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Multi-level Comprehensive Evaluation Model Based on Triangular Fuzzy Numbers and Gray Relational Theory

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***Abstract*—According to grey correlation method is used for multi-level comprehensive evaluation model of the optimal design scheme selection, the fuzzy index exists in the design scheme index, based on triangular fuzzy number of grey correlation theory is proposed on the multi-level comprehensive evaluation model. The triangular fuzzy number was introduced to the grey correlation method was used to design scheme selection of multiple products, First of all, according to the fuzzy index and the corresponding relation of triangular fuzzy number, The design scheme of the fuzzy index is transformed into the corresponding triangular fuzzy Numbers. And then according to the traditional calculating grey correlation coefficient formula, based on triangular fuzzy number of grey correlation coefficient formulas was deduced. Then using triangle fuzzy number operation rule and triangular fuzzy number ranking method, as well as the derivation of grey correlation coefficient formula of triangular fuzzy number, The triangular fuzzy number theory of grey correlation product multi-level comprehensive evaluation model of the optimal solution is established, Using grey correlation theory of multilevel fuzzy comprehensive evaluation model of scheme selection of indicators of problems have been solved. Finally, a numerical example illustrates the use of the method to solve the optimal solution process, Results show that the method for complex, multi-level haveing problems with fuzzy information, has good operability and practical consistency.**

Keywords- gray relational theory; multi-level comprehensive evaluation model; fuzzy numbers;

I. INTRODUCTION

The design scheme of production is the key to product manufacturing; it is made by the designer based on the demand of customer, the cost of products, the current production status and other factors. Therefore, there may be a variety of schemes for the same product, It has become a key aspect of product design that how to choose a best solution in many schemes. Research on this subject have become an important topic in the field of the economy[1]. Customer demand is the direct reflection of design to customer satisfaction. Grey correlation is a method of optimal scheme selection that aim for seeking the important relationship between various factors of the system. The basic idea of the algorithm is to determine the association tightness between sequences according to the geometric similarity of action sequence curve so that widely used in product scheme selection[1][2], evaluation and decision making[8][9][11][12][13]. There will be some qualitative index in product design .It is mainly due to the existence of qualitative description in customers' description of the product performance indicators (such as: small, general, etc.). Therefore, the traditional way is expressing qualitative description into quantitative value when using grey correlation method for scheme selection. This method solves the problems of qualitative indicators is difficult to calculate in the process of grey correlation calculation, but it is difficult to conform to the fuzzy characteristics of qualitative indicators. Based on this consideration, the reference[4][5][6]introducing fuzzy theory into grey correlation analysis, converting the calculation of grey correlation coefficient to solve the ratio of the relative distance of two fuzzy numbers, to get the determined value of correlation coefficient. You would get the grey correlation degree of each scheme and ideal scheme by the product of determined value and weighting coefficient. At last, you could get the best solution by using the numerical order of grey correlation degree. Despite the introduction of fuzzy number in the reference(they adapt defuzzification method when calculating the correlation coefficient, and use the distance of two point to get a certain correlation between the size of the two numbers), they did not use their own algorithm to calculate fuzzy numbers. At last, the analysis lost the meaning of introducing the theory of fuzzy when express the final evaluation result, because the final accurate value usually replace the original fuzzy description method.

In fuzzy theory, the triangle fuzzy numbers is used most widely, and it has a complete rule of operation and size comparison. Based on this, because the fuzzy language exist in the index of the product design scheme, introducing the triangular fuzzy number into the choice of product design scheme that using the grey correlation method, converting fuzziness indexes of the scheme into triangular fuzzy numbers, and then introduced into the grey correlation coefficient calculation formula. The solution formula of grey correlation coefficient is derived, and it contains the triangular fuzzy number. At last, combining the formula of triangular fuzzy number grey correlation coefficient and multi-level comprehensive evaluation model with the rules of the triangular fuzzy number operation and sorting method, working out the final fuzzy correlation coefficient, obtaining the final optimal solutions by comparing various solutions of fuzzy correlation coefficient in each scheme, and establishing the multi-level comprehensive evaluation design excellent-choosing model based on triangular fuzzy number grey correlation theory.

II. THE DEFINITION AND ALGORITHM OF TRIANGULAR FUZZY NUMBER

A. The definition of fuzzy numbers

In the domain of discourse *U*, for any $u \in U$, specify a number $\mu_{\underset{\sim}{A}}(u) \in [0,1]$, referred to as the membership:

$$\mu_{\underset{\sim}{A}} : U \to [0,1],\quad u| \to \mu_{\underset{\sim}{A}}(u)$$

B. The definition of triangular fuzzy numbers

M is set to a fuzzy number in the domain *R*, if the membership functions of *M* $\mu_M : U \to [0,1]$ can be expressed as follws

$$\mu_{\mathrm{M}}(x) = \begin{cases} \dfrac{1}{m-x}x - \dfrac{l}{m-l}, x \in [l,m] \\ \dfrac{1}{m-u}x - \dfrac{u}{m-u}, x \in [m,u] \\ 0, x \in (-\infty, l] \cup [u, +\infty) \end{cases} \tag{1}$$

The fuzzy number is called triangular fuzzy number, and, $l \le m \le u$,l and u represent the lower bound and upper bound value of triangular fuzzy number, m is the mid-value which the value of membership of triangular fuzzy number is 1. Therefore, the triangular fuzzy number M is expressed as $M(l,m,u)$.

C. The Algorithm of triangular fuzzy number

Given : The fuzzy number $M1 = (l1, m1, u1)$, $M2 = (l2, m2, u2)$, The operation rules between two fuzzy numbers is

(1) Summation Operation: $M1 + M2 = (l1 + l2, m1 + m2, u1 + u2)$

(2) Product Operation: $M1 \otimes M2 \approx (l1l2.m1m2, u1u2)$

(3) Reciprocal Operation: $\frac{1}{M} \approx (\frac{1}{u}, \frac{1}{m}, \frac{1}{l})$

According to the above three kinds of algorithms, other algorithm is deduced.

III. SORT OF TRIANGULAR FUZZY NUMBER

To sort the fuzzy number, we adapt the method of average and standard deviation proposed by Lee-Li, this method thought, relative to the fuzzy number, if a fuzzy number has higher average and lower standard deviation, it is considered rank higher. The form of a ratio distribution is applied in here, for the triangular fuzzy number $\tilde{r}_i(a,b,c), i = 1,2,3,\cdots,m$. Its formula of average $\overline{x}(\tilde{r}_i)$ and standard deviation $\sigma(\tilde{r}_i)$ as follows:

$$\overline{x}(\tilde{r}_i) = \frac{1}{4}(a + 2b + c) \tag{2}$$

$$\sigma(\tilde{r}_i) = \frac{1}{80}(3a^2 + 4b^2 + 3c^2 - 4ab - 2ac - 4bc) \tag{3}$$

The rules of comparing the size of fuzzy number is calculated as follows:

if $\overline{x}(\tilde{r}_i) > \overline{x}(\tilde{r}_j)$,then $\tilde{r}_i > \tilde{r}_j$; if $\overline{x}(\tilde{r}_i) = \overline{x}(\tilde{r}_j)$, $\sigma(\tilde{r}_i) > \sigma(\tilde{r}_j)$, then $\tilde{r}_i > \tilde{r}_j$

IV. THE GREY CORRELATION COEFFICIENT WITH TRIANGULAR FUZZY NUMBER

According to the grey system theory,the comparative sequence, $C_i(j) = \{C_i(1), C_i(2), \cdots, C_i(m)\}$,(among this,*i* (*i=1,2,…n*)represent the number of scheme,*j*(*j=1,2,…m*)represent the indicated number in each scheme, $C_i(j)$ represent the index of No.*j* in the scheme *i*; relative to the reference sequence $C_0(j) = \{C_0(1), C_0(2), \cdots, C_0(m)\}$ ($C_0(j)$, *j=1,2,…m*,represent the index of No. *j* of reference sequence) the coefficient of relational degree is shown in Eq. (4):

$$\xi_i(j) = \frac{\min_i \min_j |C_0(j) - C_i(j)| + \rho \max_i \max_j |C_0(j) - C_i(j)|}{|C_0(j) - C_i(j)| + \rho \max_i \max_j |C_0(j) - C_i(j)|} \tag{4}$$

In the Eq. (2), the index of the comparative sequence and the reference sequence is required to a certain number, however, in most of design scheme, because the human's description of the characteristics is ambiguity, for example, a description of the degree of bright color. There always exists the fuzziness index in the design scheme. For solving the correlation coefficient of design scheme which has the fuzzy number, reference[1] transformed the fuzziness index into a certain value, then calculating the result with Eq.(4).The advantage of this method is simple, easy to calculate, but, it is not consistent with fuzziness of the index. The fuzzy theory is put forward to solve the fuzziness problem. Therefore, it derives the solution method of fuzzy grey correlation coefficient method based on the fuzzy theory that combining the fuzzy theory and grey correlation theory, which solves the problem of fuzzy index existing in the scheme. Given a comparative sequence $C_i\{M_i(l_j, m_j, u_j)\}$, $M_i(l_j, m_j, u_j)$ is triangular fuzzy number,representing the index *j* in scheme *i*.The reference sequence $C_0\{M_0(l_j, m_j, u_j)\}$ ($M_0(l_j, m_j, u_j)$ is triangular fuzzy number, representing index *j* in reference sequence0).The fuzzy correlation coefficient in index $M_i\left(l_j,m_j,u_j\right)$ is shown in Eq. (5).

$$\xi_i(j)=\frac{\min_i\min_j\left|M_0\left(l_j,m_j,u_j\right)-M_i\left(l_j,m_j,u_j\right)\right|+\rho\max_i\max_j\left|M_0\left(l_j,m_j,u_j\right)-M_i\left(l_j,m_j,u_j\right)\right|}{\left|M_0\left(l_j,m_j,u_j\right)-M_i\left(l_j,m_j,u_j\right)\right|+\rho\max_i\max_j\left|M_0\left(l_j,m_j,u_j\right)-M_i\left(l_j,m_j,u_j\right)\right|} \quad (5)$$

In Eq.(5), solving the maximum, minimum and arithmetic operation of triangular fuzzy number according to its ranking method and operation rules, $\rho\in\left(0,+\infty\right)$ is resolution ratio, the smaller it is, the greater resolution capability it is,General $\rho\in\left(0,1\right)$,more general $\rho=0.5$

V. ESTABLISHING MULTI-LEVEL COMPREHENSIVE EVALUATION MODEL WITH GREY CORRELATION THEORY OF TRIANGULAR FUZZY NUMBER

Grey correlation analysis model can solve the relative metric between factors. It provides a relatively objective method to evaluate weight measure of the index. Moreover, it is not demanding to data requirement, it can be used to solve the measure problem which data size is small, the information is not complete. Ref. [1] proposes the hierarchy comprehensive evaluation based on gray correlation analysis. It does this by Eq.(4) to calculate the correlation coefficient after transform fuzziness index for the fixed value. This paper introduces the fuzzy theory, establishing the multi-level comprehensive evaluation model based on triangular fuzzy numbers gray relational theory, calculation steps are as follows:

(1)using all the index value sequence from the design scheme to determine the comparative sequence $C_i\left(j\right)=\left\{M_i\left(l_j,m_j,u_j\right)\middle|\, j=1,2,\cdots,m\right\}$, the fuzzy index in table 1 convert to the corresponding triangular fuzzy number

(2)In accordance with the principle of comparability and advanced nature, to determine the reference sequence $C_0\left(j\right)=\left\{M_0\left(l_j,m_j,u_j\right)\middle|\, j=1,2,\cdots,m\right\}$

(3)Non-dimensionalize;It is necessary to make indexes being dimensionless before the comprehensive evaluating, because, every index is hard to compare directly due to the difference of the dimension and type,as follows:

$$X_i\left(j\right)=X_i\left\{x_i\left(s_j,b_j,k_j\right)\right\}=\frac{C_i\left(j\right)}{C_j}=\frac{M_i\left(l_j,m_j,u_j\right)}{C_j},$$

$$C_j=\frac{1}{n+1}\sum_{i=0}^{n}M_i\left(l_j,m_j,u_j\right)\ (i=1,2,\ldots n\ ,j=1,2,\ldots m)\quad (6)$$

Triangular fuzzy number $x_i\left(s_j,b_j,k_j\right)$ is the composed element after dimensionless processing for the comparative sequence *i*;

(4) Taking the comparative sequence and the reference sequence of dimensionless processing into step (5) to calculating the grey correlation coefficient which has triangular fuzzy number.

(5)Judging the scheme. First of all, using the analytic hierarchy process to get the weight ω_j of the underlying evaluation index relative to the top, then, calculating the fuzzy correlation coefficient of each scheme index according to step 4, at last, calculating the overall fuzzy correlation degree of each scheme and ideal scheme, as follows:

$$\gamma_i=\sum_{j=1}^{m}\omega_j\varepsilon_i\left(s_j,m_j,u_j\right)\ (i=1,2,\cdots n)\quad (7)$$

VI. EXAMPLE

This paper also used the data in Ref.[1] as an example, taking the customers'demand on the properties of refrigerator as example, illustrating the grey correlation analysis in the application of the analytic hierarchy process. This article put this case as in Ref.[1], explaining the multi-level comprehensive evaluation model which is on the base of the triangular fuzzy number grey correlation analysis. An index structure of refrigerator customers' demand for products is a kind of multi-level structure. In view of the demand index, there are four kinds of schemes for selection, as shown in Tab.1. In the scheme, in addition to the part of parameter has specific values, part of this parameter only gives the fuzzy indexes such as "more,common"which is meeting the application of fuzzy grey theory. The application, you must convert the fuzzy index into the corresponding triangular fuzzy number, as shown in Tab. 2.First, you must quantify the qualitative indexes,because,the qualitative indexes cannot in calculation of the quantitative indexes.As shown in Tab. 2.

TABLE 1 FOUR SCHEMES DRAWN UP FOR DEMAND INDICATORS

Index of character	Scheme 1	Scheme 2	Scheme 3	Scheme 4
Total Volume	254L	225L	209L	219L
The volume of the refrigerator compartment	170L	145L	129L	147L
Volume of the freezing chamber	84L	80L	55L	72L
Ability of frozen	24kg/24h	15kg/24h	4kg/24h	20kg/24h
Rated power consumption	0.88kw.h/24h	0.69kw.h/24h	0.55kw.h/24h	0.62kw.h/24h
Price	3400RMB	3380RMB	3499RMB	3540RMB
Additional features	good	good	perfect	perfect
Performance of heat dissipated	general	perfect	good	perfect
Noise	poor	poor	bad	bad
Frosting rate	bad	poor	bad	poor

TABLE 2 FUZZY INDEX AND CORRESPONDING FUZZY TRIANGULAR FUZZY NUMBER

Index	bad	poor	general	good	perfect
The bigger the better	$\tilde{1}$	$\tilde{3}$	$\tilde{5}$	$\tilde{7}$	$\tilde{9}$
The smaller the better	$\tilde{9}$	$\tilde{7}$	$\tilde{5}$	$\tilde{3}$	$\tilde{1}$

Scheme after conversion, form the comparative sequence:

C1={254,170,84,1,0.88,3400,(6,7,8), (4,5,6), (2,3,4), (0,1,2) }

C2={225,145,80,0.625,0.69,3380, (6,7,8), (8,9,10), (2,3,4), (2,3,4)}

C3={209,129,55,0.167,0.55,3499, (8,9,10), (6,7,8), (0,1,2), (0,1,2) }

C4={219,147,72,0.833,0.62,3540, (8,9,10), (8,9,10), (0,1,2), (2,3,4) }

Selecte the reference sequence:

C0={250,170,84,1,0.55,3380, (8,9,10), (8,9,10), (0,1,2), (0,1,2)}

Apply the dimensionless method to the reference sequence and compare sequence:

C0={1.09,1.12,1.12,1.38,0.84,0.98,(0.87,1.09,1.38), (0.909,1.15,1.47),(0,0.55,2.5),(0,0.55,2.5)}

C1={1.09,1.12,1.12,1.38,1.34,0.99,(0.652,0.85,1.111),(0.45,0.64,0.88),(0.71,1.66,5),(0,0.55,2.5)}

C2={0.97,0.95,1.07,0.86,1.05,0.98,(0.65,0.84,1.111),(0 .90,1.15,1.47),(0.74,1.67,5),(0.74,1.667,5)}

C3={0.9,0.85,0.73,0.23,0.84,1.02,(0.87,1.09,1.39),(068 ,0.89,1.17),(0,0.56,2.5),(0,0.56,2.5)}

C4={0.94,0.97,0.96,1.15,0.94,1.03,(0.87,1.09,1.38),(0. 91,1.15,1.47),(0,0.55,2.5),(0.71,1.66,5)}

Bring the dimensionless reference sequence and compare sequence into Eq. (5),calculating the fuzzy correlation coefficients in each schemes, as shown in Tab 3

TABLE 3 CORRELATION COEFFICIENTS OF SCHEME

correlation coefficients	Scheme 1	scheme 2	scheme 3	scheme 4
Total Volume	(0.286,1,3.501)	(0.261,0.822,2.621)	(0.248,0.738,2.285)	(0.255,0.787,2.465)
The volume of the refrigerator compartment	(0.286,1,3.501)	(0.251,0.766,2.372)	(0.235,0.673,1.994)	(0.255,0.787,2.465）
Volume of the freezing chamber	(0.286,1,3.501)	(0.275,0.917,3.071)	(0.218,0.587,1.673)	(0.253,0.776,2.418)
Ability of frozen	(0.286,1,3.501)	(0.202,0.516,1.425)	(0.149,0.326,0.829)	(0.241,0.707,2.129）
Rated power consumption	(0.204,0.521,1.459）	(0.245,0.725,2.205)	（0.286,1,3.501）	(0.264,0.847,2.735）
Price	(0.284,0.982,3.406）	（0.286,1,3.501）	(0.277,0.933,3.149)	（0.275,0.917,3.071）
Additional features	(0.234,0.695,2.174）	(0.234,0.695,2.174）	（0.286,1,3.501）	（0.286,1,3.501）
Performance of heat dissipated	(0.194,0.52,1.541)	(0.286,1,3.501)	(0.231,0.683,2.14)	(0.286,1,3.501)
Noise	(0.095,0.333,1.167）	(0.095,0.333,1.167）	（0.286,1,3.501）	（0.286,1,3.501）
Frosting rate	（0.286,1,3.501）	(0.095,0.333,1.167）	（0.286,1,3.501）	（0.095,0.333,1.167）

Ref. [1] has calculated the weight of ten indicators using the method of hierarchical House Of Quality:

$\{\omega_i, i=1,2,\cdots,10\}$ ={0.052,0.052,0.052,0.0181,0.096,

0.068,0.124,0.126,0.123,0.125}

Bring the correlation coefficient and index weight of table 3 into formula 6, the overall correlation degree of each scheme and ideal scheme is:

$$\gamma_1 = \sum_{i=1}^{10} \omega_i \varepsilon_1 (s_i, m_i, u_i) = (0.236, 0.772, 2.598)$$

$$\gamma_2 = \sum_{i=1}^{10} \omega_i \varepsilon_2 (s_i, m_i, u_i) = (0.21, 0.656, 2.124)$$

$$\gamma_3 = \sum_{i=1}^{10} \omega_i \varepsilon_3 (s_i, m_i, u_i) = (0.244, 0.78, 2.582)$$

$$\gamma_4 = \sum_{i=1}^{10} \omega_i \varepsilon_4 (s_i, m_i, u_i) = (0.247, 0.808, 2.691)$$

According to the sorting method of triangle fuzzy number sequence: $\gamma_4 > \gamma_3 > \gamma_1 > \gamma_2$,therefore, the scheme 4 is better than the other three

VII. CONCLUSION

In this paper,method of triangular fuzzy numbers is used to convert qualitative indicators into the corresponding fuzzy quantitative data during program selection process based on gray correlation analysis.According to the arithmetic rules and sorting methods of triangular fuzzy number, combined with gray correlation analysis,each scenario fuzzy correlation calculation methods and the various options for each indicator fuzzy correlation is obtained, and by comparing the degree of overall fuzzy association the optimal solution is found.

The triangular fuzzy number based on gray correlation theory is employed to establish a multi-level comprehensive evaluation model and the use of the model is demonstrated by analyzing an instance.Compared to conventional methods, the proposed one in paper, overcoming the defect of taking qualitative indicators directly as a fixed value,is more effective to the practical situations.The method shows better maneuverability and consistency with reality for the complex, multi-level and fuzzy information invovled situations.

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