An Adaptive Independent Component Analysis Method

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Abstract.

According to the existing problem of the convention methods, an adaptive independent component analysis method is proposed. First, the signals are divided into the heavy tailed and light tailed signals according to the kurtosis. For the heavy tailed signal, the method off-line computes the score function and establishes the lookup table of the standard alpha stable distribution, and then compute the score function of the mixture signals. For the light tailed signal, the score function is estimated by the general Gaussian model. Simulated results show that, the proposed algorithm has a well performance and a lower computational complexity.

Keywords: Independent Component Analysis; Blind Source Separation; Alpha Stable Distribution; General Gaussian Model

Introduction

In recent years, independent component analysis (ICA) techniques have been widely applied in the blind source separation[1-4]. The ICA method should depend on the probability density function (PDF) of each source. The tail characteristic is the key factor of the PDF. The signal is the heavy tailed (super-Gaussian) signal if kurtosis is positive, otherwise it is light tailed (sub-Gaussian) signal. Some traditional methods rely on assumptions on the source statistics [5]. Some algorithms cannot produce the desired source when the assumptions are inaccurate [2]. Methods that employ a flexible PDF model have been introduced [5,6]. These methods usually select alternative the score function in an iterative process.

Recently, some state-of-the-art approach have well extraction effect by using the accurate density estimation method [7-12]. However, these methods is not suitable for the impulsive data, and moreover have the high computational complexity.

In this paper, an adaptive ICA method using alpha stable distribution and general Gaussian model (GGM), is introduced. The signals are divided into the heavy tailed and light tailed signals according to the kurtosis. For the standard heavy tailed and light tailed distribution, the score function lookup table are established in advance based on the alpha stable distribution and GGM, respectively; and then compute the score function of the mixture signal.

ICA model

Let $s_1, s_2, \ldots, s_N$ is the dependent source signal, where N is the signal dimension. $\mathbf{X}=\mathbf{AS}$ is the mixture signal that are mixed by an unknown, full-rank matrix $\mathbf{A}_{N\times N}$. The reconstruction of the sources is attempted through a linear projection of $\mathbf{Y}=\mathbf{WX}$.

The basic principle of most ICA frameworks is the information entropy maximization between the reconstructed signals, and the objective function is reduced to

$$J(\mathbf{W}) = \sum_{i=1}^{n} H(y_i) - \ln|\det \mathbf{W}|, \tag{1}$$

where $H(\cdot)$ denotes the entropy. The natural Riemannian gradient learning algorithm is given by [3]:

$$\frac{\mathrm{d}\mathbf{W}}{\mathrm{d}t} = \eta(\mathbf{I} - \phi(\mathbf{Y})\mathbf{Y}^T)\mathbf{W}, \tag{2}$$

where $\mathbf{I}$ is unit matrix, η is the learning rate, and the score function $ø(\mathbf{Y})$ is given by:

$$\phi(\mathbf{Y}) = [\phi_1(y_1), \ldots, \phi_n(y_n)]^T = [-\frac{p_1^{'}(y_1)}{p_1(y_1)}, \ldots, -\frac{p_n^{'}(y_n)}{p_n(y_n)}]^T, \tag{3}$$

where $p'(y_i)$ is the derivative of the PDF $p'(y_i)$ with respect to y_i.

Score Function Estimation of Heavy Tailed Signal

As mentioned above, some ICA methods are not suitable for the impulsive data that follows the heavy tailed distribution. Here, we estimate the mixture signal

PDF and score function by the Alpha stable distributions.

Alpha stable distributions are suitable for heavy tailed distribution signals such as various impulsive data [13,14]. Alpha stable distribution characteristic depends on the characteristic, symmetry, scale, and location parameters. Characteristic parameter α sets the thickness of the tails and the impulsiveness of the distribution; symmetry parameter β sets the skewness; scale parameter γ sets the dispersion around the mean; location parameter δ sets the shift of the PDF.

Alpha stable distribution is described only by the characteristic function [13]

$$\varphi(t)=\begin{cases}\exp\left\{j\delta t-\gamma|t|^{\alpha}[1+j\beta sign(t)\tan(\alpha\pi/2)\right\}, if\,\alpha\neq 1\\ \exp\left\{j\delta t-\gamma|t|^{\alpha}[1+j\beta sign(t)2\log|t|/\pi\right\}, if\,\alpha=1\end{cases}. \tag{4}$$

The PDF of a random signal Y is equal to the Fourier transform of the characteristic function

$$f(Y)_{\alpha,\beta,\gamma,\delta}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\varphi(t)e^{-jYt}dt\,. \tag{5}$$

For an standard alpha stable distribution ($\gamma=1, \delta=0$), Eq. 6 can can be simplified as:

$$f(Y)_{\alpha,\beta,1,0}=\begin{cases}\frac{1}{\pi}\int_{0}^{+\infty}e^{-t^{\alpha}}\cos(Yt+\beta(t-t^{\alpha})\tan\frac{\pi\alpha}{2})\mathrm{d}t,\alpha\neq 1\\ \frac{1}{\pi}\int_{0}^{+\infty}e^{-t^{\alpha}}\cos(Yt+\frac{2\beta}{\pi}t\log t)\mathrm{d}t,\alpha=1\end{cases}. \tag{6}$$

Therefore, a non-standard alpha stable distribution can be written as

$$f(X)_{\alpha,\beta,\gamma,0}=f(Y)_{\alpha,\beta,1,0}\,, \tag{7}$$

where $X=Y/\gamma$.

The computing time is rather large in the direct numerical integration of Eq. 6. Therefore, for the sake of decreasing the computing time, we obtain the PDF and score function of the standard alpha stable distribution by the fast Fourier transform (FFT) off-line calculation; then establish the lookup table; last estimate

α, β, γ and the score function in the lookup table.

The process is as follows:

1) Sample (α_i,β_i), i=1,2,…,m in the [0,2] of α and [-1,1] of β respectively.

2) Compute the score function of in (α_i,β_i), and establish the lookup table of the score function.

2) Compute the parameter α, β and γ of the signal Y.

3) Seek the value of $f(Y)_{\alpha,\beta,1,0}$ in the lookup table that is close to (α_i,β_i) and compute the score function of Y.

Score Function Estimation of Light Tailed Signal

In order to estimate a the PDF and score function of the light-tailed distribution, we describe the light-tailed signal using the GGM [6]. The PDF of the generalized Gaussian distribution with a shape parameter λ_θ and a scaling factor θ is represented by

$$p_\theta(y) = \frac{\theta\lambda_\theta}{2\Gamma(1/\theta)} \exp(-|\lambda_\theta y|^\theta) . \tag{8}$$

The kurtosis k_θ and shape parameter θ can be obtained by

$$k_\theta = \frac{\Gamma(5/\theta)\Gamma(1/\theta)}{\Gamma^2(3/\theta)} - 3 . \tag{9}$$

In order to obtain $p_\theta(y)$ in Eq. 8, we need compute θ. However, θ is unknown but k_θ is known in the Eq .9. Therefore, we establish off-line a lookup table of θ. In the iterative process, we seek the value of θ in the table that is close to k_θ.

Simulation Experiments

In order to investigate the performance of the proposed adaptive ICA algorithm, the blind separation was attempted with the Extended InfoMax ICA [5], Kernel ICA [7], and our algorithm. The kurtosis of source signals were 0.261,0, -0.3778, -0.1278 from up to down, as shown in Fig. 1. The sample size was 1000, and A was a 4×4 random matrix. Fig. 2 are the mixture signals.

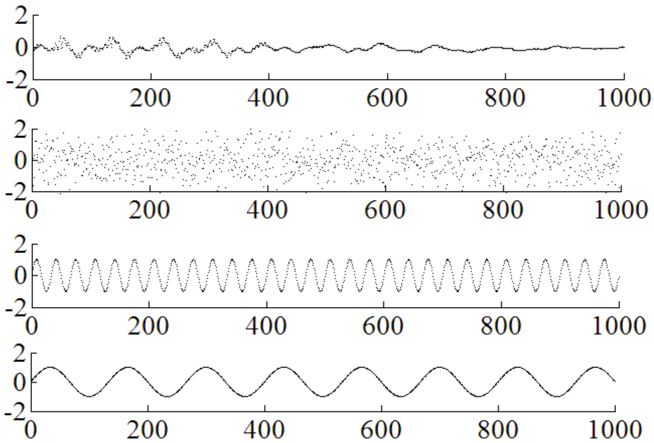

Fig. 1. The source signals

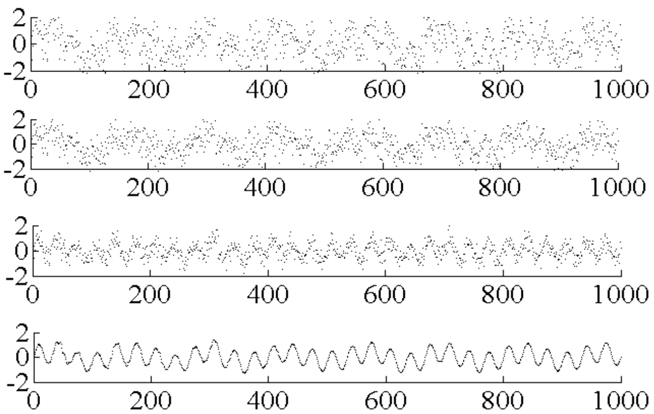

Fig. 2. The mixture signals

The weight matrix was initialized as the identity matrix. The learning rate η was 0.05. Fig. 3 are the reconstructed signals.

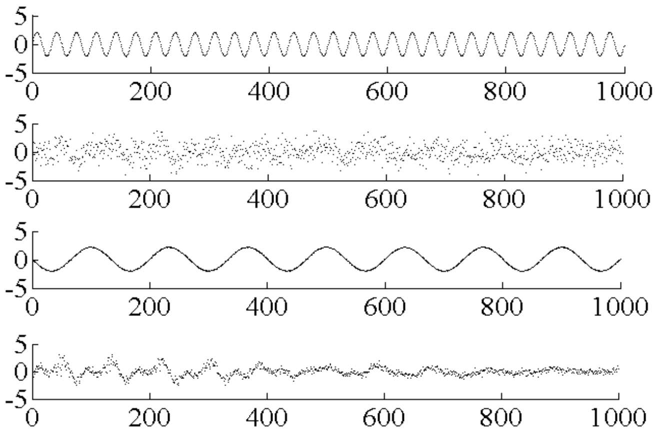

Fig. 3. The reconstructed signals

The separation performance was evaluated in terms of median signal-to-interference ratio (SIR) defined as [8]

$$SIR = 10\log\left(\sum_{m=1}^{M} S_m^2 \Big/ \sum_{m=1}^{M} (Y_m - S_m)^2\right), \qquad (10)$$

where M is the sample size. The results show that the proposed ICA algorithm has a well separation performance with different sample sizes, as presented in Fig. 4.

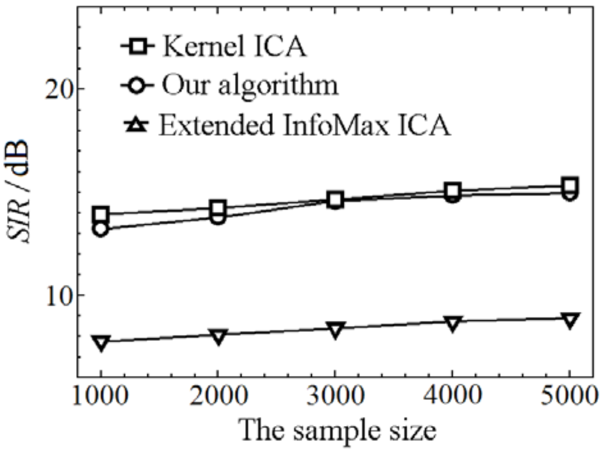

Fig. 4. Results of the separation performance

The CPU time of the three ICA algorithms is shown in Fig. 5 with various sample sizes. Here, the sample dimension is 6. It is noticed that our algorithm is faster than Kernel ICA.

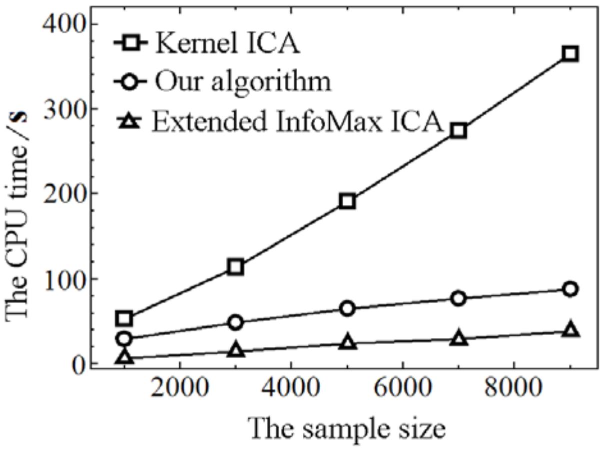

Fig. 5. Running time in terms of CPU seconds

Summary

We proposed an adaptive ICA method that is blind to the distribution of the original signal. Simulated results show a well convergence property and a lower computational complexity of the proposed method. The method is suitable for the analysis of real-world mixtures, especially for the heavy tailed signal.

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