

Effective Detuning of a Three-Level Atom Interacted with the Laguerre-Gaussian Laser Beam

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Abstract—We derive the general analytic expression of the effective detuning of the linearly polarized light field from the Λ -configuration three-level atomic mode. The detuning amending term resulting from the three-level atomic mode is not related with the type of light field, which is determined by the two lower level split and the ratio of the two lower level dipole matrix element to the total dipole matrix element in the three-level atomic mode. For a linearly polarized Laguerre-Gaussian laser beam, we show that there is same frequency amending for the axial, radial and azimuthal frequency detuning. We define an effective two-level atomic mode to substitute the three-level atomic mode, in which it can be more practical and convenient in consideration of the light detuning and Doppler shift from atomic transition frequency.

Keywords—three-level atomic mode, laguerre-Gaussian laser beam, light detuning, Doppler shift

I. INTRODUCTION

In 1936, Beth first experimentally observed the deflection of a quartz wave plate suspended from a thin quartz fiber with circularly polarized light passing through it [1]. In 1992, Allen et al. observed the torque on suspended cylindrical lenses arising from the reversal of the helicity of a Laguerre-Gaussian mode with well-defined orbital angular momentum [2]. Recently, the transfer of angular momentum from a light beam carrying angular momentum to absorbing macroscopic particles trapped in an optical tweezer causing them to rotate has been experimentally and theoretically studied [3-6]. Furthermore, an atom moving in a Laguerre-Gaussian beam experiences a torque and an azimuthal shift in its resonant frequency in addition to the usual axial Doppler shift and recoil shift [7-8]. Where they considered a two-level atom which is interacted with the Laguerre-Gaussian beam and obtained the axial, radial and azimuthal Doppler shift as a complementarity for the total detuning of the beam frequency from the atomic resonant frequency.

In this paper, we extend the two-level atomic mode to the three-level atomic mode because the three-level atom has been received considerable attention for creating many new quantum effects [9-10]. First we consider a Λ -configuration three-level atom in a light beam and derive the general expression of the detuning of the beam from the three-level atom. Then taking a linearly polarized

Laguerre-Gaussian for an example, we derive the total detuning of the Laguerre-Gaussian beam from the three-level atom. Finally, for the three-level atomic mode we derive an effective detuning and an effective two-level atomic mode.

II. CALCULATION AND DISCUSSION

We consider a Λ -configuration three-level atom with one upper level $|3\rangle$ and two lower levels $|1\rangle$, $|2\rangle$ with frequency split Δ_g , as shown in Fig.1, which is interacted with an arbitrary light field with the frequency ω_L . The atomic transition between $|1\rangle$ and $|3\rangle$, $|2\rangle$ and $|3\rangle$ are assumed to be optically allowed, while the transition between $|1\rangle$ and $|2\rangle$ is forbidden, which can be resulted from the D₁, D₂ line of alkali-metal atoms.

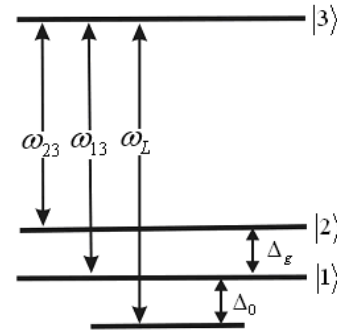


Figure 1. The Λ -configuration three-level atom diagram

The Hamiltonian of our system is written as [9]

$$H = \mathbf{P}^2/2m + U(\mathbf{R}) + \hbar\omega_1|1\rangle\langle 1| + \hbar\omega_2|2\rangle\langle 2| + \hbar\omega_3|3\rangle\langle 3| + \hbar\omega_L a^\dagger a + H_{\text{int}} \quad (1)$$

Where \mathbf{P} and \mathbf{R} are the momentum vector and the position vector of the atomic center of mass with atomic mass m , and $U(\mathbf{R})$ is trapping potential. Where $\hbar\omega_1$, $\hbar\omega_2$, $\hbar\omega_3$ are eigenenergies of levels $|1\rangle$, $|2\rangle$, $|3\rangle$, respectively, and $\omega_{13} = \omega_3 - \omega_1$, $\omega_{23} = \omega_3 - \omega_2$ are atomic transition frequencies. a^\dagger (a^-) is the creation (annihilation) operator of light field ω_L . The detuning of the laser frequency

from atomic transition ω_{13} is Δ_0 . The last part H_{int} is the interaction Hamiltonian, taking electronic dipole and rotating wave approximating, which in the interaction picture may be given by

$$H_{\text{int}} = -i\hbar \left\{ e^{-i\Delta_0 t} \pi^+ a f_1(\mathbf{R}, t) - f_1^*(\mathbf{R}, t) a^+ \pi e^{i\Delta_0 t} \right\} - i\hbar \left\{ e^{-i(\Delta_0 + \Delta_g)t} \sigma^+ a f_2(\mathbf{R}, t) - f_2^*(\mathbf{R}, t) a^+ \sigma e^{i(\Delta_0 + \Delta_g)t} \right\} \quad (2)$$

Where transition operators $\hat{\pi}^+$ and $\hat{\sigma}^+$ represent transitions from $|1\rangle$ to $|3\rangle$ and $|2\rangle$ to $|3\rangle$ respectively, while transition operators $\hat{\pi}$ and $\hat{\sigma}$ represent transitions from $|3\rangle$ to $|1\rangle$ and $|3\rangle$ to $|2\rangle$ respectively. The coupling parameters $f_1(\mathbf{R}, t)$ and $f_2(\mathbf{R}, t)$ can be written as

$$f_1(\mathbf{R}, t) = G_1(\mathbf{R}) e^{i[\theta(\mathbf{R}) + \Delta_0 t]} \quad (3.a)$$

$$f_2(\mathbf{R}, t) = G_2(\mathbf{R}) e^{i[\theta(\mathbf{R}) + (\Delta_0 + \Delta_g)t]} \quad (3.b)$$

Where $G_1(\mathbf{R}) = \frac{1}{\hbar} \mathbf{D}_{13} \cdot \boldsymbol{\varepsilon}(\mathbf{R})$ and $G_2(\mathbf{R}) = \frac{1}{\hbar} \mathbf{D}_{23} \cdot \boldsymbol{\varepsilon}(\mathbf{R})$ correspond to position dependent Rabi frequency, $\boldsymbol{\varepsilon}$ and $\theta(\mathbf{R})$ are the light field amplitude and phase respectively, $\mathbf{D}_{13}(\mathbf{D}_{23})$ is the dipole matrix element between levels $|1\rangle$ ($|2\rangle$) and $|3\rangle$. The total coupling parameter $f(\mathbf{R}, t)$ can be written as

$$f(\mathbf{R}, t) = f_1(\mathbf{R}, t) + f_2(\mathbf{R}, t). \quad (4)$$

The time evolution of the total coupling parameter is derivable from the Heisenberg equation of motion, which is given as [7]

$$\frac{df(\mathbf{R}, t)}{dt} = \frac{i}{\hbar} [H, f(\mathbf{R}, t)] = \frac{1}{2m} (\mathbf{P} \cdot \nabla f(\mathbf{R}, t) + \nabla f(\mathbf{R}, t) \cdot \mathbf{P}). \quad (5)$$

Here we have assumed that the coupling parameter $f_1(\mathbf{R}, t)$ and $f_2(\mathbf{R}, t)$ commute with $U(\mathbf{R})$.

From Eq.(5), we may derive the frequency detuning operator δ including Doppler and recoil frequency shift, etc, which by considering the leading order is given by[7]

$$\delta = -\frac{i}{2m} \left(\frac{\mathbf{P} \cdot \nabla f(\mathbf{R}, t) + \nabla f(\mathbf{R}, t) \cdot \mathbf{P}}{f(\mathbf{R}, t)} \right)_0. \quad (6)$$

Here the subscript zero denotes operators at the initial time $t=0$. Eq.(6) can be further written as

$$\delta = \left[-i \frac{\nabla f(\mathbf{R}, t) \cdot \mathbf{v}}{f(\mathbf{R}, t)} - \frac{\hbar \nabla^2 f(\mathbf{R}, t)}{2mf(\mathbf{R}, t)} \right]_0. \quad (7)$$

For the case of a linearly polarized along the x -axis plane wave, we assume $\mathbf{D}_{13} = D_{13} \hat{\mathbf{x}}$, $\mathbf{D}_{23} = D_{23} \hat{\mathbf{x}}$, $\boldsymbol{\varepsilon}(\mathbf{R}) = \varepsilon \hat{\mathbf{x}}$, $\theta(\mathbf{R}) = -\mathbf{k} \cdot \mathbf{R}$, and obtain

$$\left[-i \frac{\nabla f(\mathbf{R}, t) \cdot \mathbf{v}}{f(\mathbf{R}, t)} \right]_0 = \Delta_0 - \mathbf{k} \cdot \mathbf{v} + \frac{D_{23}}{D_{13} + D_{23}} \Delta_g \quad (8.a)$$

$$\left[-\frac{\hbar \nabla^2 f(\mathbf{R}, t)}{2mf(\mathbf{R}, t)} \right]_0 = \frac{\hbar k^2}{2m} - \left(\frac{\hbar k \Delta_0}{mv} + \frac{\hbar k}{mv} \frac{D_{23} \Delta_g}{D_{13} + D_{23}} \right) + \frac{\hbar \Delta_0^2}{2mv^2} + \frac{\hbar (2\Delta_0 \Delta_g + \Delta_g^2)}{2mv^2} \frac{D_{23}}{D_{13} + D_{23}} \quad (8.b)$$

We know from Eqs.(8.a), (8.b) that there are amending terms resulting from three-level atomic mode and related with two lower level split Δ_g , D_{13} and D_{23} for the Doppler shift $-\mathbf{k} \cdot \mathbf{v}$ and recoil shift $\hbar k^2 / 2m$.

Then we consider the case of atoms moving in a Laguerre-Gaussian beam which can be readily produced in the laboratory [10]. In the paraxial approximation, assuming the Laguerre-Gaussian beam traveling in the $-z$ direction and polarized in the x direction and $\mathbf{D}_{13} = D_{13} \hat{\mathbf{x}}$, $\mathbf{D}_{23} = D_{23} \hat{\mathbf{x}}$, we derive the total coupling parameter $f(\mathbf{R}, t) = f_1(\mathbf{R}, t) + f_2(\mathbf{R}, t)$, and $f_1(\mathbf{R}, t)$, $f_2(\mathbf{R}, t)$ are expressed as

$$f_1(\mathbf{R}, t) = -i\omega_L D_{13} u_{pl}(r, z) \exp(-ikz) \exp(-il\phi) \exp(i\Delta_0 t) \quad (9.a)$$

$$f_2(\mathbf{R}, t) = -i\omega_L D_{23} u_{pl}(r, z) \exp(-ikz) \exp(-il\phi) \exp[i(\Delta_0 + \Delta_g)t] \quad (9.b)$$

where the complex scalar function $u_{pl}(r, z)$ describing the distribution of the field amplitude is given by

$$u_{pl}(r, z) = (-1)^p \frac{C}{w(z)} \left(\frac{r\sqrt{2}}{w(z)} \right)^l L_p^l(2r^2/w^2(z)) \times \exp[-i(2p+l+1)\tan^{-1}(z/z_R)] \times \exp\left[\frac{-ikr^2 z}{2(z^2 + z_R^2)} \right] \times \exp[-(r^2/w^2(z))] \quad (10)$$

Where L_p^l are associated Laguerre polynomials, z_R is the Rayleigh range, $w^2(z) = 2(z_R^2 + z^2)/kz_R$ is the beam width at distance z from the beam waist. The integer indices l and p are quantum numbers characterizing the mode and l is the orbital angular momentum quantum number. From Eqs.(9) and (10), we obtain the expression of $\theta(\mathbf{R})$

$$\theta(\mathbf{R}) = -\frac{kr^2 z}{2(z^2 + z_R^2)} - l\phi - (2p + l + 1) \tan^{-1}(z/z_R) - kz. \quad (11)$$

In Eq.(7), the recoil shift (i.e. the second term) is so smaller than the Doppler shift in the first term that the recoil term can be not considered further. Therefore, we only consider the first term including the Doppler shift of the Laguerre-Gaussian beam from the three-level atom:

$$\delta_{LG} = \left[-i \frac{\nabla f(\mathbf{R}, t) \cdot \mathbf{v}}{f(\mathbf{R}, t)} \right]_0 = \delta_{LG}^{(1)} - i\delta_{LG}^{(2)}. \quad (12)$$

Where

$$\delta_{LG}^{(1)} = \left[\frac{f_1(\mathbf{R}, t) \nabla[\theta(\mathbf{R}) + \Delta_0 t] \cdot \mathbf{v} + f_2(\mathbf{R}, t) \nabla[\theta(\mathbf{R}) + (\Delta_0 + \Delta_g) t] \cdot \mathbf{v}}{f(\mathbf{R}, t)} \right]_0 \quad (13.a)$$

$$\delta_{LG}^{(2)} = \left[\frac{\nabla G_1(\mathbf{R}) \cdot \mathbf{v} \times \exp i[\theta(\mathbf{R}) + \Delta_0 t] + \nabla G_2(\mathbf{R}) \cdot \mathbf{v} \times \exp i[\theta(\mathbf{R}) + (\Delta_0 + \Delta_g) t]}{f(\mathbf{R}, t)} \right]_0 \quad (13.b)$$

The imaginary part $\delta_{LG}^{(2)}$ in Eq. (12) is determined by the change of the electric field amplitude of the Laguerre-Gaussian beam, which is not related with the final detuning or Doppler shift and is not considered further.

The term $\delta_{LG}^{(1)}$ can be further written as

$$\delta_{LG}^{(1)} = \nabla \theta(\mathbf{R}) \cdot \mathbf{v} + \left(\Delta_0 + \frac{D_{23}}{D_{13} + D_{23}} \Delta_g \right) \times (\nabla t \cdot \mathbf{v}). \quad (14)$$

In Eq.(14), the term Δ_0 is the Laguerre-Gaussian beam detuning from the three-level atomic transition ω_{13} . The term $\nabla \theta(\mathbf{R}) \cdot \mathbf{v}$ is the Doppler shift same as the result from the two-level atomic mode, which is given by [7]

$$\nabla \theta(\mathbf{R}) \cdot \mathbf{v} = \left[-k + \frac{kr^2}{2(z^2 + z_R^2)} \left(\frac{2z^2}{(z^2 + z_R^2)} - 1 \right) - \frac{(2p+l+1)z_R}{(z^2 + z_R^2)} \right] v_z - \left(\frac{kzr}{z^2 + z_R^2} \right) v_r - \left(\frac{l}{r} \right) v_\phi \quad (15)$$

Where v_z , v_r and v_ϕ are the axial, radial and azimuthal velocity components of the atom. We know from Eq.(14) that the detuning amending term $\Delta_{am} = D_{23}\Delta_g / (D_{13} + D_{23})$ results from the three-level mode, which is determined by two lower level split Δ_g and the ratio of dipole matrix element D_{23} to the total dipole matrix element $D_{23} + D_{13}$. From Eq.(14), we also find that there is same frequency amending for the axial, radial and azimuthal frequency detuning. Therefore, we can define an effective detuning of Laguerre-Gaussian beam from the three-level atom, which is given by

$$\Delta_{eff} = \Delta_0 + \Delta_{am} = \Delta_0 + \frac{D_{23}}{D_{13} + D_{23}} \Delta_g. \quad (16)$$

According to the effective detuning of Laguerre-Gaussian beam from the three-level atom, we also define an effective two level-atomic mode. In such effective two level atomic mode, the upper level is $|3\rangle$ and the lower level locates between levels $|1\rangle$ and $|2\rangle$ with a frequency interval Δ_{am} from level $|1\rangle$. When $\Delta_0 \gg \Delta_{am}$, that is the case of large detuning Δ_0 compared with Δ_g , we can neglect the amending frequency in the effective detuning. Therefore, the three-level atomic mode can be simplified as the two-level atomic mode. When the detuning Δ_0 is about the order of Δ_g , the detuning amending term Δ_{am} plays an important role for the effective detuning and Doppler shift. In particular, when $\Delta_0 = 0$, the laser

frequency is resonant with ω_{13} , but the laser frequency is detuned Δ_{am} from the effective two-level atomic mode. When $\Delta_0 = -\Delta_{am}$, the laser frequency is resonant with the effective two-level atomic mode.

To discuss further quantitatively about the detuning amending term $\Delta_{am} = D_{23}\Delta_g / (D_{13} + D_{23})$, we consider the relation between Γ and D [10]

$$\Gamma = \omega^3 D^2 / (3\pi\epsilon_0 \hbar c^3), \quad (17)$$

here D is the dipole matrix element, Γ is the spontaneous transition rate, ω is atomic resonance frequency, ϵ_0 is the free space permittivity, \hbar is Planck constant over 2π , c is the speed of light in vacuum, and we derive

$$\Delta_{am} = \frac{D_{23}}{D_{13} + D_{23}} \Delta_g = \frac{\omega_{13} \sqrt{\omega_{13} \Gamma_{23}}}{\omega_{13} \sqrt{\omega_{13} \Gamma_{23}} + \omega_{23} \sqrt{\omega_{23} \Gamma_{13}}} \Delta_g. \quad (18)$$

Considering a hyperfine structure three-level ^{85}Rb atom, $|1\rangle \leftrightarrow 5S_{1/2}, F=2$ state, $|2\rangle \leftrightarrow 5S_{1/2}, F=3$ state, $|3\rangle \leftrightarrow 5P_{3/2}$ state, $\Delta_g = 3.04\text{GHz}$, $\lambda = 0.78\mu\text{m}$, $\Gamma_{13} = 0.74\Gamma_3$, $\Gamma_{23} = 0.26\Gamma_3$, we calculate Eq.(18) and derive the approximate value of the amending term

$$\Delta_{am} = \frac{D_{23}}{D_{13} + D_{23}} \Delta_g \approx 0.37\Delta_g \approx 1.12\text{GHz}. \quad (19)$$

In particular, for $\lambda = 0.78\mu\text{m}$ and $v = 1\text{m/s}$, the axial Doppler shift kv is about 1.3MHz , which is much smaller than Δ_{am} , and radial and azimuthal Doppler shift are also about MHz much smaller than Δ_{am} [7]. Therefore, the Doppler shift can be distinguished obviously from the detuning amending term. If the two lower levels are two magnetic sublevels and the level splitting Δ_g can be modified in a wide range by changing magnetic field, we find the detuning amending term Δ_{am} can reach the order of Doppler shift. Therefore, the Doppler shift is easily affected by the detuning amending term Δ_{am} , which should be well distinguished from detecting the Doppler shift.

From Eqs. (7), (8.a), (14), we find that the effective detuning Δ_{eff} derived from the three-level atom in the plane beam or Laguerre-Gaussian beam is not related with the form of light field amplitude and phase, that is, the expression of the effective detuning in Eq.(16) is a general expression for every kind of the light field. The origination of the effective detuning is the three-level atomic mode.

III. CONCLUSIONS

In conclusion, we have derived the general analytic expression of the effective detuning $\Delta_{eff} = \Delta_0 + D_{23}\Delta_g / (D_{13} + D_{23})$ of the linearly polarized light field from the Λ -configuration three-level atomic mode. The detuning amending term $\Delta_{am} = D_{23}\Delta_g / (D_{13} + D_{23})$ resulting from the three-level mode is not related with the form of light field, which is decided by two lower level split Δ_g and the ratio of dipole matrix element D_{23} to the total dipole matrix element $D_{23} + D_{13}$. For a linearly polarized Laguerre-

Gaussian laser beam, we find that there is same frequency amending for the axial, radial and azimuthal frequency detuning. We define an effective two-level atomic mode in which the upper level is $|3\rangle$ and the lower level locates between levels $|1\rangle$ and $|2\rangle$ and has the frequency interval Δ_{am} from level $|1\rangle$. The effective detuning plays an important role in detecting the laser detuning and Doppler shift from atomic transition frequency.

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