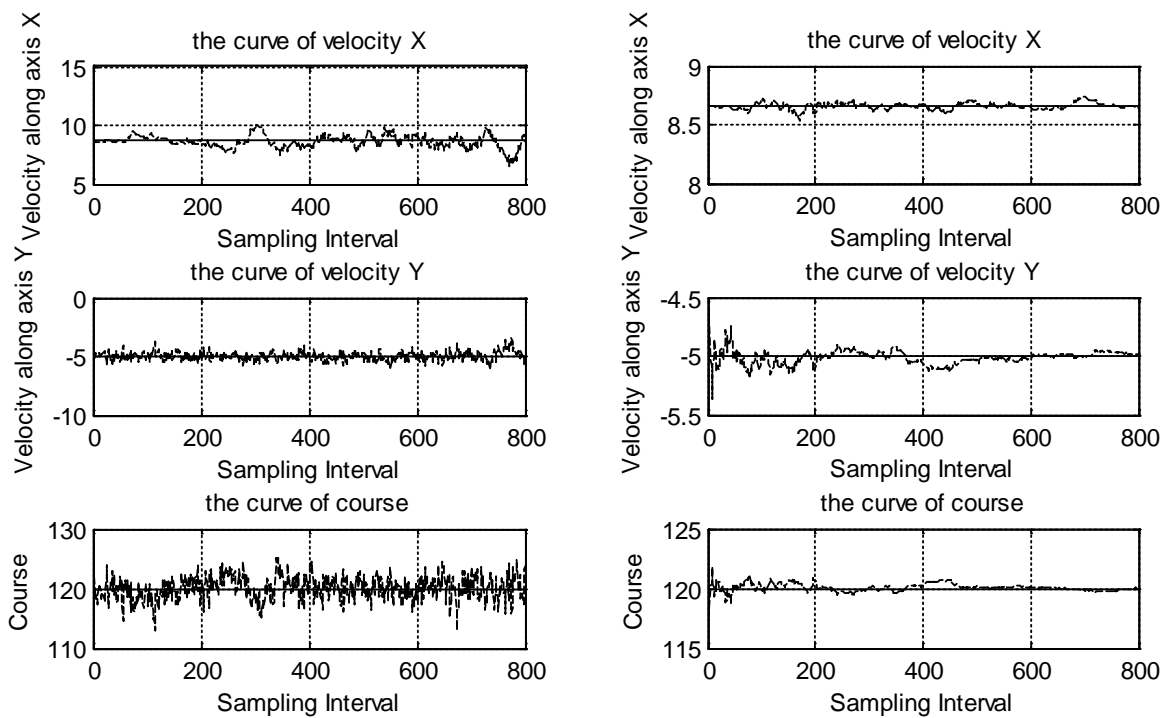


Fig.1 the Estimated Curves of Extended Kalman Filter and Adaptive Kalman Filter

Table 1 Parameters of the mean square error

A\P	Vx[kn]	Vy[kn]	C[d]	X[m]	Y[m]
EKF	0.4455	0.1855	1.0496	62.8289	24.3941
AEKF	0.2672	0.1059	0.2606	35.3824	11.3296

Initial conditions: assume that the target and observation platform both keep constant direct movement. If the speed of target is 18 knot, the course 120 degree, the initial position of 0 degrees, located in due north direction observation platform, the speed of the observation platform is 6 knot, and the course is 0 degree. In the initial time, the interval distance of both is 20 km.

Through the simulation curves compared in Fig.1, the extended Kalman filter cannot suppress linearizing error increases. Therefore its performance will gradually become poor with the accumulation of filter, but the adaptive Kalman filter can finally get good filtering effect through making real-time estimation on statistical characteristics of state noise and observation noise and making up for the error caused by linearization. Table 1 figures out that the effect of adaptive Kalman filter is relatively better than regular extended Kalman filter, and the parameters of target have better stability as well as filtering accuracy. The main specific reason is regular extended Kalman filtering which adopts fixed system noise matrix and measurement noise matrix. But when there is a big difference between the actual system and the selected matrix, filtering error will increase even divergent, while adaptive Kalman filter is estimating the matrix to reduce the model error through the statistical characteristics of process noise and measurement noise real-time in current system, to restrain filtering divergence, improve the filtering accuracy and stability.

## Conclusions

Working in a passive mode, the result of instability, divergence, deviation, slow convergence and low convergence precision was easy to appear when tracking. And the BOTMA can destruct the concealment of the observation platform because it needs to do some motor. To solve the above problems, the paper puts forward out an adaptive Kalman filtering algorithm based on Doppler frequency, which makes use of measurement of azimuth sequence and Doppler line-spectrum frequency shift. The simulation results show that the algorithm compensates and makes up for the errors caused by the linearization of the system state model and observation model through estimating the statistical characteristics of the system state noise and observation noise real-time, to improve the stability and convergence precision of the filter, and gain good filtering effect.

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## References

- [1] Xin Xiong, Xinhua Zhang, Chengzhi Gao, Haijie Lu and Ying Lan: Ship Science and Technology, Vol.32 (2010) No.7, p.140.
- [2] Simon Dan, S. Shmaliy Yuriy: Automatica, Vol.49 (2013) No.6, p.1892.
- [3] <http://www.cs.unc.edu/~welch/kalman/>.
- [4] Chong Wang and Qingjun Zeng: Electronic Design Engineering, Vol.21 (2013) No.15, p.75.
- [5] Tang W and Jiang Y: IEEE International Conference on Mechatronic Sciences, Electric Engineering and Computer (MEC) (Shenyang, China, Dec 20-22, 2013), p.335.
- [6] Hongwei Bian, Zhihua Jin, Junpu Wang and Weifeng Tian: Journal of Shanghai Jiaotong University, Vol.40 (2006) No.6, p.1000.
- [7] Xiaoli Li and Guangda Hu: Journal of System Simulation, Vol.20 (2008) No.3, p.590.
- [8] Rui Zhang and Linan Shi: Aerospace Control, Vol.30 (2012) No.3, p.12.
- [9] Karasalo M. and Hu X.: Automatica, Vol.47 (2011) No.8, p.1785.