

# Symmetric graphs of order $4p$ of valency prime

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## Abstract.

A graph is symmetric or arc-transitive if its automorphism group acts transitively on vertices, edges and arcs. Let  $p, q$  be odd primes with  $p, q \geq 5$  and  $X$  a  $q$ -valent symmetric graph of order  $4p$ . In this paper, we proved that  $X \cong K_{4p}$  with  $4p-1=q$ ,  $X \cong K_{2p, 2p}-2pK_2$  with  $2p-1=q$ , the quotient graph of  $X$  is isomorphic to  $K_{p, p}$  and  $p=q$ , or  $K_{2p}$  and  $2p-1=q$ .

**Keywords:** *Symmetric graph; Transitive graph; quotient graph; complete graph; prime*

## 1 Introduction

Throughout this paper we denote by  $Z_n$  the cyclic group of order  $n$  and by  $Z_n^*$  the multiplicative group of  $Z_n$  consisting of numbers coprime to  $n$ . Let  $D_{2n}$  be the dihedral group of order  $2n$ , and let  $A_n$  and  $S_n$  be the alternating and symmetric group of degree  $n$ , respectively. All graphs are assumed to be finite, simple and undirected, but with an implicit orientation of the edges when appropriate. For a graph  $X$ , let  $V(X)$ ,  $E(X)$ ,  $A(X)$  and  $\text{Aut}(X)$  be the vertex set, the edge set, the arc set and the automorphism group of  $X$ , respectively. A graph  $X$  is said to be vertex-transitive, edge-transitive or arc-transitive (symmetric) if  $\text{Aut}(X)$  acts transitively on  $V(X)$ ,  $E(X)$ , or  $A(X)$ , respectively.

To end this section, we introduce the so called quotient graph of a graph  $X$ . Let  $G \leq \text{Aut}(X)$  acts imprimitively on  $V(X)$ . Then  $G$  has a complete block system  $\Sigma = \{B_0, B_1, \dots, B_{n-1}\}$  on  $V(X)$ . The quotient graph  $X_\Sigma$  of  $X$  relative to  $\Sigma$  is defined to have vertex set and edge set as following:

$$V(X_\Sigma) = \Sigma,$$

$$E(X_\Sigma) = \{\{B_i, B_j\} \mid \text{there exist } v_i \in B_i, v_j \in B_j \text{ such that } \{v_i, v_j\} \in E(X)\}$$

In particular, if  $N$  is a normal subgroup of  $G$  then the set of orbits of  $N$  on  $V(X)$  is a complete block system of  $G$ . In this case, the quotient graph of  $X$  relative to the orbits of  $N$  is also called the quotient graph of  $X$  relative to  $N$ , denoted by  $X_N$ . It is easy to see that if  $X$  is edge-transitive then the valency of  $X_N$  is a divisor of the valency of  $X$ .

One of the standard problems in the study of symmetric graphs is to classify such graphs of certain orders. Let  $p$  be a prime. The classification of symmetric

graphs of order  $p$ ,  $p^2$  and  $2p$  were given in [1-2]. Wang and Xu [3] classified the symmetric graphs of order  $3p$ . In particular, constructing and classifying the symmetric graphs of small order is currently one of active topics in algebraic graph theory [4, 5, 6]. Let  $p, q$  be odd primes with  $p, q \geq 5$  and  $X$  a  $q$ -valent symmetric graph of order  $4p$ . In this paper, we proved that  $X \cong K_{4p}$  with  $4p-1=q$ ,  $X \cong K_{2p, 2p-2pK_2}$  with  $2p-1=q$ , the quotient graph of  $X$  is isomorphic to  $K_{p,p}$  and  $p=q$ , or  $K_{2p}$  and  $2p-1=q$ .

## 2 Preliminary Results

Cheng and Oxley [2] classified the connected symmetric graphs of order  $2p$  for a prime  $p$ . To extract a classification of connected  $q$ -,  $2q$ - and  $4q$ -valent symmetric graphs of order  $2p$  for a prime  $q \geq 5$ , we need to define some graphs. Let  $V$  and  $V'$  be two disjoint copies of  $Z_p$ , say  $V = \{i \mid i \in Z_p\}$  and  $V' = \{i' \mid i \in Z_p\}$ . Let  $r$  be a positive integer dividing  $p-1$  and  $H(p, r)$  the unique subgroup of  $Z_p^*$  of order  $r$ . Define the graph  $G(2p, r)$  to have vertex set  $V \cup V'$  and edge set  $\{xy' \mid x, y \in Z_p, y-x \in H(p, r)\}$ . Clearly,  $G(2p, p-1) \cong K_{p,p} - pK_2$ , the complete bipartite graph of order  $2p$  minus a one factor. Furthermore, assume that  $r$  is an even integer dividing  $p-1$ . Then the graph  $G(2, p, r)$  is defined to have vertex set  $V \cup V'$  and edge set  $\{xy, x'y, xy', x'y'\} \mid x, y \in Z_p, y-x \in H(p, r)\}$ .

### Proposition 2.1.

[2, Theorem 2.4 and Table 1] Let  $p, q$  be odd primes with  $q \geq 5$  and let  $X$  be a connected edge-transitive graph of order  $2p$ . Then  $X$  is symmetric. Furthermore, if  $X$  has valency  $q$  then one of the following holds:

- (1)  $X \cong K_{2p}$ , the complete graph of order  $2p$ , and  $2p-1=q$ ;
- (2)  $X \cong K_{p,p}$ , the complete bipartite graph of order  $2p$ , and  $p=q$ ;
- (3)  $X \cong G(2p, q)$  with  $q \mid (p-1)$  and  $(p, q) \neq (11, 5)$ , and  $\text{Aut}(X) \cong (Z_p \rtimes Z_q) \rtimes Z_2$ ;
- (4)  $X \cong G(2 \cdot 11, 5)$  and  $\text{Aut}(X) \cong \text{PSL}(2, 11) \rtimes Z_2$ .

If  $X$  has valency  $2q$  then  $X$  is bipartite and one of the following holds:

- (5) For  $2q < p-1$ ,  $X \cong G(2p, 2q)$  with  $2q \mid (p-1)$  and  $\text{Aut}(X) \cong (Z_p \rtimes Z_{2q}) \rtimes Z_2$ ;
- (6) For  $2q=p-1$ ,  $X \cong K_{p,p} - pK_2$  and  $\text{Aut}(X) \cong Z_p \rtimes Z_2$ .

If  $X$  has valency  $4q$  then one of the following holds:

- (7)  $X$  is non-bipartite,  $X \cong G(2, p, 2q)$  with  $2q \mid p-1$ ; for  $2q < p-1$ ,  $\text{Aut}(X) \cong \frac{Z_p^p}{Z_{2q}} \rtimes (Z_p \rtimes Z_{2q})$  and for  $2q=p-1$ ,  $\text{Aut}(X) \cong \frac{Z_p^p}{Z_{2q}} \rtimes S_p$ .
- (8)  $X$  is bipartite and  $X \cong G(2p, 4q)$  with  $4q \mid (p-1)$ ; for  $4q < p-1$ ,  $\text{Aut}(X) \cong (Z_p \rtimes Z_{4q}) \rtimes Z_2$  and for  $4q=p-1$ ,  $X \cong K_{p,p} - pK_2$  and  $\text{Aut}(X) \cong S_p \rtimes Z_2$ .

The socle of a finite group  $G$ , denoted by  $\text{soc}(G)$ , is the product of all minimal normal subgroups of  $G$ . One may extract the following results from [7, Table 3].

### Proposition 2.2.

Let  $p$  be a prime and  $G$  a primitive group of degree  $n$ .

- (1) For  $n = p$ ,  $G$  is either solvable with a normal Sylow  $p$ -subgroup or non-solvable with the following table, where  $d$  and  $k$  denote the degree and transitive multiplicity, respectively.

Table 1 Sylow p-subgroup or non-solvable with data

Soc(G)	d	k	comment
$A_p$	p	p-2	$G=A_p$
$A_p$	p	p	$G=S_p$
$PSL(2, 2^{2^s})$	$p=2^{2^s}+1$	3	$S>0$
$PSL(n,q)$	$p=(q^n-1)/(q-1)$	2	$n>3, n$ odd
$PSL(2,11)$	11	2	
$M_{11}$	11	4	
$M_{23}$	23	4	

(2) For  $n=2p$ , either  $G$  is 2-transitive or  $p=5$ .

(3) For  $n=4p$ , either  $G$  is 2-transitive or  $p=7, 13$ , or  $17$ .

Also, one may see Proposition 2.2(1) and 2.2(2) from [8, Corollary 3.5B] and [2, Theorem 1.1], respectively. Moreover, if  $G$  is primitive, but not 2-transitive of degree  $2p$  then  $p=5$  and  $G \cong A_5$  or  $S_5$ . If  $G$  is primitive, but not 2-transitive of degree  $4p$  then  $G \cong A_8, S_8, PSL(2,8)$  or  $PGL(2,7)$  for  $p=7$ ,  $G \cong \text{Aut}(PSL(3,3))$  for  $p=13$ , or  $G$  is isomorphic to a subgroup between  $PSL(2,16)$  and  $PGL(2,16)$  for  $p=17$ .

**Proposition 2.3.** Let  $G$  be a 2-transitive permutation group of degree  $2p$ . Then  $\text{soc}(G) \cong A_{2p}, M_{22}$  or  $PSL(2, r^n)$ .

## MAIN RESULT

Let  $p, q \geq 5$  be odd primes with  $q \geq 5$ . In this section we classify the  $q$ -valent symmetric graphs of order  $4p$ . The mainly ideas for this paper come from two situation which is “Primitive” and “Non-Primitive”, figure 1 showed the process of the method

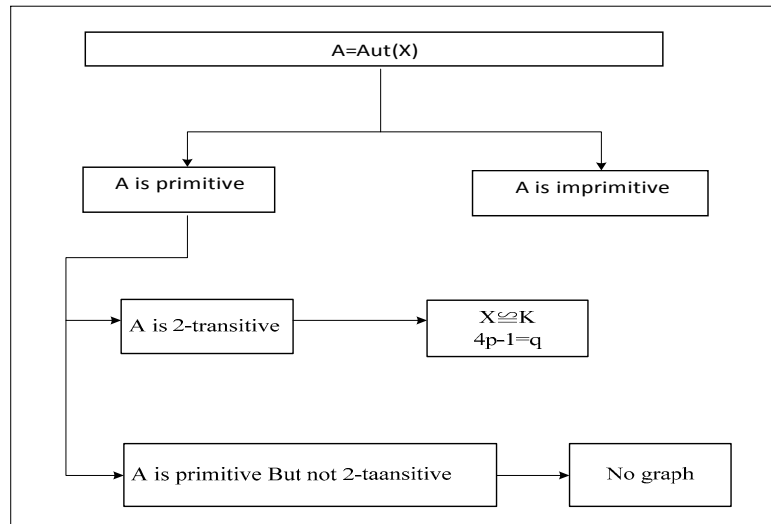


Figure 1 process of the method

**Theorem 3.1.** Let  $p, q$  be odd primes with  $p, q \geq 5$  and  $X$  a  $q$ -valent symmetric graph of order  $4p$ . Then  $X \cong K_{4p}$  with  $4p-1=q$ ,  $X \cong K_{2p,2p} - 2pK_2$  with  $2p-1=q$ , the quotient graph is isomorphic to  $K_{p,p}$  and  $p=q$ , or  $K_{2p}$  and  $2p-1=q$ .

**Proof.** Let  $A = \text{Aut}(X)$ . Since  $X$  is symmetric graph of valency  $q$ ,  $q \mid |A_\alpha|$  for some  $\alpha \in V(X)$ . First assume that  $A$  is 2-transitive on  $V(X)$ . Then  $X \cong K_{4p}$  with  $4p-1=q$ . Now assume that  $A$  is primitive, but not 2-transitive. By Proposition 2.2, we have  $A \cong A_8, S_8, \text{PGL}(2,7), \text{PSL}(2,8)$  with  $p=7$ , or  $A \cong \text{Aut}(\text{PSL}(3,3))$  with  $p=13$ , or  $\text{PSL}(2,16) \leq A \leq \text{P}\Gamma\text{L}(2,16)$  with  $p=17$ . Suppose that  $A = \text{PGL}(2,7), \text{PSL}(2,8)$  or  $\text{Aut}(\text{PSL}(3,3))$ . Then  $|A_\alpha| = 2^2 \cdot 3, 2 \cdot 3^2$  or  $2^3 \cdot 3^3$ , implying that  $q=3$ , a contradiction. Suppose that  $A = A_8$ , or  $S_8$ . Then  $|A_\alpha| = 2^4 \cdot 3^2 \cdot 5$  or  $2 \cdot 3^2 \cdot 5$ , implying that  $q=5$ . It is impossible because the subdegrees of  $A$  are 1, 12, 15. Suppose that  $\text{PSL}(2,16) \leq A \leq \text{P}\Gamma\text{L}(2,16)$ . Then  $|A_\alpha| = 2^2 \cdot 3 \cdot 5, 2^3 \cdot 3 \cdot 5$  or  $2^4 \cdot 3 \cdot 5$ , implying that  $q = 5$ . By Magma[9], the subdegrees of  $A$  are 1, 12, 15, 20, 20, and 1, 12, 15, 20, 20, and 1, 12, 15, 40, respectively. It is impossible. Thus, we may assume that  $A$  is imprimitive.

Let  $B$  be a non-trivial block of  $A$  on  $V(X)$ . Since  $|V(X)|=4p$ , we have  $|B|=p, 4, 2$  or  $2p$ . It follows that  $B = \{B^a \mid a \in A\}$  is a complete block system of  $A$  on  $V(X)$ . Consider the quotient graph  $X_B$  relative to  $B$  and let  $K$  be the kernel of  $A$  on  $B$ . Then  $K$  is a normal subgroup of  $A$ , implying that  $K_\alpha$  is a normal subgroup of  $A_\alpha$ . Since  $X$  has valency  $q$ ,  $A_\alpha$  is primitive on  $N(\alpha)$ . It follows that  $K_\alpha = 1$  or  $q \mid |K_\alpha|$ . We can get the complete graph by using Primitive group in “Non-Primitive” situation. Figure 2 showed the structure of the method. For the “Non-Primitive” situation, we can get the remaining graphs from four different lengths.

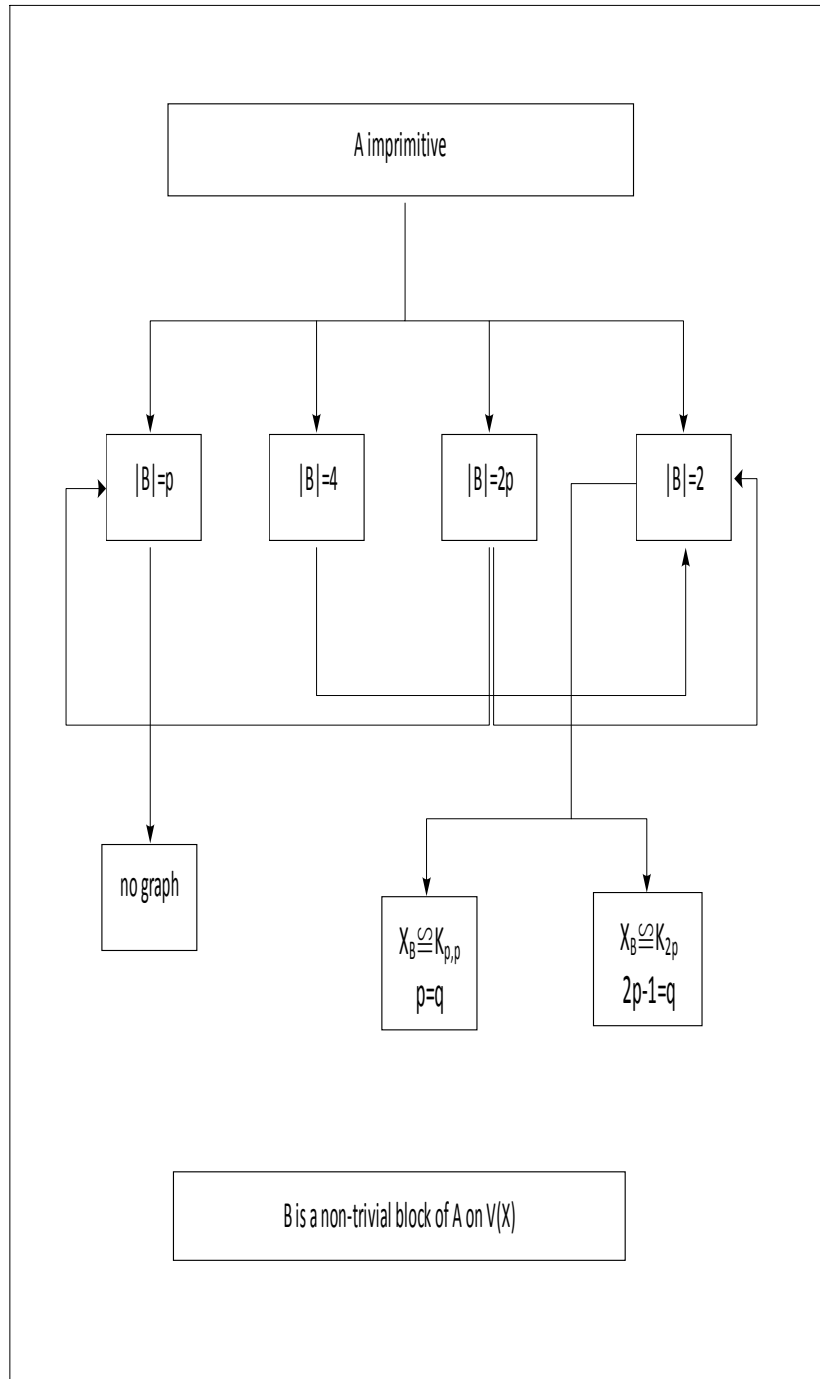


Figure 2 Structure of the method

**Case I:**  $|B|=p$ .

In this case,  $|X_B|=4$  and  $A/K \leq S_4$ . Thus,  $pq \mid |K|$ . It follows that  $X$  is unconnected, a contradiction. In particular,  $A$  has no non-trivial normal  $p$ -subgroup.

**Case II:**  $|B|=4$ .

In this case,  $K$  is a  $\{2,3\}$ -group. It follows that  $K_\alpha = 1$  and  $|K| \leq 4$ . Since  $A/K$  is transitive on  $X_B$ , by Proposition 2.2,  $A/K$  has a normal  $p$ -subgroup or  $A/K$  is 2-transitive on  $V(X_B)$ . Suppose that  $A/K$  has a normal  $p$ -subgroup, say  $M/K$ . Then  $M$  is a normal subgroup of  $A$  and the Sylow  $p$ -subgroup  $P$  of  $M$  is characterized in  $M$ , implies that  $P$  is a normal subgroup of  $A$ , contrary to Case I. Thus,  $A/K$  is 2-transitive on  $V(X_B)$ , implying that  $X_B \cong K_p$ . Assume that  $k$  is the number of the edges between two blocks. Then  $(p-1)k = 4q$ , it follows that  $k=1$  and  $p-1=4q$ , or  $k=2$  and  $p-1=2q$ . By Proposition 2.2,  $\text{soc}(A/K) \cong A_p$ ,  $\text{PSL}(2, 2^{2^s})$  with  $p=2^{2^s}+1$ ,  $\text{PSL}(m,r)$  with  $p=(r^m-1)/(r-1)$ ,  $\text{PSL}(2,11)$ ,  $M_{11}$  or  $M_{23}$ . If  $\text{soc}(A/K) \cong A_p$  then  $|A/K|$  is divisible by  $1/2(p!)$ . From elementary number theory it is well known that there exists a prime  $t$  between  $q$  and  $2q$ . Since  $p-1 = 2q$  or  $4q$ , one has  $t \mid |A|$ , which is impossible because  $|A| = 4pqn$  where each prime divisor of  $n$  is a divisor of  $q!$ . If  $\text{soc}(A/K) \cong \text{PSL}(2, 2^{2^s})$  then  $p-1 = 2^{2^s} \neq 2q$  or  $4q$ , which is clearly impossible [10]. If  $\text{soc}(A/K) \cong \text{PSL}(m,r)$  then  $p=(r^m-1)/(r-1)=r^{m-1}+\dots+r+1$  and  $m \geq 3$  is odd. It follows that  $r(1+r) \mid (p-1)$ , which is also impossible because  $p-1=2q$  or  $4q$  and  $q \geq 5$  (note that  $4 \cdot 5 + 1 = 21$  is not a prime). Thus,  $\text{soc}(A/K) \cong \text{PSL}(2,11)$ ,  $M_{11}$  and  $p=11$ , or  $M_{23}$  and  $p=23$ . If  $|K|=2$  or  $4$ , then  $K$  is imprimitive on  $B$  since  $|B|=4$ . Let  $\Delta$  be a non-trivial block of  $K$  acting on  $B$ . Then  $\Delta$  is non-trivial block of  $A$ ,  $|\Delta|=2$  and  $\Sigma = \{\Delta^a \mid a \in A\}$  is a complete block system of  $A$ . (see case IV) Thus, we may assume that  $K=1$ . It follows that  $A \cong \text{PSL}(2,11)$ ,  $\text{PGL}(2,11)$ ,  $M_{11}$  or  $M_{23}$  because  $|\text{Out}(\text{PSL}(2,11))|=2$ , and  $|\text{Out}(M_{11})| = |\text{Out}(M_{23})| = 1$  (see [11]). Note that  $|\text{PSL}(2,11)| = 2^2 \cdot 3 \cdot 5 \cdot 11$ ,  $|M_{11}|=2^4 \cdot 3^2 \cdot 5 \cdot 11$  and  $|M_{23}|=2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 23$ . Since  $|V(X)|=4p$ , if  $A \cong \text{PSL}(2,11)$  or  $\text{PGL}(2,11)$  then  $A_u$  is a subgroup of  $A$  of order 15 or 30, respectively, which is not true. Similarly,  $A \neq M_{11}$  or  $M_{23}$  because  $M_{11}$  and  $M_{23}$  have no subgroups of order  $2^2 \cdot 3^2 \cdot 5$  and  $2^5 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$ , respectively.

**Case III:**  $|B|=2p$ .

In this case,  $X$  is a bipartite graph with bipartite sets  $B$  and  $B'$ . Let  $A^*$  be the subgroup of  $A$  fixing  $B$  and  $B'$  setwise. Then  $|A:A^*| = 2$  and  $A^*$  is a normal subgroup of  $A$ . Assume that  $A^*$  is imprimitive on  $B$ . Note that  $A^*$  is the block stabilizer of  $B$  in  $A$ . Thus, every non-trivial block of  $A^*$  on  $B$  is also a block of  $A$ , which has size 2 or  $p$  (see Case I and Case IV). Now assume that  $A^*$  is primitive. Since  $|B|=2p$ , by Proposition 2.2,  $A^*$  is 2-transitive on  $B$ . Then every vertex in  $B \setminus \{u\}$  has the same number of neighbors  $N(u)$ , say  $m$ . It follows that  $q(q-1)=(2p-1)m$ , implying that  $q \mid 2p-1$  and  $m \mid q-1$ . By Proposition 2.3,  $\text{soc}(A^*) \cong A_{2p}$ ,  $M_{22}$  or  $\text{PSL}(2, r^n)$ . If  $\text{soc}(A^*) \cong A_{2p}$  and  $q < 2p-1$  then there exists a prime  $t$  between  $q$  and  $2q$  such that  $t \mid |A^*|$ , contrary to the fact that  $|A| = 4pqn$  and each prime of  $n$  dividing  $q!$ . If  $q=2p-1$  then  $X \cong K_{2p,2p} - 2pK_2$ . If  $\text{soc}(A^*) \cong M_{22}$

then  $q=7$  since  $|M_{22}| = 2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$ . It follows that  $m=2$ . Since  $M_{22}$  is 3-transitive on  $B$ , we have  $A^*_{u,v}$  is transitive on  $B \setminus \{u,v\}$ . However,  $A^*_{u,v}$  fixes  $\{u',v'\}$  setwise, and  $|N(N(u) \setminus \{u',v'\}) \setminus \{u,v\}| = 10 < 20$ , contrary to the fact that  $A^*$  is 3-transitive on  $B$ . If  $\text{soc}(A^*) \cong \text{PSL}(2, r^n)$  then  $2p-1=r^{n-1}$ . Since  $q|2p-1$  and  $q^2|2p-1$ , we have  $n=2$  and  $r=q$ . It follows that  $2p-1=q$  and  $X \cong K_{2p, 2p-2pK_2}$ .

**Case IV:**  $|B|=2$ .

In this case,  $K$  is 2-group and  $|X_B|=2p$ . Since  $K_u=1$  or  $q \mid |K_u|$ , we have  $|K| \leq 2$ . Since  $X$  is symmetric,  $X_B$  is symmetric. Let  $X_B$  have valency  $d$  and let  $l$  be the number of edges between two blocks. Then  $2q=d \cdot l$ . It follows that  $l=1$  and  $d=2q$ , or  $l=2$  and  $d=q$ .

First assume that  $X_B$  has valency  $2q$ . By Proposition 2.1,  $X_B$  is symmetric, and  $PK/K \leq A/K$ , or  $X_B \cong K_{p,p} \cdot pK_2$  with  $p=2q$ . Let  $2q=p-1$  and  $L=(A/K)^*$ . By Proposition 2.2,  $L$  has a normal  $p$ -subgroup or  $\text{soc}(L)=A_p, \text{PSL}(2, 2^{2^s}), \text{PSL}(n,r), \text{PSL}(2,11), M_{11}$  or  $M_{23}$ . If  $\text{soc}(L)=A_p$  then there exists a prime  $t$  between  $q$  and  $2q$  such that  $t \mid |L|$ , a contradiction. If  $\text{soc}(L)=\text{PSL}(2, 2^{2^s})$  or  $\text{PSL}(n,r)$  then  $p-1 \neq 2q$ , a contradiction. Suppose that  $\text{soc}(L)=M_{11}$  or  $M_{23}$ . If  $|K|=2$  then  $K \in Z(A^*)$ , it is impossible because  $\text{Mult}(M_{11}) = \text{Mult}(M_{23})=1$ . If  $K=1$  then  $A^* = M_{11}$  or  $M_{23}$  because  $\text{Out}(M_{11}) = \text{Out}(M_{23})=1$ . However,  $M_{11}$  and  $M_{23}$  has no subgroup of index 22 or 46, respectively, a contradiction. Thus,  $\text{soc}(L)=\text{PSL}(2,11)$ , implying that  $L=\text{PSL}(2,11)$  or  $\text{PGL}(2,11)$ . Note that  $X_B$  is a Cayley graph on  $D_{2 \cdot 11}$ . If  $|K|=2$  then  $X$  is a Cayley graph on  $Q_{4p}$ . Furthermore,  $B$  is a subgroup of order 2 of  $Q_{4p}$ . It is impossible because  $Q_{4p}$  has unique an involution and  $X$  has no edges in  $B$ . If  $K=1$  then  $A^*=\text{PSL}(2,11)$  or  $\text{PGL}(2,11)$ . However,  $\text{PSL}(2,11)$  and  $\text{PGL}(2,11)$  both have no subgroup of index 22, a contradiction.

Now assume that  $X_B$  has valency  $q$ . By Proposition 2.1,  $X_B \cong K_{2p}$  with  $2p-1=q$ ,  $K_{p,p}$  with  $p=q$ , or  $G(2p,q)$  with  $q|p-1$ . Suppose that  $X_B \cong G(2p,q)$  and  $(p,q) \neq (11,5)$ . Then  $PK/K < A/K$ , implying that  $P < A$ , a contradiction. If  $X_B \cong G(2 \cdot 11,5)$  then  $A/K \leq \text{PSL}(2,11) \rtimes Z_2$ . If  $A/K \cong (Z_{11} \rtimes Z_5)$  then the Sylow 11-subgroup of  $A$  is normal in  $A$ , a contradiction. Thus,  $A/K_B \cong \text{PSL}(2,11) \rtimes Z_2$ . If  $|K|=2$  then  $X$  is a Cayley graph on  $Q_{4 \cdot 11}$ , a contradiction. If  $K=1$  then  $A=\text{PSL}(2,11) \rtimes Z_2$ . It is impossible because  $\text{PSL}(2,11)$  has no subgroup of index 22. Thus,  $X_B \cong K_{p,p}$  and  $p=q$ , or  $X_B \cong K_{2p}$  and  $2p-1=q$ .

## Conclusion

We give the classification of symmetric graphs of order  $4p$  of valency prime in the paper. It is proved that if such graph exists, it must be complete graph of order  $4p$  or complete bipartite graph minor 1- factor, or the quotient graph is isomorphic to complete bipartite graph or complete graph of order  $2p$ . The two former graphs must be symmetrical graphs, but the two later graphs should be further verified. We guess that both of these two graphs are symmetric. In addition, the classification of symmetric graphs of order of  $p, 2p$  and  $3p$  have been completely

given. For the symmetric graphs of order  $4p$ , only the classification of symmetric graphs of order  $4p$  of valency prime is done. We hope to use the theorem of non-primitive block to study the classification of symmetric graphs of order  $4p$  with general valency.

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## References

- [1] C.Y. Chao, On the classification of symmetric graphs with a prime number of vertices: *Trans. Amer. Math. Soc.* Vol.158(1971),p.247-256.
- [2] Y. Cheng, J. Oxley, On weakly symmetric graphs of order twice a prime: *J. Combin. Theory B* Vol.42(1987),p.196-211.
- [3] R.J. Wang, M. Y. Xu, A classification of symmetric graphs of order  $3p$ : *J. Combin. Theory B* Vol. 58(1993),p:197-216.
- [4] Y.T. Li and Y.Q. Feng, The cubic graphs of order  $4p$ , *Chinese Annals of Mathematics: Series A* Vol.30(2009), p.677-684.
- [5] M.Y. Xu and S.J. Xu, The symmetry properties of Cayley graphs of small valencies on the alternating group  $A_5$ : *Science in China, Ser A* Vol.47(2004),p. 593-604.
- [6] J.H. Kwak, Y.S. Kwon and J.M. Oh, Infinitely many one-regular Cayley graphs on dihedral groups of any prescribed valency: *J. Combin. Theory B* Vol. 98(2008) p.585-598.
- [7] M.W. Liebeck, J. Saxl, Primitive permutation groups containing an element of large prime order: *J. London. Math. Soc.* Vol. 31(1985),p. 237-249.
- [8] J.D. Dixon, B. Mortimer: *Permutation Groups* (Springer-Verlag, New York, 1996).
- [9] W. Bosma, C. Cannon and C. Playoust, The MAGMA algebra system I: The user language: *J. Symbolic Comput* Vol.24(1997),p. 235-265.
- [10] X.G. Fang, C.H. Li and M.Y. Xu, On edge-transitive Cayley graphs of valency 4, *Euro-pean J. Combin* Vol.25(2004) ,p.1107-1116.
- [11] H.J. Conway, R.T. Curtis, S.P. Norton and R.A. Parker, R.A Wilson: *Atlas of Finite Groups*(Oxford University Press, Oxford, 1985).