# Identification of Complex Blocks in the Corner of Underground Chamber 

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Keywords: underground chamber, corner, complex block, block theory.


#### Abstract

The combination of free planes in the corner of an underground chamber is complex. If the fractures cross the roof and walls, or floor and walls at the same time, a complex key block may exist. The theorem of finiteness in the classical block theory applies only to the convex block, and not to the non-convex. Based on the analysis of the system of spatial planes and the system of blocks, the operations of intersection and union between the half-spaces of surface planes are represented explicitly with two symbols. All the possible finite convex blocks can be analysed by enumeration at first. Then according to the theory of united blocks and the principles that the codes of fractures keep the same all the time, a complete combination of a non-convex block is determined. Further, cutting along the free planes which include concave intersections, the new combination, in which the convex blocks are connected only at the cutting planes, is optimized. Finally, the identification of complex block in the corner of an underground chamber is implemented. The case of practical engineering shows that, the method is correct and effective, and lay a solid theoretical foundation for the further study on the cases of edges or intersecting tunnel. The method also have some practical values to guarantee the safety of underground engineering.


## Introduction

The engineering of underground chambers exist widely in many areas of infrastructure, such as building, transportation, defences and so on. Collapse, rock fall and other geological disasters are the outstanding problems affecting the safety of underground chamber engineering. The essence is the stability of surrounding rocks, and also one of the core problems of rock mechanics.

In general, roof fall and collapse usually occur in the roof and walls of underground chamber, and mostly in the form of convex blocks. These problems have been settled maturely by the classical block theory which have learnt from the thoughts of limit equilibrium [1-3]. At the same time, the advantage of key block theory is that it has a strict mathematical basis, less input parameters, rapid geometric analysis and practical computing results. Block theory have been applied widely in many large projects at home and abroad, such as Three Gorges and so on [4]. However, There may exist key blocks crossing roof and walls at the same time in the edges, corners of underground chambers. These key blocks are often in the form of non-convex key blocks which include concave intersections of free planes [5-9]. The classical block theory applies only to the convex blocks, and not to the non-convex.

Aimed at the characteristics of complex combination between roof and walls in the corner of underground chamber, the spatial planes system and rock blocks system are analysed at first. Then a non-convex block can be represented with a symbolic method. Finally, the identification algorithm of complex block in the corner of underground chamber is designed and implemented.

## Spatial Planes System and Rock Blocks System in the Corner of Underground Chamber

## Spatial Planes System

The simplest form of a corner in an underground chamber consists of at least three free planes by artificial excavation. Rock mass is cut into blocks along with the fractures and free planes. The formed block system can only include convex blocks, and also have non-convex blocks possibly.

Since the fractures among the internal of rock mass may have curved surface, experiences show that most of them are flat planes, and the assumption plane can also meet the requirements of designs. Although most of excavation planes of chamber, especially the roofs, are designed to be curved surfaces, these surfaces can be represented with a series of flat planes connected with each other. Therefore, both fractures inside the rock mass and excavation planes can be attributed to a model of spatial plane system.
(1) Combination of free planes formed by artificial excavation.

The simplest form of a corner in underground chamber consists of at least three free planes of artificial excavations, i.e., a roof and two walls or a floor and two walls respectively, as shown in Fig. 1 and Fig. 2. From the view of the internal of solid rock mass, these three free planes intersected with each other, i.e., roof intersects with the two walls concavely, the two walls intersect with each other concavely, and a concave angle is formed. The geometric characteristics of the angle is that it consists of three planes which intersect concavely on one vertex and three edges. The excavation pyramid is the union of the solid half-spaces of these three free planes.
(2) Combination of fractures formed by nature.

Fractures are dispersed among the internal of rock mass, and its shape and size are difficult to obtain accurately [10]. The thought that fracture are assumed to be infinite appears a little conservative but to be safe. Therefore, the assumption is still adopted here. It has been proved mathematically that the intersections between infinite planes must be convex. Thus the joint pyramids are still the operations of intersections between fractures. It is not hard to find that, the intersection between fractures and free planes are also convex.


Fig.1: Operations of Corners on Roof and Walls


Fig.2: Operations of Corners on Floor and Walls

## Rock Blocks System

Rock masses can be cut into rock blocks with different size and different shape by fractures. If a block is exposed on a free planes, it may slide along or fall from some fractures while a certain mechanical condition is required, and the key block is formed at that time.
(1) The system of convex blocks.

If only one free plane is intersected with fractures, the block formed must be convex. The problems of finiteness and removability on convex blocks have been studied maturely in the classical block theory.

As stated above, the fractures have been assumed to be infinite. The operation of intersection between half-spaces of fractures can be represented explicitly with a symbol "i". The methods on block denotations in the classical bloc theory can be viewed to be omitted with the symbol by default.
(2) The system of non-convex blocks.

If the fractures intersect with at least two free planes and concave intersections are included between the two free planes, non-convex blocks possibly exist in the block system. Obviously, the corner of underground chamber is composed of at least three free planes intersected concavely with each other. Hence, if the fractures cross the roof and walls of underground chamber at the same time, a complex non-convex block may exist in the corner of the underground chamber.

The free planes in the corer intersected concavely with each other. The operation of union of solid half-spaces of free planes can be represented with a symbol "u" explicitly. Thus, the complex
relationships of combination between these surface half-spaces can be expressed clearly.

## Identification Algorithm of Complex Blocks in the Corner of Underground Chamber

## The Theory of United Blocks

A non-convex block can be viewed as a combination of several simple convex blocks. Thus, an important criterion can be introduced to be the basis of algorithm. That is, if all the convex blocks are finite, the non-convex block is also finite; conversely, if at least one simple convex block is infinite, the non-convex is also infinite.

The combination between surface half-spaces of non-convex block can be represented with a symbolic method. Then the algorithm of identification on the non-convex block can be designed according to the relationship between the whole and the parts.

## Complete Combination of Complex Blocks

Any non-convex block with complex shape can be viewed as a combination of several simple convex blocks. However, for the same non-convex block, it may have different combinations.

As a matter of experience, the fractures developed by group are less than five groups. Along the a limited number of free planes, all the convex blocks that may be combined to a non-convex block can be analyzed by enumeration. For a block system composed of $M$ fractures and ${ }^{N}$ free planes, while all the fractures intersect with each other convexly and free planes contains concave intersections, the space can be divided into $4^{(M+N)}$ combinations theoretically. In the actual project, two kinds of codes 0 and 1 of fractures are usually considered only, and three kinds of codes 0,1 and 2 are also only considered for free planes that contain concave intersections. In addition, the value of $M$ or $N$ is not more than five. On this occasion, the number of combinations to be considered is: ... Subtracting is because the joint blocks without free planes should be removed, i.e., the number of free planes needed to be analyzed is: $C_{N}^{1}+C_{N}^{2}+\cdots+C_{N}^{N}$.

## Optimized Combination of Complex Blocks

According to the theorems of finiteness on the convex block and non-convex block, a finite nonconvex block in the corner can be composed of a series of finite convex blocks. However, these convex blocks are overlapped with each other. A further optimized combination needs to be analyzed.

Step 1: Cut the space into many combinations by the plane system.
Step 2: Search out all the finite convex blocks in the block system.
Step 3: Combined the convex blocks which have the same codes of fractures.
Step 4: Cut along any concave edge, and one convex block can be $2^{M}$ determined from the group obtained from step 3; the code of half-space on the other side of cutting planes can be deduced.

Step 5: if there have been no concave intersections between free planes after cutting from step 4, the remaining block can be determined and the identification is finished; otherwise, go on cutting like the step 4 till no concave combinations of free planes.

When the identification is implemented, the volume of the non-convex block is the algebraic summation of the ones of each convex blocks, and the area is the algebraic sum subtracting the two times areas of cutting planes. Once each convex block is drawn, the shape of the non-convex block is determined.

## Case Study

## Project Overview

Combined with an example, the algorithm of identification of complex block in the corner of underground chamber can be validated. The corner of the underground chamber is composed of three fractures $P_{1}, P_{2}, P_{3}$ and three free planes $P_{4}, P_{5}, P_{6}$, and the free planes are intersected concavely with each other, as shown in Table 1 and Fig. 3.

Table 1: The Spatial Planes System in the Corner of Underground Chamber

| $P_{i}$ | $\alpha_{i}$ | $\beta_{i}$ | $X_{i}$ | $Y_{i}$ | $Z_{i}$ | $\varphi_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P_{1}$ | $75^{\circ}$ | $300^{\circ}$ | 0.0 | 1.0 | 0.0 | $30^{\circ}$ |
| $P_{2}$ | $75^{\circ}$ | $60^{\circ}$ | 1.0 | -0.866 | 0.0 | $25^{\circ}$ |
| $P_{3}$ | $75^{\circ}$ | $180^{\circ}$ | -1.0 | -0.866 | 0.0 | $35^{\circ}$ |
| $P_{4}$ | $55^{\circ}$ | $300^{\circ}$ | 0.0 | 1.0 | 0.0 | -- |
| $P_{5}$ | $55^{\circ}$ | $60^{\circ}$ | 1.0 | -0.866 | 0.0 | -- |
| $P_{6}$ | $55^{\circ}$ | $180^{\circ}$ | -1.0 | -0.866 | 0.0 | -- |

## Identification of Complex Blocks

According to the above method, the space can be divided into $2^{3} \times 3^{3}=216$ combinations by the spatial plane system in the corner of the underground chamber. The number of combinations to be considered is: $C_{3}^{1}+C_{3}^{2}+C_{3}^{3}=7$, i.e., three kinds of single free plane, three kinds of double free planes, one kind of tri free planes, and the case without free planes need not to be considered. Hence, the number of combinations is reduced to $2^{3}\left(3^{3}-1\right)=208$. By the theorem of finiteness on the convex blocks in the classical block theory, 68 kinds of convex blocks can be obtained.

According to the principle that the codes of fractures keep the same all the time and the criterion on the non-convex block, all the possible non-convex block system can be combined, shown in Table 2.

For a finite non-convex block which is composed of 19 finite convex blocks, it can be optimized with only three convex blocks which are connected at the cutting free planes, as shown in Fig. 4.


Fig.3: A Corner of an Underground Chamber


Fig.4: Complex Block in the Corner

Table 2: Complex Blocks System in the Corner of Underground Chamber

| No. | Finite non-convex blocks in the corner of underground chamber |
| :--- | :--- |
|  | $(000001,000010,000011,000012,000021,000100$ |
| 1 | $000101,000102,000110,000112,000111,000120$, |
|  | $000121,000122,000201,000210,000211,000212,000221)$ |
| 2 | $(001010,001100,001110,001120,001210)$ |
| 3 | $(010001,010100,010101,010102,010201)$ |
| 4 | $(011100,011101,011102,011110,011120)$ |
| 5 | $(100001,100010,100011,100012,100021)$ |
| 6 | $(101010,101011,101012,101110,101210)$ |
| 7 | $(110001,110011,110021,110101,110201)$ |
|  | $(111000,111001,111002,111010,111011,111012$, |
| 8 | $111020,111021,111022,111100,111101,111102$, |
|  | $111110,111120,111200,111201,111202,111210,111220)$ |



Fig.5: Stereographic Projection of Spatial Planes


Fig.6: Sphere Analysis of Joint Pyramid

The removability of the finite non-convex block can be analysed by the stereographic projection and the block sphere method, shown in Fig. 5 and Fig. 6 respectively. If only gravity is considered, the sliding modes, sliding directions, net sliding forces and factors of safety can be obtained in Table 3, where the direction of projection is $(0,0,1)$ and the vector of resultant is $(0,0,-1)$.

Table 3: Modes, Directions, Net Forces and Factors of Safety of Complex Blocks

| No. | JP | Mode | Direction | Net Force | FoS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 000 | -1 | -- | +0.000000 | 0.000000 |
| 2 | 111 | 0 | $(0.000,0.000,-1.000)$ | +1.000000 | 0.000000 |
| 3 | 011 | 1 | $(-0.598,0.345,-0.723)$ | +0.816639 | 0.154577 |
| 4 | 101 | 2 | $(0.598,0.345,-0.723)$ | +0.845360 | 0.124847 |
| 5 | 110 | 3 | $(0.000,-0.690,-0.723)$ | +0.784360 | 0.187470 |
| 6 | 001 | 12 | $(0.000,0.886,-0.464)$ | +0.431908 | 0.510001 |
| 7 | 010 | 13 | $(-0.767,-0.443,-0.464)$ | +0.331136 | 0.624300 |
| 8 | 100 | 23 | $(0.767,-0.443,-0.464)$ | +0.378977 | 0.570037 |

## Conclusions

The corner of underground chamber usually appears in the form of concave angle. The combinations of free planes of the concave angle is complex, and the simplest concave angle is composed of at least three planes which are intersected concavely with each other. If the fractures cross the roof and walls at the same time, the non-convex block may exists in the corner. The theorem of finiteness in the classical block theory only applies to the convex block, but not to the non-convex. In this paper, two symbols are used to represent the complex combination of solid halfspaces of free planes. Then on the basis of united block theory, all the finite convex blocks in the system can be searched out with an enumeration method. Next, according to the principle that the codes of fractures keep the same all the time, the finite non-convex blocks can be combined with the related convex blocks. Further, the combination is optimized with the convex blocks which are connected only at the cutting free planes. The practical example shows that, the algorithms of identification of the complex blocks in the corner of underground chamber are correct and effective. It has an active significance on several cases, such as edges and intersecting tunnels and so on. These are also directions of further study.

## Acknowledgements

The study is supported by the Distinguished Research Project of North China University of Water Resources and Electric Power (40274) and the project of the National Natural Science Foundation of China (41402269, 51409102).

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