Nonlinear Process Fault Detection Method using Slow Feature Analysis

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Abstract.

Slow feature analysis (SFA) is a method that extracts the invariant or slowly varying features from an input signal based on a nonlinear expansion of it. This paper introduces SFA into industrial process monitoring. It overcomes the innate drawback of principal component analysis (PCA) that it fails to draw the more complex features or underlying nonlinear structure of the industrial process signals. Moreover, the invariance and slowness indicate the intrinsic properties of data. Thus the extracted information is interesting for data analysis. For the purpose of fault detection, two statistics are constructed: the T2 statistic and the SPE statistic. Then, these two statistics are applied to perform process monitoring. Simulations are run on the Tennessee Eastman (TE) process and the results illustrate the effectiveness of the proposed method.

Keywords: Slow Feature Analysis; Fault Detection; Process Monitoring; TE Process; PCA

Introduction

The prosperous development of computer science and its related fields has brought a fundamental evolution to modern industry during these decades. The industrial process control system has become more integrated, intellectualized and complicated. The conventional method of modeling the actual industrial processes for control is less practical and effective [1]. Meanwhile, the data-driven methods have been applied in industry process very successfully [2].Principal component analysis is a classical multivariate statistical method was introduced into process monitoring [3]. PCA is a linear method to extract the uncorrelated components from the input data. However, in practical the process data are usually of high nonlinearity. In this paper slow feature analysis [4] is applied to tackle this problem. SFA can be used for feature extraction, dimensionality reduction, and invariance learning [5]. It performs a nonlinear transformation on the input signal and draws the slowest varying components, which are senior representation of the input and indicate the intrinsic properties of the original system or the source. The core of SFA algorithm is eigenvalue decomposition [4].Its nature is to search for the optimal linear combination that satisfies the objective function in the nonlinearly expanded feature space. Its nonlinearity is realized through introducing the higher order polynomials of the input variables in. The order of the polynomial reflects the richness of the feature space. More nonlinear the extension is, more process intrinsic information the outputs will convey.

This paper applies SFA for fault detection. With a good selection of the nonlinear extension of the input signal, SFA can catch the inherent and nonlinear information of the system very well. Therefore, the effect of fault detection can be expected to be improved. The simulation results demonstrate this expectation.

Slow Feature Analysis

Slow feature analysis is proposed by Wiskott [4] to extract the slow features from fast varying complex signal. Slow features refer to the underlying slowly varying components of a signal and characterize its intrinsic properties. SFA obtains global optimization, which are the slowest features according to the changing rate in ascending order.

The learning problem is formulated as follows [4]:

Given a multidimensional input signal $\mathbf{x}(t) = [x_1(t), \ldots, x_n(t)]^T$ with t indicating time and n the dimension. The goal is to find an input-output function $\mathbf{g}(\mathbf{x}) = [g_1(\mathbf{x}), \ldots, g_m(\mathbf{x})]^T$, so that the generated m-dimensional output signal $\mathbf{y}(t) = [y_1(t), \ldots, y_m(t)]^T$ with $y_i(t) := g_i(\mathbf{x}(t))$ satisfies:

$$\Delta(y_i) := \langle \dot{y}_i^2 \rangle \text{is minimal for each } i \in \{1, \ldots, m\}, \quad (1)$$

under the constraints

$$\langle y_i \rangle = 0 \quad (2)$$

$$\langle y_i^2 \rangle = 1 \quad (3)$$

$$\forall\, i < j: \quad \langle y_i y_j \rangle = 0, \quad (4)$$

where$\langle\cdot\rangle$ denotes the temporal average and $\dot{y}_i$ is the first derivative of y_i. Equation (1) describes that the objective of the learning problem is to minimize the temporal fluctuation of the output. The constraints Eq.(2) and Eq.(3) eliminate the trivial solution $y_i = const$. Constraint Eq.(4)guarantees the decorrelation of the output signals.

The learning problem is in general difficult to solve. Therefore, the output functions $g_i$are constraint to be a linear combination of a finite set of nonlinear functions as follows:

$$g_i(x) := \sum_{p=1}^{P} w_{ip} h_p(x). \tag{5}$$

The function vector **h** = $[h_1, \ldots, h_p]^T$ expands the input signal nonlinearly to form a new signal **z**(t), which is defined as **z**(t) := **h(x**(t)**)**.So the i-thcomponent can be expressed as:

$$y_i(t) = g_i(x(t)) = w_i^T h(x(t)) = w_i^T z(t). \tag{6}$$

Therefore, the learning objective can be reformulated:

$$\min \Delta(y_i) := \langle \dot{y}_i^2 \rangle = w_i^T \langle \dot{z}\dot{z}^T \rangle w_i. \tag{7}$$

Suppose **h** is chosen such that the mean of $\mathbf{z}(t)$ is zero and the variance of $\mathbf{z}(t)$ is one.This can be achieved simply be a whitening step based on an arbitrary $\mathbf{h}'$. So the constraints can be found with:

$$\langle y_i \rangle = w_i^T \underbrace{\langle z \rangle}_{=0} = 0 \tag{8}$$

$$\langle y_i^2 \rangle = w_i^T \underbrace{\langle zz^T \rangle}_{=I} w_i = w_i^T w_i = 1 \tag{9}$$

$$\forall\, i < j : \qquad \langle y_i y_j \rangle = w_i^T \underbrace{\langle zz^T \rangle}_{=I} w_j = w_i^T w_j = 0.$$

(10)

The constraints can be fulfilled if $\mathbf{W}$=$[\mathbf{w}_1, \ldots, \mathbf{w}_m]^{\mathbf{T}}$isorthogonal matrix.Thus, the normalized vector that minimizes $\Delta(y_1)$ is the normed eigenvector of matrix $\langle \dot{\mathbf{z}}\dot{\mathbf{z}}^{\mathbf{T}} \rangle$that corresponds to the smallest eigenvalue [6]. The normed eigenvector of the second smallesteigenvalue gives the component with second smallest Δ value, i.e. $\Delta(y_2)$, and so on. Principal component analysis is applied to complete this step.

Fault Detection based on SFA

As the orthonormal set of vectors is derived from PCA, the fault detection procedure can be inherited from PCA. Two statistics are constructed:

$$T^2 = z^T W_s \Lambda^{-1} W_s^T z, \quad W_s \in R^{P\times r}$$

(11)

$$SPE = \|\tilde{\boldsymbol{z}}\|^2 = \left\|(\boldsymbol{I_P} - \boldsymbol{W_s}\boldsymbol{W_s}^T)\boldsymbol{z}\right\|^2, \qquad \boldsymbol{I_P} \in R^{\boldsymbol{P}\times\boldsymbol{P}}$$

(12)

$\boldsymbol{W_s}$is the reduced matrix of $\boldsymbol{W}$ and consists of r selected eigenvectors. $\boldsymbol{\Lambda}$is a diagonal matrix with r smallest eigenvalues as its diagonal entries. T^2reflects the process changes through the norm vibration of the slow features. SPEexpresses the distance of a sample to the feature space and reflects the deviation of a measured value from the model.

Simulation and Result

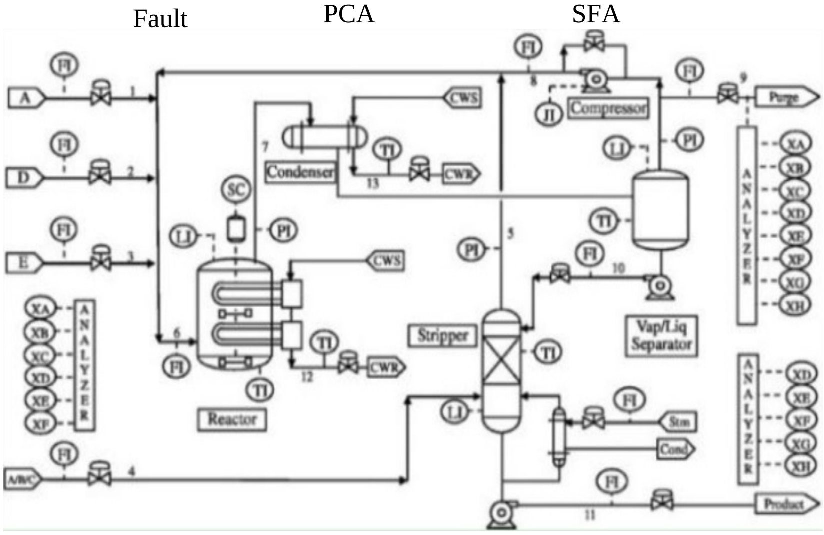
TE process experimental platform [7,8] was developed by Downs and Vogel in 1993. It can simulate the complicated industrial conditions in reality very well.

The training set consists of 500 normal samples, each of which contains 33 variables. The specific definition of each variable can be referred to in [7]. Firstly, perform SFA on the training dataset. Then tests are done on the test datasets. Each test dataset has 960 samples of 33 dimensions. The first 160 samples of the test dataset are normal data, while the following 800 samples are fault data. The detection results of SFA are listed in Table 1 with a comparison with that of PCA.

Figure 1 TE Process

Table 1Comparison of Fault Detection Accuracy between PCA and SFA

Fault	PCA	SFA



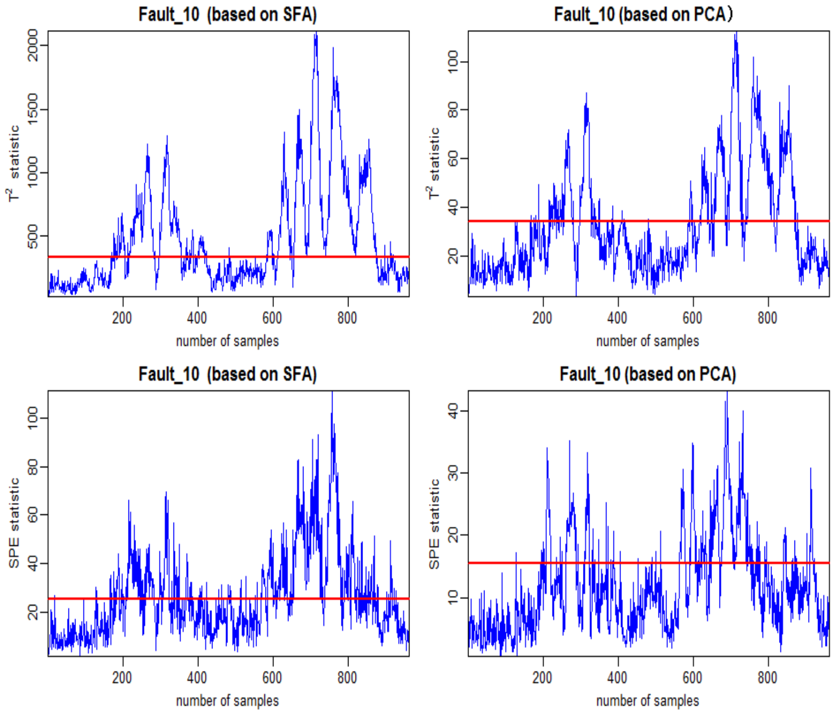


Figure 2 Comparison of Fault Detection Effectiveness between PCA and SFA when Fault 10 happens

Table 1 indicates that the T2 statistic of SFA has a better performance than that of PCA. As to SPE statistic, each has advantage. Fig. 2 illustrates the feasibility and effectiveness of the SFA method for fault detection

Conclusion

This paper proposes a fault detection method based on SFA to overcome the weakness of conventional PCA method that it deals only with linear models. SFA applies to both linear and nonlinear mixed models. It draws the slowly varying information from the process data and uses this information to detect fault in the industrial process. Slowness characterizes the industrial process inherently. The simulation results on the TE process have shown the feasibility and effectiveness of the SFA method for fault detection. In the future, two aspects of work can be done to optimize the process monitoring performance based on SFA. Firstly, use a kernel-based nonlinear extension of input signal, which enriches the feature space to a great extent. Thus the extracted signals contain more intrinsic information of the industrial process. Then better results can be expected. Secondly, the outputs of SFA are uncorrelated, but not independent. It means the higher order information is not taken into consideration. Therefore, efforts can be made to incorporate the goal of independence in the scheme of SFA algorithm.

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