A Calibration Method of Channel Error of Digital Measurement System

Based on the One Element Linear Regression

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***Abstract*—Systematic error of digital measurement system (DSM) mainly exists in the analog measurement channels. The cause of systematic errors is circuit status and parameters of OPA, DAC and reference circuit which drift with changes in temperature and time deviate from the standard values. In this paper, firstly the causes of DMS errors are analyzed and errors calibration formula based on the one element linear regression is derived. Secondly equivalent formulas of zero drift, multiplying power changes and cascade in analog measurement channels are derived. Finally calibration of analog measurement channels is implemented in an engineering example.**

Keywords-one element linear regression; channel errors; calibration method

I INTRODUCTION

One of the main ways to improve the accuracy of the measurement device is reducing the systematic error of digital measurement system (DSM). In both of individual components and entire digital measurement device, systematic errors can be reduced. In this method, firstly, the size of the linearity error is calculated. Then the correction value is introduced into the signal processing circuit. This method reduces the sampling rate of digital measurement system (DMS). However, it is adaptable and can be widely applied. This paper introduces a internal calibration method of zero drift and multiplying power changes in DMS based on the one element linear regression.

II CHANNEL ERRORS AND CALIBRATION METHOD

Ideal transfer function for the first-order linear system is as follows:

$$y = kx + b \tag{1}$$

If step-characteristic caused by the quantization is not considered, the causes of difference between the actual transfer function and the ideal transfer function are three system error components. First one is additive error Δa which has nothing to do with being measured. Second one is multiplicative error Δ m which is proportional to the measured x. The last one is nonlinear error Δh which is determined by the component characteristics and all kinds of external factors. Because Δh has been partially compensated while Δa and Δm are corrected, so error Δh is rarely alone corrected. In addition, because of its uncertainty, Δh corrected alone will complicate the digital signal processing circuit and reduce sampling rate. When considering Δa and Δm, the transfer function of a first-order linear system is as follows:

$$\hat{y} = kx + b + \Delta a + \Delta m = k\left(1+\alpha\right)x + b + \Delta a \tag{2}$$

In the formula,α= △ m/(kx) is the relative multiplicative error.

In the actual measurement system, multiplying power changes coefficient k is often change with the difference of temperature and time, so for calibration system must eliminate Δ error and the influence of the k at the same time.

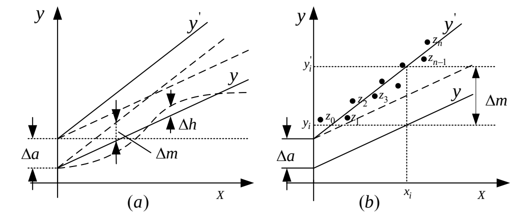

Fig.1: Linear error of the system

In the first order linear system, n sample points x_0, x_1, x_2 ...... x_{n-1}, x_n are selected evenly in valid values range of X. Output values of the sample points $\hat{y}_0$, $\hat{y}_1$, $\hat{y}_2$ ...... $\hat{y}_{n-1}$, $\hat{y}_n$ are measured. We get the measurement point coordinates $z_0(x_0,\hat{y}_0)$, $z_1(x_1,\hat{y}_1)$, $z_2(x_2,\hat{y}_2)$ ...... $z_{n-1}(x_{n-1},\hat{y}_{n-1})$, $z_n(x_n,\hat{y}_n)$.

Suppose $Y = k + bx + \varepsilon$ is linear regression model of this group of data. $E(\varepsilon)=0$ is mathematical expectation of ε, and $D(\varepsilon)=\sigma^2$ is variance of ε in the formula. Defines $\hat{k}$ and $\hat{b}$ are points estimation of k and b, then linear equation $\hat{y} = \hat{k}x + \hat{b}$ is the empirical regression equation of Y to X. Determine the values of the equation coefficient $\hat{k}$ and $\hat{b}$. Let $\varphi = \Sigma(y_i^{'} - k^{'}x_i - b^{'})^2$ for $k^{'}$ and $b^{'}$ partial derivatives respectively equal to zero, shown as below:

$$\frac{\partial\varphi}{\partial\hat{k}} = -2\Sigma x_i(\hat{y}_i - \hat{k}x_i - \hat{b}) = 0 \tag{3}$$

$$\frac{\partial \varphi}{\partial \hat{b}} = -2\Sigma(\hat{y}_i - \hat{k}x_i - \hat{b}) = 0 \quad (4)$$

Eq. (3). and Eq. (4) are linear equations group, so it has a unique solution. Through formula transformation, solutions of equations are given

$$\hat{k} = \frac{\Sigma x_i \Sigma \hat{y}_i - n\Sigma x_i \hat{y}_i}{(\Sigma x_i)^2 - n\Sigma x_i^2} \quad (5)$$

$$\hat{b} = \frac{\Sigma x_i \Sigma x_i \hat{y}_i - \Sigma x_i^2 \hat{y}_i}{(\Sigma x_i)^2 - n\Sigma x_i^2} \quad (6)$$

$\hat{k}$ and $\hat{b}$ are substituted into Eq. $\hat{y} = \hat{k}x + \hat{b}$:

$$\hat{y} = \frac{\Sigma x_i \Sigma \hat{y}_i - n\Sigma x_i \hat{y}_i}{(\Sigma x_i)^2 - n\Sigma x_i^2} x + \frac{\Sigma x_i \Sigma x_i \hat{y}_i - \Sigma x_i^2 \hat{y}_i}{(\Sigma x_i)^2 - n\Sigma x_i^2} \quad (7)$$

Eq. (7) is unary linear regression equation of actual system transfer function.

The result of Eq. (7) minus Eq. (1) is

$$\hat{y} - y = (k(1+\alpha)x + b + \Delta a) - (kx+b) = \alpha x + \Delta a$$

$$= \frac{\Sigma x_i \Sigma \hat{y}_i - n\Sigma x_i \hat{y}_i}{(\Sigma x_i)^2 - n\Sigma x_i^2} x + \frac{\Sigma x_i \Sigma x_i \hat{y}_i - \Sigma x_i^2 \hat{y}_i}{(\Sigma x_i)^2 - n\Sigma x_i^2} - (kx+b) \quad (8)$$

Eq. (8) is linear error value of the actual transfer function. When the input is x_n, the system actual output value is $\hat{y}_n$. $\hat{y}_n$ can be corrected by Eq. (8), then we can get ideal result y:

$$y = kx + b = (k(1+\alpha)x + b + \Delta a) - (\alpha x + \Delta a) =$$

$$(k(1+\alpha)x + b + \Delta a) - (\hat{k}x + \hat{b} - (kx+b))$$

$$= \hat{y} - ((\frac{\Sigma x_i \Sigma \hat{y}_i - n\Sigma x_i \hat{y}_i}{(\Sigma x_i)^2 - n\Sigma x_i^2} - k)x + \frac{\Sigma x_i \Sigma x_i \hat{y}_i - \Sigma x_i^2 \hat{y}_i}{(\Sigma x_i)^2 - n\Sigma x_i^2} - b)$$

$$= \hat{y} - (hx + c) \quad (9)$$

Where $h = \frac{\Sigma x_i \Sigma \hat{y}_i - n\Sigma x_i \hat{y}_i}{(\Sigma x_i)^2 - n\Sigma x_i^2} - k$ and

$c = \frac{\Sigma x_i \Sigma x_i \hat{y}_i - \Sigma x_i^2 \hat{y}_i}{(\Sigma x_i)^2 - n\Sigma x_i^2} - b$ are called channel linear error correction coefficients.

III ZERO DRIFT AND MULTIPLYING POWER CHANGES OF ANALOG MEASUREMENT CHANNELS

Systematic error of DSM mainly exists in the analog measurement channels. The cause of systematic errors is circuit status and parameters of OPA, DAC and reference circuit that drift with changes in temperature and time deviate from the standard values. This kind of deviation and drift are mainly reflected on the zero drift and multiplying power changes. Zero drift U refers to when input is zero, output is not zero. It will change with time and temperature changes, and cause the additive error on the system. Multiplying power changes refers to the ratio k of input and output will change with dk when input signal changes, and cause multiplicative error on the system.

Dc parameters of the OPA ,which are the main reason of U and dk, have input offset voltage, input offset current, input offset current, open loop gain and closed loop gain resistance. The equivalent single-stage voltage sampling circuit in DMS analog channel is shown as Fig. 2.

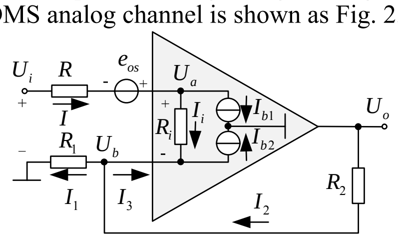

Fig.2: Equivalent voltage sampling circuit in DMS

In Fig. 2, e_{os} is input offset voltage. I_{b1} and I_{b2} are input offset current. R_1 and R_2 are closed loop gain resistance. R is input balance resistance. R_i is equivalent resistance of amplifier. U_i is input voltage of amplifier. U_o is onput voltage of amplifier.

$$U_i = IR - e_{os} + I_i R_i + I_1 R_1 \quad (10)$$

$$I = \frac{U_o}{AR_i} + I_{b1} \quad (11)$$

$$I_1 = \frac{1}{R_1 + R_2}(U_o + \frac{U_o R_2}{AR_i} - R_2 I_{b2}) \quad (12)$$

In the formula, A is open loop gain of amplifier.

Eq. (11) and Eq. (12) are substituted into Eq.(10). $I_{os}=I_{b2}-I_{b1}$, $R=R_1R_2/(R_1+R_2)$ and $I_iR_i=U_o/A$ (where I_{os} is called offset current) are considered, then:

$$U_i = \frac{R}{R_2}(2\frac{R_2}{AR_i} + \frac{R_2}{AR} + 1)U_o - RI_{os} - e_{os} \quad (13)$$

Let $\alpha = -(\frac{2\frac{R_2}{AR_i} + \frac{R_2}{AR}}{2\frac{R_2}{AR_i} + \frac{R_2}{AR} + 1})$, $k = \frac{R_2}{R}$ and

$U_{os} = RI_{os} + e_{os}$, then:

$$U_o = k(1+\alpha)U_i + k(1+\alpha)U_{os} \quad (14)$$

We can learn two things from Eq. (14):

1). The gain of the amplifier is k(1+α). If the amplifier open-loop gain A>>1+R1/R2, Ri>>R2, and α→1, then gain of the actual OPA is close to gain of the ideal OPA. Gain of the actual OPA will drift with temperature changes because A, R_i, R_1 and R_2 will change with temperature changes

2). The influence of input offset voltage e_{os} and offset current I_{os} to the amplifier output is equivalent to zero drift voltage U_{oa} superimposed on input signal. From Eq. $U_{os}=e_{os}+RI_{os}$ we can know that offset voltage e_{os} directly affects amplitude of U_{os}, and offset current I_{os} affect amplitude of U_{os} through input balance resistance R. The equivalent of the zero drift voltage will change with temperature changes because e_{os}, I_{os}, R_1 and R_2 are functions of temperature,

There are offset voltage and offset current in input port of instrumentation amplifier, DAC, voltage reference source, etc. Analysis method of zero drift and multiplying power changes in instrumentation amplifier, DAC, voltage reference source, etc. is similar to the above.

IV SERIES EQUIVALENT CIRCUIT OF ANALOG MEASUREMENT CHANNEL

Sections 2 and 3 have introduced the linear error model and correction method of single-stage transfer function. Various analog components in analog channel are often series connection. Fig. 3 is a kind of depth measurement processing circuit schematic. This circuit is composed of five sub circuits in series.

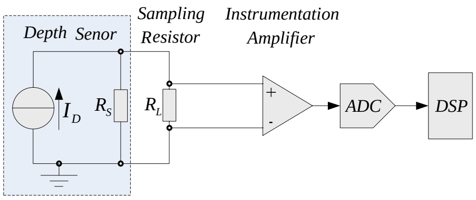

Fig. 3: Depth measurement processing circuit schematic

Depth sensor is a current source. Output current size is proportional to the water pressure size perceived by pressure sensor. Actually there is a source resistance R_S in parallel with current source. The source resistance has shunted part of the current, and introduces multiplicative error into circuit. Output current size of depth sensor in zero water depth will change with altitude of using location changing, so additive error is introduced into circuit.

The sampling resistor R_L has some impedance tolerance and temperature drift, so multiplicative error is introduced into circuit.

There are offset voltage and offset current in input port of instrumentation amplifier, DAC, voltage reference source, etc. They are equivalent to zero drift and ratio change which introduce additive error and multiplicative error into circuit.

In addition, different type depth sensors possess different current value range with the same water depth range. For a same depth solver of DSP, it is equivalent to introduce multiplicative error into circuit.

Therefore, there are linear errors in five sub circuits of depth measurement processing circuit. How can we calculate equivalent system error of five sub circuits in series?

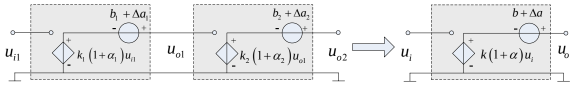

Fig. 4: Equivalent Analytical Part of Series Parts

K_1 and k_2 are ideal gains of element 1 and element 2, respectively. U_{os1} and U_{os2} are zero drift voltages of output port, respectively. Element 1 and element 2 are series connection. Output voltages U_{o1} and U_{o2} of element 1 and element 2 are Eq. (7) and Eq. (8), respectively.

$$u_{o1} = k_1(1+\alpha_1)u_{i1} + b_1 + \Delta a_1 \tag{15}$$

$$\begin{aligned} u_{o2} &= k_2(1+\alpha_2)u_{o1} + b_2 + \Delta a_2 = k_2(1+\alpha_2)(k_1(1+\alpha_1)u_{i1} + b_1 + \Delta a_1) + b_2 + \Delta a_2 \\ &= k_1k_2(1+\alpha_1+\alpha_2+\alpha_1\alpha_2)u_i + k_2b_1 + b_2 + k_2\Delta a_1 + k_2\alpha_2 b_1 + k_2\alpha_2\Delta a_1 + \Delta a_2 \\ &= k\,(1+\alpha\,)u_{i1} + b\, + \Delta a \end{aligned} \tag{16}$$

Therefore, two elements in series can be equivalent to one element. Ideal gain of the element is $k = k_1k_2$, relative multiplicative error of the element is $\alpha = \alpha_1 + \alpha_2 + \alpha_1\alpha_2$ and zero drift voltage of the element is $\Delta a\ = k_2\Delta a_1 + k_2\alpha_2 b_1 + k_2\alpha_2\Delta a_1 + \Delta a_2$. Similarly, transfer function of five elements in series can be equivalent to one element .Channel error of entire system can be expressed with additive error and multiplicative error, as shown in Eq. (1). Calibration formula of linear error is still applicable to multilevel series circuit.

V ENGINEERING EXAMPLE

This error calibration method is applied on underwater synchronous beacon transmitter. The beacon adopts the principle of synchronous distance measurement. Beacon transmitter and beacon measurement device have the same standard time reference. Beacon measurement device estimates the location and depth of beacon transmitter by measuring the amount of depth pulse delay.

The beacon transmitter adopts the depth measurement principle is shown in Fig.3. Analog measurement channel convert output signal of depth sensor into voltage can be measured within the range. ADC convert analog signal into 16-bits wide digital signal. Digital measurement values are linear calculated by DSP. Then, amount of depth CW pulse transmitted by beacon transmitter can be generated. Channel error exists in measurement system, and can be calibrated as following steps:

1) The method is realized in excel. Correction formula is edited into the background function. Input/output parameter name and its value are set in the table of excel. This way of design has good user experience and is convenience for the operators.

2) Calculate the ideal transfer function coefficient k and b.

3) Select n sample points x_0, x_1, $x_2 \ldots\ldots x_{n-1}$, x_n evenly in range of 0m-150m. One by one depth, use pressure gauge to pressure on the depth of the sensor, and get the amount of depth beacon delay $\hat{y}_0$, $\hat{y}_1$, $\hat{y}_2 \ldots\ldots \hat{y}_{n-1}$, $\hat{y}_n$ with an oscilloscope.

4) Fill the coordinates of sample points $z_0(x_0, \hat{y}_0)$, $z_1(x_1, \hat{y}_1)$, $z_2(x_2, \hat{y}_2) \ldots\ldots z_{n-1}(x_{n-1}, \hat{y}_{n-1})$, $z_n(x_n, \hat{y}_n)$ in corresponding blanks of linear error correction interface in excel, and systematic linear error correction coefficients h and can be calculated.

5) Write parameters h and c into registers of DSP.

6)Once again , use pressure gauge to pressure on the depth of the sensor, and get the amount of depth beacon delay $\hat{\hat{y}}_0$, $\hat{\hat{y}}_1$, $\hat{\hat{y}}_2$ ...... $\hat{\hat{y}}_{n\text{-}1}$, $\hat{\hat{y}}_n$ with an oscilloscope. Calculate the error value after correction, and verify the effect of this calibration method. Complete the depth signal error correction.

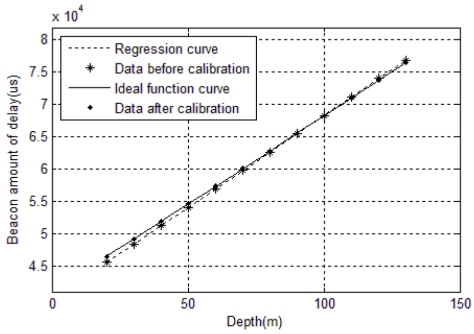

Fig.5: The channel error calibration curves of depth signal processing

The channel error curves of depth signal processing before and after calibration are shown as Fig.5. The measurement results are analyzed. Before calibration, the standard deviation between measurement values and theoretical values was 1.772 meters, while after calibration the standard deviation between measurement values and theoretical values has reduced to 0.1meters.

VI CONCLUSION

Digital measuring system (DMS) is generally used in underwater acoustic measurement, radar stations, IC automatic test and CNC machine tools and other fields. DMS has strict requirements on the accuracy of the measurement. This paper introduces calibration method of zero drift and multiplying power changes in DMS from the perspective of reduce the DMS analog channel tool measurement error. Under the premise of considering the measurement time, measurement accuracy of the DMS has been improved. In addition, reliability and stability of the measurement equipment have been improved.

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