Consensus and Obstacle Avoidance of Partially-informed Multi-agent System

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***Abstract*—In this paper, we study the collective obstacle avoidance problem when only part of agents is informed of obstacle information. The control protocol is proposed to lead second order agents trend to state consensus in the existence of smooth convex obstacles. Both the position and velocity of agents will trend to the same. The moving obstacles which have varying acceleration can also be avoid, even without acceleration sensor. We prove the convergence and collision avoidance of multi-agent system. A simulation demonstrates the effect of the proposed control protocol.**

Keywords-collective movement, partial informed, obstacle avoidance, consensus, multi-agent system.

I. INTRODUCTION

Multi-agent group is a complex network system. The features of control distribution, local interaction, and self-organization indicate that simple behavior of a agent in the group may lead to an integral phenomenon. The overall performance is insensitive to the failure or fault of the individual agent. The most typical instances include the intelligent consensus behavior, swarming behavior and gathering behavior[1,2,3]. In the nature, the information that an agent can get is always limit[4]. Considering the limitations of obstacle sensors, we study the problem of collective obstacle avoidance for multi-agent system when only part of agents can detect obstacle shape, position and moving states[5,6].

To distinguish the actual agent and obstacle-agent, the actual agents are called α agent, obstacle-agent is called β agent which are generated by α agent. α agent generate virtual β agents passively for all near obstacles. Only when an α agent move close to an obstacle, the relevant virtual β agent can be calculated. So β agent may change all the time. The obstacle avoidance rule for a single agent is defined as follows:

(1) Find obstacle O_k in the detecting radius of α agent i, and get the center of obstacle q_k^o.

(2) Generate a virtual β agent on the boundary of obstacle O_k. The location of generated β agent is denoted as $q_{i,k}^{\beta}$. It satisfies

$$q_{i,k}^{\beta} = \arg\min_{x \in O_k} \| x - q_i^{\alpha} \|_2 \tag{1}$$

where the boundary of O_k is sphere or hypersphere.

II. COLLECTIVE OBSTACLE AVOIDANCE OF SECOND ORDER AGENTS

A. Control Algorithm and Results Statement

Consider the second order kinematics of agents as

$$\begin{cases} \dot{q}_i = p_i, \\ \dot{p}_i = u_i, \quad i = 1, \cdots, N, \end{cases} \tag{2}$$

where $q_i \in \mathrm{R}^n$ and $p_i \in \mathrm{R}^n$ are the position vector and velocity vector, $u_i \in \mathrm{R}^n$ is the control input with related to the acceleration of ith agent. The objective of the control algorithm is to make agents swarm and bypass every virtual obstacle-agent $o_k, k = 0, 1, \cdots$ with switched local interaction, and there is no collision happens.

The control protocol for α agent i avoiding H obstacles moving with unique velocity is defined as

$$u_i^{\alpha} = -\sum_{j=1}^{N} a_{ij}(q_i^{\alpha} - q_j^{\alpha}) - \xi_i \sum_{\beta i, k \in N_i^{\beta}(t)} \nabla_{(q_i^{\alpha} - q_{i,k}^{\beta})} \psi^{\beta}(\| q_i^{\alpha} - q_{i,k}^{\beta} \|_2)$$
$$-\xi_i c_0 (q_i^{\alpha} - q_o - \chi\theta) - \sum_{j=1}^{N} a_{ij}(p_i^{\alpha} - p_j^{\alpha}), \tag{3}$$

where c_0 is positive constant. If α agent i has obstacle sensor, $\xi_i = 1$, otherwise $\xi_i = 0$. If $j \in N_i^{\alpha}(t), j \neq i$, $a_{ij} = 1$, otherwise $a_{ij} = 0$.

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The collective reference point CRP is relatively static with obstacle, so $p_o = p_k^o, k = 1, \cdots, H$. Since $q_i^\alpha - q_{i,k}^\beta = (1-\tau)(q_i^\alpha - q_k^o)$, the relative position and relative velocity between α agent and CRP are $\hat{q}^l pha_i = q_i^\alpha - q_o - \chi\theta, \hat{p}_i^\alpha = p_i^\alpha - p_o = p_i^\alpha - p_k^o$. Then the control input can be rewritten as,

$$\hat{u}_i^\alpha = -\sum_{j=1}^N a_{ij}(\hat{q}_i^\alpha - \hat{q}_j^\alpha) - \xi_i \sum_{\beta i, k \in N_i^\beta(t)} \nabla_{(q_i^\alpha - q_k^o)} \psi^\beta(\|(1-\tau)(q_i^\alpha - q_k^o)\|_2)$$

$$-\xi_i c_0 \hat{q}_i^\alpha - \sum_{j=1}^N a_{ij}(\hat{p}_i^\alpha - \hat{p}_j^\alpha), \tag{4}$$

Theorem 3.2: Consider N α agents with second order dynamics (2) encounter H obstacles with a unique velocity. Using control protocol in (3) for multi-agent system, the following statement holds:

(1) (Convergence) The α agents which are unaware of obstacle information can converge to a same place, if they keep connected with any aware α agents. Their velocities trend to consensus.

(2) (Collision Avoidance) If the derivative of potential function is big enough, α agents will not collide with obstacle.

B. Convergence Proof

The energy function of multi-agent system is defined as

$$\Upsilon(\hat{q}_i^\alpha, \hat{p}_i^\alpha) = \frac{1}{2}\sum_{i=1}^N \sum_{j=1}^N a_{ij} \|\hat{q}_i^\alpha - \hat{q}_j^\alpha\|_2^2 + \frac{1}{2}\sum_{i=1}^N \xi_i c_0 (\hat{q}_i^\alpha)^T(\hat{q}_i^\alpha)$$

$$+\sum_{i=1}^N \xi_i \sum_{\beta_{i,k} \in N_i^\beta(t)} \psi^\beta(\|(1-\tau)(q_i^\alpha - q_k^o)\|_2)$$

$$+\frac{1}{2}\sum_{i=1}^N (\hat{p}_i^\alpha)^T(\hat{p}_i^\alpha) \tag{5}$$

Clearly, it is a positive semi-definite function with related to relative vector $\hat{q}_i^\alpha$ and $\hat{p}_i^\alpha$.

The derivative of potential function is

$$\dot{\Upsilon}(\hat{q}_i^\alpha, \hat{p}_i^\alpha) = \sum_{i=1}^N \sum_{j=1}^N a_{ij}(\hat{q}_i^\alpha - \hat{q}_j^\alpha) \cdot \hat{p}_i^\alpha + \sum_{i=1}^N \xi_i c_0 (\hat{q}_i^\alpha) \cdot \hat{p}_i^\alpha$$

$$+\sum_{i=1}^N \xi_i \sum_{\beta_{i,k} \in N_i^\beta(t)} \nabla_{(q_i^\alpha - q_k^o)} \psi^\beta(\|(1-\tau)(q_i^\alpha - q_k^o)\|_2) \cdot \hat{p}_i^\alpha$$

$$+\sum_{i=1}^N (\hat{p}_i^\alpha)^T(\hat{u}_i^\alpha)$$

$$= -\sum_{i=1}^N (\hat{p}_i^\alpha)^T \sum_{j=1}^N a_{ij}(\hat{p}_i^\alpha - \hat{p}_j^\alpha)$$

$$= -(\hat{p}^\alpha)^T (L \otimes I_n)(\hat{p}^\alpha) \tag{6}$$

where L is the Laplacian matrix, $\hat{p}^\alpha$ is the column stack of $\hat{p}_i^\alpha$. Suppose the adjacent matrix is defined as $A = [a_{ij}]$, elements of the relevant diagonal matrix $\Delta(A)$ is $\sum_{j=1}^N a_{ij}$. We can get Laplacian matrix as $L = \Delta(A) - A$. Since $(\hat{p}^\alpha)^T(L \otimes I_n)(\hat{p}^\alpha)$ is positive semi-definite, $-(\hat{p}^\alpha)^T(L \otimes I_n)(\hat{p}^\alpha) \le 0$. Hence, $\dot{\Upsilon}(\hat{q}^\alpha, \hat{p}^\alpha) \le 0$, the multi-agent system converges. If the initial value of total energy is Υ_0, $\Upsilon(t) \le \Upsilon_0$ for $t \ge 0$.

According to LaSalle Invariant Principle, each unaware agent which is connected with an aware agent will converge to the largest invariant set

$$S = \left\{(q^\alpha, p^\alpha) : -(\hat{p}^\alpha)^T(L \otimes I_n)(\hat{p}^\alpha) = 0\right\}. \tag{7}$$

We have $\hat{p}^\alpha = 0$ with the property of SOS. Then, $p_i^\alpha = p_o$ which implies the velocity of α agent trend to p_o.

C. Collision Avoidance Analysis

If α i can detect an obstacle, according to the energy function between α agents and obstacle, $\max \psi^\beta(\|(1-\tau)(q_i^\alpha - q_k^o)\|_2) = +\infty$. Since $\Upsilon_0 < \max \psi^\beta(\|(1-\tau)(q_i^\alpha - q_k^o)\|_2)$, agents which are aware of obstacles will not collide with obstacles.

The α agents which are not aware of obstacle information can only follow the movement of aware α agents. Since the energy function between agents and obstacles is defined as $\Upsilon_0 < \psi^\beta(r_{safe})$, the minimum distance between aware α agents and obstacle is no less than r_{safe}. With the constraint of $r_{safe} - r_k \ge N^2 r_\alpha$, agents group will not collide with obstacle.

III. SIMULATION

Simulation that fifty α agents which are partially informed avoid three smooth convex obstacles in 3D space is demonstrated. Fifteen agents are randomly chosen to be informed about obstacle information. Three circles are located at $q_{OA}=[7,6,10], q_{OB}=[9,10.5,10], q_{OC}=[7,13.5,10]$ with velocity $\bar{p}_o^{\beta}=[0.01,0,0]^T$ and acceleration 0. The objective direction is $\theta=[1,0,0]$. The initial position is randomly chosen in box $[0.2,4]\times[0.4,20]\times[8,12]$. The initial velocity is randomly chosen in box $[-0.01,0.01]\times[-0.01,0.01]\times[-0.01,0.01]$. The communication radius is $r_\alpha=2$, the repulsion radius to obstacles is $r_\beta=2$, and $c_0=0.5$. The initial position of CRP is chosen as $q_o+\chi\theta=[18,10,10]^T$ with $\chi=9.33$. Then, the simulation results is shown in Fig. 1.

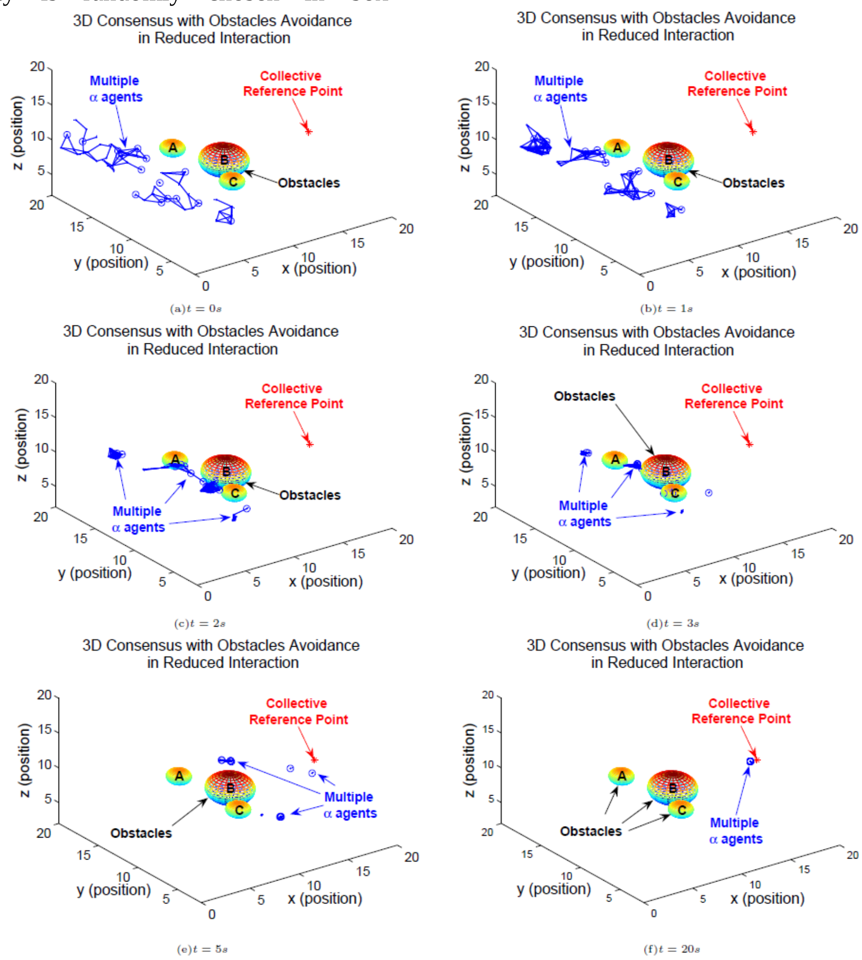

Figure 1. Consensus and Obstacle Avoidance of Partially-informed multi-agent system in 3D space.

IV. Summary

In this paper, the obstacle avoidance problem of multi-agent system when only a small fraction of agents are informed of the obstacle is focused. A collective obstacle avoidance protocol in switched topology for agents with second-order kinematics is designed. The obstacle is moving with varying acceleration, but the sensor to detect obstacle acceleration is not necessary for agents here. We prove that agents which cannot detect obstacle will move and avoid obstacle either, if they keep a linkage with anyagent informed of obstacle information. No collision happens between agents and obstacles.

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