

# Multi-objective Fuzzy Optimization Design of Helical Gear Drive

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**Abstract.** fuzzy factors in the helical gear reducer design are analyzed, and a multi-objective optimization model with minimum volume and maximum bearing capacity is established in this paper. Optimal level cut-set method is introduced to conduct comprehensive evaluation on constraining fuzziness and convert the fuzzy constraint into general constraint according to the actual conditions and requirements; for multi-objectives, single-objective optimal solutions are fuzzified to form fuzzy sets to establish the membership functions of the fuzzy sets by taking the fuzziness between the single objectives and between the single objectives and multiple objectives into consideration, and multi-objective optimal solutions are obtained by maximizing the membership functions of the fuzzy set intersection. For an actual helical gear reducer, the fuzzy theory and method are applied to obtain results.

## Preface

Volume, weight and bearing capacity are the important indexes for evaluating the performance of helical gear reducer and these indexes depends on the design and selection of the transmission parameters such as module  $m$ , width of gear  $b$ , teeth number of gear  $z_1$ , helix angle  $\beta$  etc. The optimization design solutions targeted at minimum volume in the prior art<sup>[1]</sup> cannot reflect the actual operation conditions, as they do not take the fuzzy factors in the design into consideration and have single objective, and by adopting such solutions, the real optimization solution is highly possible to be missed to obtain the satisfactory comprehensive effects. In this paper, the multi-objective fuzzy optimization model with minimum volume and maximum bearing capacity is established by combining with the optimization techniques and adopting the theory and method of fuzzy mathematics. Optimal level cut-set method is introduced to conduct comprehensive evaluation on constraining fuzziness and convert the fuzzy constraint into general constraint according to the actual conditions and requirements; for multi-objectives, single-objective optimal solutions are fuzzified to form fuzzy sets to establish the membership functions of the fuzzy sets by taking the fuzziness between the single objectives and between the single objectives and multiple objectives into consideration, and multi-objective optimal solutions are obtained by maximizing the membership functions of the fuzzy set intersection.

## Establishing multi-objective fuzzy optimization

### Objective functions and design variables

The objective of the optimization is the confining dual-objective model with minimum volume and maximum bearing capacity under the condition of ensuring transmission ratio and various confining conditions, and the minimum volume is taken as the optimization objective, at the premise of meeting the application requirements, which means<sup>[2]</sup>:

$$F_1(x) = 0.785m^2bz_1^2(1+i^2)/\cos^2\beta \quad (1)$$

Where:  $m$  – modules of gear

$b$  – width of gear

$i$  – transmission ratio

$z_1$  –teeth number of gear

$\beta$  --helix angle

For bearing capacity, the input torque T1 can be taken as objective function:

$$F_2(x) = T1 \quad (2)$$

Obviously,  $F1(x)$  and  $F2(x)$  are both subjected to the confining effects of the parameters  $m$ ,  $b$ ,  $z1$  and  $T1$ , thus, the design variable can be taken as:

$$X = [m \ b \ z1 \ T1 \ \beta]^T = [x1 \ x2 \ x3 \ x4 \ x5]^T \quad (3)$$

So, the objective function can be established as:

$$F_1(x) = 0.785x1^2x2x3^2(1+i^2)/\cos^2x5 \quad (4)$$

$$F_2(x) = x4 \quad (5)$$

### Confining conditions

From the perspectives of actual conditions of the gear drive, the confining conditions can be divided into stress constraints and boundary constraints. Stress constraints include constraint of contact strength of gear tooth face and constraint of bending strength of tooth root, and the boundary constraints include the constraints on value taking for modules, width of gear and teeth number of gear. For these constraints, the determination of the stress constraints has significant fuzziness and they shall be deemed as fuzzy constraints, while other constraints can be taken as general constraints.

#### 1) Stress constraints

The constraint of contact strength of gear tooth face and constraint of bending strength of tooth root are respectively shown as follows<sup>[3]</sup>:

$$g_1(x) = Z_E \cdot Z_H \cdot Z_B \sqrt{\frac{2kT_1(u+1)}{bd_1^2u}} - \left[ \frac{\sigma}{\sim} \right]_H \leq 0 \quad (6)$$

$$g_2(x) = \frac{2kT_1}{d_1mb} Y_{Fa} Y_{sa} - \left[ \frac{\sigma}{\sim} \right]_F \leq 0 \quad (7)$$

Where:  $ZE$  – elastic factor of material,

$ZH$  – region factor,

$ZB$  – helix angle factor,

$k$  – load coefficient,

$T1$  – torque of worm gear, N·mm

$\left[ \frac{\sigma}{\sim} \right]_H$  – allowable contact stress with fuzziness

$YFa$  – tooth form coefficient of gear

$Ysa$  – correct coefficient of gear

$\left[ \frac{\sigma}{\sim} \right]_F$  – allowable bending stress with fuzziness

#### 2) Boundary constraints

For module limit ( $1.5 \leq m \leq 4$ ) (8)

For limit of width of gear ( $30 \leq b \leq 80$ ) (9)

For limit of number of teeth of gear ( $18 \leq z1 \leq 28$ ) (10)

For torque limit ( $T_{min} \leq T1 \leq T_{max}$ ) (11)

Helix angle limit ( $10^\circ \leq \beta \leq 20^\circ$ ) (12)

Thus, the multi-objective optimization module of reducer is obtained by:

Solving  $X = [m \ b \ z1 \ T1 \ \beta]^T = [x1 \ x2 \ x3 \ x4 \ x5]^T$  to make  $\min F1(x)$  and  $\max F2(x)$  meet the following requirement:

$$S.T. \ g_u(x) \leq 0 \ (u=1, 2, \dots \text{ and } 7) \quad (13)$$

### Dealing with fuzzy constraints [4]

For the confining conditions  $g1(x)$  and  $g2(x)$ , the values of allowable stresses  $\left[ \frac{\sigma}{\sim} \right]_H$  and  $\left[ \frac{\sigma}{\sim} \right]_F$  are taken within certain range, and the determination of the specific values depends on various fuzzy factors. By assuming the upper and lower limits of  $\left[ \frac{\sigma}{\sim} \right]_H$  and  $\left[ \frac{\sigma}{\sim} \right]_F$  as  $\left[ \frac{\sigma}{\sim} \right]_H^+$ ,  $\left[ \frac{\sigma}{\sim} \right]_H^-$ ,  $\left[ \frac{\sigma}{\sim} \right]_F^+$  and  $\left[ \frac{\sigma}{\sim} \right]_F^-$ ,

$\begin{bmatrix} - \\ \sigma \end{bmatrix}_F$  and  $\begin{bmatrix} \sigma \\ - \end{bmatrix}_F$ , level cut-set is introduced, and the corresponding allowable stresses are respectively as follows:

$$\begin{bmatrix} \sigma \\ - \end{bmatrix}_H = \begin{bmatrix} - \\ \sigma \end{bmatrix}_H - \left( \begin{bmatrix} - \\ \sigma \end{bmatrix}_H - \begin{bmatrix} \sigma \\ - \end{bmatrix}_H \right) V \quad (14)$$

$$\begin{bmatrix} \sigma \\ - \end{bmatrix}_F = \begin{bmatrix} - \\ \sigma \end{bmatrix}_F - \left( \begin{bmatrix} - \\ \sigma \end{bmatrix}_F - \begin{bmatrix} \sigma \\ - \end{bmatrix}_F \right) V \quad (15)$$

$V \in [0,1]$

Obviously, different V values will lead to different optimization results. The V values reflect different fortification levels, and the determination of its specific value is affected by various fuzzy factors, so the fuzzy comprehensive evaluation method can be adopted for the determination:

1) It is assumed that  $U = (u_1, u_2, \dots, u_m)$  (16)

is the factor value affecting the value taking. These factors are generally design and manufacture levels, material quality, significance, application conditions, damage and loss, etc. and they are all fuzzy.

2) It is assumed that  $V = (v_1, v_2, \dots, v_n)$  (17)

is the alternative set, from which the V value can be selected.

3) Single factor evaluation matrix  $R_{m \times n}$  is established, and its element  $r_{ij}$  indicates the membership degree of the No. j element in the alternative set when evaluation is conducted based on the No. i factor.

4) Fuzzy comprehensive evaluation

As the factor weight set is assumed as  $A = (a_1, a_2, \dots, a_m)$  (18)

The fuzzy comprehensive evaluation set is  $B = (b_1, b_2, \dots, b_n)$  (19)

$$b_j = \sum_{i=1}^m a_i r_{ij} \quad (j = 1, 2, \dots, n) \quad (20)$$

The optimal level cut-set can be obtained as follows based on the weighted average method:

$$V^* = \frac{\sum_{j=1}^n b_j V_j}{\sum_{i=1}^n b_i} \quad (21)$$

## Solution of optimization module

The solution steps of multi-objective optimization model by adopting fuzzy theory and method as determined by Formula (16) are as follows:

1) The optimal values  $f_1^*$  and  $f_2^*$  as well as the worst values  $\bar{f}_1$  and  $\bar{f}_2$  of the objectives are obtained by general single objective optimization method, and obviously, the following can be obtained:

$$f_1^* \leq f_1(x) \leq \bar{f}_1 \quad (22)$$

$$\bar{f}_2 \leq f_2(x) \leq f_2^* \quad (23)$$

2) The optimal solutions are fuzzified to establish the membership functions of the optimal solutions:

$$\tilde{N}_1(x) = \left[ \frac{\bar{f}_1 - f_1(x)}{\bar{f}_1 - f_1^*} \right]^q \quad (24)$$

$$\tilde{N}_2(x) = \left[ \frac{f_2(x) - \bar{f}_2}{f_2^* - \bar{f}_2} \right]^q \quad (25)$$

Where:  $\tilde{N}_1(x)$  and  $\tilde{N}_2(x) \in [0,1]$

3) Instrumental variable  $a$  is introduced to convert the multi-objective issue into the single-objective issue, which means that:

$X = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ a]^T$  is solved to make max  $a$  meet the following:

$$\begin{aligned} \text{S.T. } gu(x) &\leq 0 \quad (u=1, 2, \dots, 7) \\ a &\leq \tilde{N}_1(x) \quad \text{and} \quad \tilde{N}_2(x) \end{aligned} \quad (26)$$

### Calculation example analysis

For a motor-driven helical gear reducer, with motor power  $P=40\text{kw}$ , speed  $n=1470\text{rpm}$ , transmission ratio  $i=3.3$  and middling shock load, for which two-way drive is adopted, the little gear is made from 20CrMnTi, the large gear is made from 20Cr, 56~62HRC is adopted, the value taking range of  $\left[\begin{smallmatrix} \sigma \\ \sim \end{smallmatrix}\right]_H$  is<sup>[3]</sup> 1450~1540MPa, that of  $\left[\begin{smallmatrix} \sigma \\ \sim \end{smallmatrix}\right]_F$  is 448~504MPa, that of T1 is 250000~280000N.mm, and the other fuzzy conditions are high design and manufacture levels, good application conditions, large damage and loss, high-quality materials, and high significance of reducer. The cylindrical worm reducer is designed herein based on the minimum volume and maximum bearing capacity.

The optimal level cut-set  $V^*$  is first determined based on the above-mentioned method to convert the fuzzy constraint into general constraint:

The factor set is  $U=(\text{high design and manufacture levels, good application conditions, large damage and loss, high-quality materials, and high significance of reducer})$

The alternative set is  $V=(0.0, 0.1, 0.2, \dots, 0.9 \text{ and } 1.0)$ .

The single factor evaluation matrix is:

$$R = \begin{bmatrix} 0.0 & 0.0 & 0.2 & 0.6 & 0.8 & 1.0 & 0.9 & 0.6 & 0.3 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.6 & 0.8 & 1.0 & 0.9 & 0.6 & 0.3 & 0.1 & 0.0 \\ 0.1 & 0.2 & 0.6 & 0.9 & 1.0 & 0.8 & 0.6 & 0.3 & 0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.1 & 0.2 & 0.3 & 0.6 & 0.8 & 1.0 & 0.9 & 0.7 \\ 0.0 & 0.0 & 0.2 & 0.6 & 0.8 & 1 & 0.9 & 0.6 & 0.3 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.6 & 0.8 & 1 & 0.9 & 0.6 & 0.3 & 0.1 & 0.0 \end{bmatrix}$$

The factor weight set is:

$$A=(0.25, 0.23, 0.20, 0.20, 0.10 \text{ and } 0.02)$$

The optimal level cut-set can be obtained as  $V^*=0.54$  based on the weighted average method, and is introduced into the formulas (14), (15), (6) and (7) to convert the fuzzy constraint into general constraint.

The best point and worst point obtained from optimization of single objective are:

$$\begin{aligned} f_1^* &= 1196167\text{mm}^3 & \tilde{f} &= 9293107\text{mm}^3 \\ f_2^* &= 280000\text{Nmm} & \tilde{f}_2 &= 250000\text{Nmm} \end{aligned}$$

The established membership functions are as follows (for  $q=1$ ):

$$\begin{aligned} \tilde{N}_1(x) &= \left[ \frac{9293107 - f_1(x)}{9293107 - 1196167} \right] \\ \tilde{N}_2(x) &= \left[ \frac{f_2(x) - 250000}{280000 - 250000} \right] \end{aligned}$$

The above said results are introduced into the formula (26), and the optimal solutions are ( $a>0.98$ ).

$X=[2.5 \quad 60 \quad 18 \quad 280000 \quad 15^0 44' 24'']$ ,  $F1=1304910 \text{ mm}^3$  and  $F2=280000\text{Nmm}$ . The volume is reduced by 25.2% and the bearing capacity is improved by 7.1% when compared with the original design parameters<sup>[3]</sup>  $[3 \quad 50 \quad 19 \quad 260000 \quad 18^0 53' 16'']$  ( $F1 = 1744713 \text{ mm}^3$ ).

The comprehensive optimization effect is very remarkable.

### Conclusion

1) Fuzzy comprehensive evaluation and fuzzy membership function methods are applied in the paper respectively to solve the confining conditions and multi-objective fuzziness issues in the cylindrical worm reducer optimization design to make the design results more consistent with the

actual conditions, with good comprehensive effect.

2) The results of fuzzy optimization are closely related to the determination of the factor weight set. Grade I fuzzy comprehensive evaluation method is adopted in the paper, though Grade II or even Grade III fuzzy comprehensive evaluation method should be adopted in determination of factor weight set to reduce or eliminate the anthropogenic factors such as experience and point of view which are closely related, as the work amount of calculation will be significantly increased.

3) The calculation example of the paper gives the specific fuzzy conditions. Changes to these conditions will lead to different optimization results (ordinary optimization design method cannot keep the optimization results unchanged in case of changes to fuzzy conditions). Therefore, this fuzzy optimization design is more suitable for the design with many factors and more specific conditions.

## References

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