Dynamic Behavior of Traveling Wave Solutions for a class of nonlinear partial differential equations

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Abstract

Applying bifurcation method, some new exact traveling wave solutions of a class nonlinear partial differential equations generated by the Jaulent-Miodek hierarchy models are obtained, which corresponding to the kink wave, anti-kink wave solutions of its traveling wave system.

Keywords: Bifurcation, Kink wave solution, Unbounded wave solutions.

Introduction

In this paper, we discuss system

$$u_{xt}+\frac{1}{4}u_{xxxx}-\frac{3}{2}u_x^2u_{xx}+\frac{3}{16}u_{yy}+\frac{3}{4}u_{xx}u_y-\alpha u_{zz}=0, \qquad (1.1)$$

where α is constant[1]. The traveling wave system of (1.1) is derived[5]. Wazwaz [1] has obtained by Hirota bilinear method multiple soliton solutions which were formally derived. In this paper, we research the travel wave solutions of (1.1) by bifucation method of dynamical systems [3,5].

$$\frac{d\varphi}{d\xi}=y,$$

$$\frac{dy}{d\xi} = -\frac{(3r^2 - 16kc - 16\alpha)}{4k^4}\varphi - \frac{3r}{2k^2}\varphi^2 + 2\varphi^3 \tag{1.2}$$

where $u_\xi = \varphi$, $\xi = kx + ry + z - ct$.

The traveling wave solutions of (1.1) corresponding to peridioc wave solutions and solitary wave solutions, have been found in [5] completely. In this paper, we will research other traveling wave solutions which closely related to kink wave solutions of (1.2). This paper is organized as follows. In section 2, all the possible travel wave solutions of the system (1.1) are given, which correspond to solitary wave solutions of the system (1.2). Finally, the physical significance of solutions of (1.1) are given.

Exact traveling wave Solutions of the system (1.1)

Lemma 1

the system (1.2) has kink (anti-kink) wave solutions as follows (see Fig1(1,2,3,7,8,9)):

(1) when $r = -2\sqrt{kc+\alpha}$,

$$\varphi_1(\xi) = \frac{\sqrt{kc+\alpha}\,(-1 + \tanh(\frac{\sqrt{kc+\alpha}\,\xi}{2k^2}))}{2k^2}, \tag{2.1a}$$

$$\varphi_{1'}(\xi) == \frac{\sqrt{kc+\alpha}\,(-1 - \tanh(\frac{\sqrt{kc+\alpha}\,\xi}{2k^2}))}{2k^2}. \tag{2.1b}$$

(2) when $r = 2\sqrt{kc+\alpha}$,

$$\varphi_2(\xi) = \frac{\sqrt{kc+\alpha}\,(1 + \tanh(\frac{\sqrt{kc+\alpha}\,\xi}{2k^2}))}{2k^2}, \tag{2.2a}$$

$$\varphi_{2'}(\xi) = \frac{\sqrt{kc+\alpha}\,(1 - \tanh(\frac{\sqrt{kc+\alpha}\,\xi}{2k^2}))}{2k^2}. \tag{2.2b}$$

(3) when $r=0, kc+\alpha<0$,

$$\varphi_3(\xi)=\frac{\sqrt{-2(kc+\alpha)}\tanh(\frac{\sqrt{-2(kc+\alpha)}\xi}{k^2})}{k^2}, \tag{2.3a}$$

$$\varphi_{3'}(\xi)=-\frac{\sqrt{-2(kc+\alpha)}\tanh(\frac{\sqrt{-2(kc+\alpha)}\xi}{k^2})}{k^2}. \tag{2.3b}$$

Theorem 1

From lemma2.1, six unbounded exact traveling wave solutions (see Fig1(4,5,6,10,11,12)) of (1.1) corresponding to the kink wave solutions of (1.2) are obtained as follows:

$$u_1(\xi)=-\frac{1}{2}(\frac{\sqrt{kc+\alpha}\xi}{k^2}+\ln(1-\tanh^2(\frac{\sqrt{kc+\alpha}\xi}{k^2}))), \tag{2.4a}$$

$$u_{1'}(\xi)=\frac{1}{2}(-\frac{\sqrt{kc+\alpha}\xi}{k^2}+\ln(1-\tanh^2(\frac{\sqrt{kc+\alpha}\xi}{k^2}))), \tag{2.4b}$$

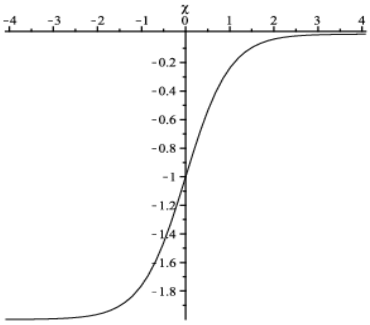
$$u_2(\xi)=-\frac{1}{2}(-\frac{\sqrt{kc+\alpha}\xi}{k^2}+\ln(1-\tanh^2(\frac{\sqrt{kc+\alpha}\xi}{k^2}))), \tag{2.5a}$$

$$u_{2'}(\xi)=\frac{1}{2}(\frac{\sqrt{kc+\alpha}\xi}{k^2}+\ln(1-\tanh^2(\frac{\sqrt{kc+\alpha}\xi}{k^2}))), \tag{2.5b}$$

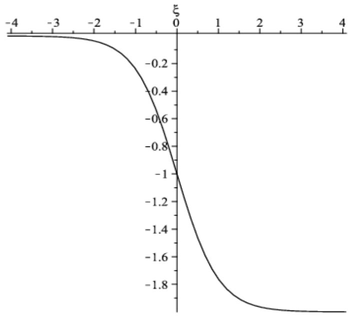
$$u_3(\xi)=-\ln(1-\tanh^2(\frac{\sqrt{-2kc-2\alpha}\xi}{2k^2})), \tag{2.6a}$$

$$u_{3'}(\xi)=\ln(1-\tanh^2(\frac{\sqrt{-2kc-2\alpha}\xi}{2k^2})). \tag{2.6b}$$

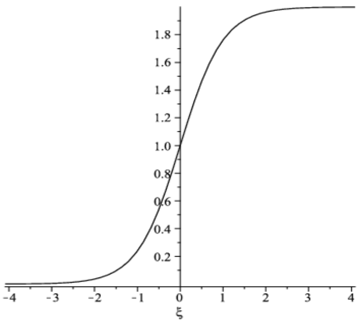
Proof : By lemma1 and $u_\xi=\varphi$, theorem 1 are easily proved.



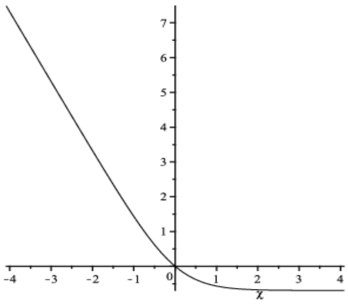
(1) φ_1



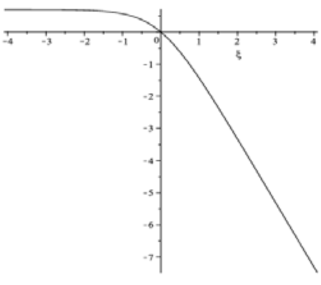
(2) $\varphi_{1'}$



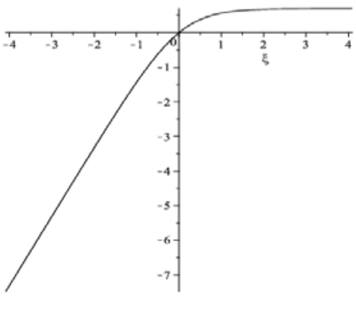
(3) φ_2



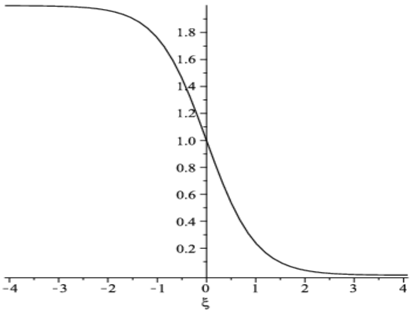
(4) u_1

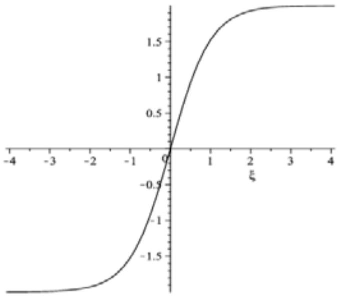


(5) $u_{1'}$

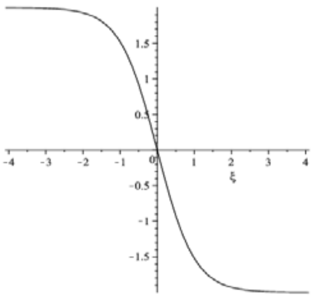


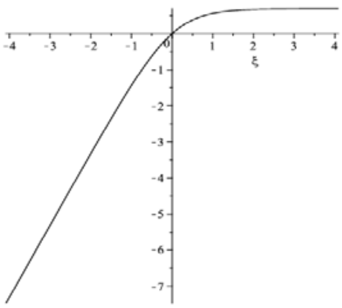
(6) u_2



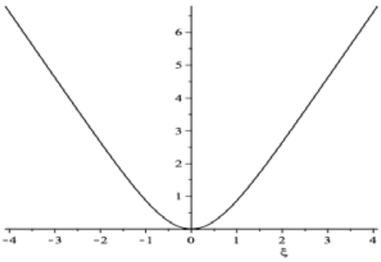


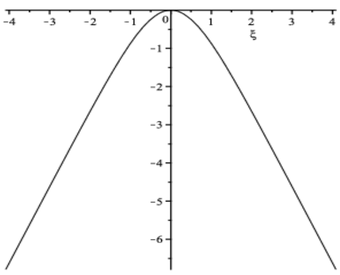
(7) $\varphi_{2'}$ (8) φ_3





(9) $\varphi_{3'}$ (10) $u_{2'}$





(11) u_3 (12) $u_{3'}$

Fig1:The wave solutions of the system (1.1) corresponding to the kink wave solutions of the system (1.2).

Conclusions

In this paper, a class of traveling solutions of (1.1) which are integration of all kink wave solutions are obtained. These solution are neither solitary wave solutions nor kink wave solutions. All the integration of all kink wave solutions are unbounded wave solutions. Our results are significant to analyze the integrate of traveling wave solutions.

References

[1] A. M. Wazwaz, Multiple soliton solutions for some (3+1)-dimensional nonlinear models generated by the Jaulent–Miodek hierarchy, Appl. Math. Lett. 21 (2012), no. 25, 1936--1940.

[2] A. M. Wazwaz, Partial Differential Equations and Solitary Waves Theorem, Springer and HEP, Berlin, 2009a.

[3] J. B. Li, Y. Zhang, X. H. Zhao, On a Class of Singular Nonliner Traveling Wave Equations (II): An Example of Gckdv Equations, Internat. J. Bifur. Chaos Appl.Sci. Engrg. 54 (2009), no. 19, 1995–2007.

[4] J. B. Li and G. R. Chen, Exact Traveling wave solutions and their bifurcations for the Kudryashov-Sinelshchikov equation, Internat. J. Bifur. Chaos Appl. Sci. Engrg.12 (2012), no. 22(5), 1250118-1-19.

[5] Y. P. Ran and J. Li, Bifurcation Method to Analysis of Traveling Wave Solutions for (3 + 1)-Dimensional Nonlinear Models Generated by the Jaulent-Miodek Hierarchy,The Bulletin of the Iranian Mathematical Society. _ _ _ (2014), in press.

[6] R. Hirota, The Direct Method in Soliton Theory, Cambridge Univ. Press, Cambridge,2004.