

Dynamic Circuit Analysis Based on Computer

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Abstract—This essay put forward an analysis solution on the basis of the computer software-Matlab, from the output and input of the dynamic circuit system's mathematic establishment to the solution of the mathematic model of differential equation. It make use of the high quality numerical of calculation function, drawing function and each tool kit comes with the function. Therefore, it can overcome the impossibility that the traditional time domain, frequency--domain analysis method brought miscellaneous operation and the hand calculation of high order circuit. Taking a three order dynamic circuit for example, we can get the same result with using the traditional method and the method based on computer of analyzing this circuit.

Keywords-Dynamic circuit; Computer; Matlab software; Numerical calculation

I. INTRODUCTION

Both in the circuit analysis and signal processing, dynamic circuit analysis is an vital content, many designments of electrical equipment, the performance index are considered according to the sine steady-state. In the course of analyzing the circuit, phasor method was general used in dynamic sinusoidal steady state circuit analysis, using phasor make the process of solving differential equation sine time function more simplified to the plural of phasor algebra. There are two methods in signal processing, to begin with, the time domain analysis method, establishing a dynamic input and output mathematical model of the differential equation of electric circuit to solve differential equation response. Besides, "s" domain analysis method, the principle of the phasor method and this method are similar, However, "s" domain analysis have no limitation of incentive form. This method, to establish dynamic circuit "s" domain model first, and then work out the response in the "s" domain algebraic equations[1]. No matter what method be used, the analysis and the solving process is very complex, large and complicated computation makes the analysis of high order dynamic circuit analysis failed to be accomplished. With the rapid development of computer technology, all kinds software tool of processing mathematical calculation were created at the same time, Matlab is a senior interactive applications with powerful computing function and which is suitable for analysis design and the complex calculation in various fields of engineering application. Using numerical calculation function, drawing function of Matlab software and its equipped various toolbox avoid the complex computation of the traditional analysis method. The course of establishing the mathematical model of high order dynamic circuit input and

output and solving the mathematical model of differential equation becomes more easier[2].

II. THE ESTABLISHMENT OF THE MATHEMATICAL MODEL OF DYNAMIC CIRCUIT

A The traditional method of establishing time domain model

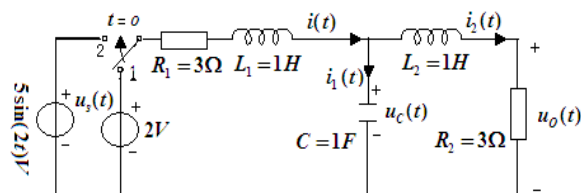


Fig.1 Third-order dynamic circuit

According to the ohm's law and kirchhoff's law, form the third-order dynamic circuit in figure 1, we can list the equation shown in (1):

$$\begin{cases} i(t) - i_1(t) - i_2(t) = 0 \\ i(t)R_1 + L_1 \frac{di(t)}{dt} + u_c(t) = u_s(t) \\ u_c(t) = L_2 \frac{di_2(t)}{dt} + i_2(t)R_2 \\ C \frac{du_c(t)}{dt} = i_1(t) \\ u_o(t) = i_2(t)R_2 \end{cases} \quad (1)$$

Eliminate $i(t)$ 、 $i_1(t)$ 、 $i_2(t)$ 、 $u_c(t)$,and get a dynamic mathematical model of the circuit shown in figure 1:

$$CL_1L_2u_o'''(t) + C(R_1L_2 + R_2L_1)u_o''(t) + (CR_1R_2 + L_1 + L_2)u_o'(t) + (R_1 + R_2)u_o(t) = R_2u_s(t)$$

After applying the various parameters, the circuit can be simplified as the third order linear differential equation with constant coefficient:

$$u_o'''(t) + 6u_o''(t) + 11u_o'(t) + 6u_o(t) = 3u_s(t) \quad (2)$$

B. The traditional method of establishing "s" domain model

Draw the figure 1 "s" domain model, as shown in figure 2.

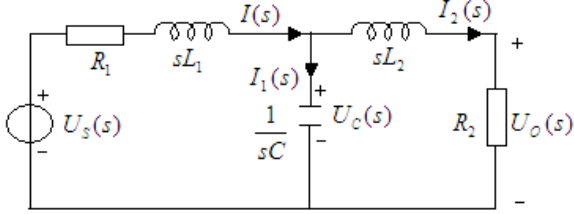


Fig.2 The "s" domain model of third-order dynamic circuit
List equations shown in (3):

$$\begin{cases} I(s)(R_1 + sL_1 + \frac{1}{sC}) - I_2(s) \frac{1}{sC} = U_s(s) \\ -I(s) \frac{1}{sC} + I_2(s)(R_2 + sL_2 + \frac{1}{sC}) = 0 \\ U_o(s) = I_2(s)R_2 \end{cases} \quad (3)$$

Eliminate $I(s)$ 、 $I_2(s)$, then:

$$U_o(s)[CL_1L_2s^3 + C(R_1L_2 + R_2L_1)s^2 + (CR_1R_2 + L_1 + L_2)s + (R_1 + R_2)] = R_2U_s(s)$$

Therefore, the system function is:

$$H(s) = \frac{U_o(s)}{U_s(s)} = \frac{R_2}{CL_1L_2s^3 + C(R_1L_2 + R_2L_1)s^2 + (CR_1R_2 + L_1 + L_2)s + (R_1 + R_2)} \quad (4)$$

Apply the parameters and get the same differential equation as (2).

$$u_o'''(t) + 6u_o''(t) + 11u_o'(t) + 6u_o(t) = 3u_s(t) \quad (5)$$

In fact, these two traditional kinds methods of establishing the model have certain difficulty, the process of the elimination of (1) and (3) is not simple, with complex computation and large amount of calculation, easily making mistake.

C. The method of establishing model based on computer

Take Zero-state Laplace Transform of each equation in (1), then get (6):

$$\begin{cases} I(s) - I_1(s) - I_2(s) = 0 \\ I(s)R_1 + L_1sI(s) + U_C(s) = U_s(s) \\ U_C(s) = L_2sI_2(s) + I_2(s)R_2 \\ CsU_C(s) = I_1(s) \\ U_o(s) = I_2(s)R_2 \end{cases} \quad (6)$$

Coordinate (6) as the matrix form of (7):

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 \\ R_1 + L_1s & 0 & 0 & 1 & 0 \\ 0 & 0 & L_2s + R_2 & -1 & 0 \\ 0 & -1 & 0 & Cs & 0 \\ 0 & 0 & R_2 & 0 & -1 \end{bmatrix} \begin{bmatrix} I(s) \\ I_1(s) \\ I_2(s) \\ U_C(s) \\ U_o(s) \end{bmatrix} = \begin{bmatrix} 0 \\ U_s(s) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

Abbreviation:

$$AX=Y. \quad (8)$$

Using Matlab program M file, and substitute R、r、L、1、u、uo for R₁、R₂、L₁、L₂、U_s(s)、U_o(s).

```
%lyy1
syms s u R r L C
A=[1,1,1,0,0;R+s*L,0,0,1,0;0,0,s*L+r,1,0;0,1,0,C*s,0;0,0,r,0,-1];B=[0,u,0,0,0]';
X=A\B; uo=X(5,1)
The result after circulated:
uo
=1/(R+R*C*s^2*I+R*C*s*r+s*L+s^3*L*C*I+s^2*L*C*r+s*I+r)*r*conj(u)
```

Then:

$$U_o(s) = \frac{R_2}{CL_1L_2s^3 + C(R_1L_2 + R_2L_1)s^2 + (CR_1R_2 + L_1 + L_2)s + (R_1 + R_2)} U_s(s)$$

The system function is:

$$H(s) = \frac{U_o(s)}{U_s(s)} = \frac{R_2}{CL_1L_2s^3 + C(R_1L_2 + R_2L_1)s^2 + (CR_1R_2 + L_1 + L_2)s + (R_1 + R_2)} \quad (9)$$

Just as (4), get the same mathematic model as (5).

III. THE SOLUTION OF DIFFERENTIAL EQUATION

A. The method of time domain system to solve differential equation

After the construction of the system mathematical model, dynamic analysis of the circuit was converted to solve differential equations. The method of time domain analysis system is a important method in the system analysis of solving differential equation, the principle is dividing the system response into zero input response and zero state

response.

1) Zero input response

To use $u_{Ozi}(t)$ represent zero input response of this system, it abide by homogeneous equation.

$$u_{Ozi}'''(t) + 6u_{Ozi}''(t) + 11u_{Ozi}'(t) + 6u_{Ozi}(t) = 0, \text{ and}$$

$$u_{Ozi}''(0_-) = 0, \quad u_{Ozi}'(0_-) = 0, \quad u_{Ozi}(0_-) = 1.$$

"-1", "-2", "-3" is the three natural frequencies of the system, so we can assume:

$$u_{Ozi}(t) = c_{zi1}e^{-t} + c_{zi2}e^{-2t} + c_{zi3}e^{-3t} \quad (10)$$

According to:

$$\begin{cases} u_{Ozi}(0_-) = u_O(0_-) = 1 = c_{zi1} + c_{zi2} + c_{zi3} \\ u_{Ozi}'(0_-) = u_O'(0_-) = 0 = -c_{zi1} - 2c_{zi2} - 3c_{zi3} \\ u_{Ozi}''(0_-) = u_O''(0_-) = 0 = c_{zi1} + 4c_{zi2} + 9c_{zi3} \end{cases} \Rightarrow \begin{cases} c_{zi1} = 3 \\ c_{zi2} = -3 \\ c_{zi3} = 1 \end{cases}$$

Therefore, the zero input response is:

$$u_{Ozi}(t) = (3e^{-t} - 3e^{-2t} + e^{-3t})\varepsilon(t). \quad (11)$$

2) Zero state response

To use $u_{Ozs}(t)$ represent zero state response of this system, it abide by equation.

$$u_{Ozs}'''(t) + 6u_{Ozs}''(t) + 11u_{Ozs}'(t) + 6u_{Ozs}(t) = 3u_s(t),$$

and $u_{Ozs}''(0_-) = u_{Ozs}'(0_-) = u_{Ozs}(0_-) = 0, u_s(t) = 5\sin(2t).$

Assume the solution of homogeneous of zero state response according to the natural frequency:

$$u_{Ozsh}(t) = c_{zs1}e^{-t} + c_{zs2}e^{-2t} + c_{zs3}e^{-3t}. \quad (12)$$

For free items of the equation is $15\sin(2t)$ when $t \geq 0_+$, so we can assume the particular solution is:

$$u_{Op} = A\sin(2t) + B\cos(2t). \quad (13)$$

Apply(13) into

$$u_{Op}'''(t) + 6u_{Op}''(t) + 11u_{Op}'(t) + 6u_{Op}(t) = 15\sin(2t)$$

Coordinate:

$$(14A - 18B)\cos(2t) + (-14B - 18A)\sin(2t) = 15\sin(2t) \Rightarrow A = -27/52, B = -21/52$$

$$\therefore u_{Op}(t) = -\frac{27}{52}\sin(2t) - \frac{21}{52}\cos(2t). \quad (14)$$

So the zero state response is:

$$u_{Ozs}(t) = c_{zs1}e^{-t} + c_{zs2}e^{-2t} + c_{zs3}e^{-3t} - \frac{27}{52}\sin(2t) - \frac{21}{52}\cos(2t). \quad (15)$$

According

$$\begin{cases} u_{Ozs}(0_-) = 0 = c_{zs1} + c_{zs2} + c_{zs3} - \frac{21}{52} \\ u_{Ozs}'(0_-) = 0 = -c_{zs1} - 2c_{zs2} - 3c_{zs3} - \frac{27}{26} \\ u_{Ozs}''(0_-) = 0 = c_{zs1} + 4c_{zs2} + 9c_{zs3} + \frac{21}{13} \end{cases} \Rightarrow \begin{cases} c_{zs1} = 3 \\ c_{zs2} = -3.75 \\ c_{zs3} = 1.1538 \end{cases}$$

Therefore, the zero state response of the system is:

$$u_{Ozs}(t) = [3e^{-t} - 3.75e^{-2t} + 1.1538e^{-3t} - \frac{27}{52}\sin(2t) - \frac{21}{52}\cos(2t)]\varepsilon(t)$$

$$= [3e^{-t} - 3.75e^{-2t} + 1.1538e^{-3t} + 0.66\sin(2t - 142^\circ)]\varepsilon(t). \quad (16)$$

For the system complete response is the plus of the zero input response and the zero state response, so the complete response is:

$$u_O(t) = u_{Ozi}(t) + u_{Ozs}(t) = [6e^{-t} - 6.75e^{-2t} + 2.1538e^{-3t} + 0.66\sin(2t - 142^\circ)]\varepsilon(t). \quad (17)$$

3) Solve differentia equation by "s" domain Laplace transform

Because "s" domain Laplace transform method can transform the differential equation for algebra, it greatly reduce the difficulty of getting solution, so in the traditional method "s" domain Laplace transform method is a kind of optimization method to solve differential equations[4].

Take Laplace transform of (5):

$$[s^3U_O(s) - s^2u_O(0_-) - su_O'(0_-) - u_O''(0_-)] + 6[s^2U_O(s) - su_O(0_-) - u_O'(0_-)] + 11[sU_O(s) - u_O(0_-)] + 6U_O(s) = 15U_S(s)$$

Coordinate

$$u_{O'}(0_-) = 0, u_{Ozs}'(0_-) = 0, u_{Ozs}(0_-) = 1, U_S(s) = \frac{2}{s^2 + 4};$$

$$U_O(s) = \frac{s^2 + 6s + 11}{(s+1)(s+2)(s+3)} + \frac{30}{(s+1)(s+2)(s+3)(s^2+4)}$$

$$= \frac{U_{Ozi}(s)}{U_{Ozs}(s)} = \frac{3}{(s+1)} + \frac{-3}{(s+2)} + \frac{1}{(s+3)} + \frac{3}{(s+1)} + \frac{-3.75}{(s+2)} + \frac{1.1538}{(s+3)} - \frac{27}{52} \frac{2}{s^2+4} - \frac{21}{52} \frac{s}{s^2+4}$$

Apply the Inverse Laplace transform:

$$u_{Ozi}(t) = L^{-1}[U_{Ozi}(s)] = (3e^{-t} - 3e^{-2t} + e^{-3t})\varepsilon(t)$$

$$u_{Ozs}(t) = L^{-1}[U_{Ozs}(s)] = [3e^{-t} - 3.75e^{-2t} + 1.1538e^{-3t} - \frac{27}{52}\sin(2t) - \frac{21}{52}\cos(2t)]\varepsilon(t)$$

$$u_O(t) = u_{Ozi}(t) + u_{Ozs}(t) = [6e^{-t} - 6.75e^{-2t} + 2.1538e^{-3t} + 0.66\sin(2t - 142^\circ)]\varepsilon(t).$$

B. The solution of differential equation based on computer

This two methods of solving differential equation (especially time domain method) are involved in a large number of complex mathematical analysis and mathematical calculation, especially when the system natural frequency are non-integer, its uneasy to get correct results even fail to accomplish operation. However, through the application of Matlab software, all problems can be simple.

1) Symbol function method

It is a way of using Matlab to write the files to solve symbol differential equation, this method can not only get zero input response, zero state response and the complete response of the symbol function type, but can also draw the response waveform.

```
%lyy2
eq='D3y+6*D2y+11*Dy+6*y=0';cond='y(0)=1,Dy(0)=0
,D2y(0)=0';
ans=dsolve(eq,cond);yzi=simplify(ans) %zero input
response
eq='D3y+6*D2y+11*Dy+6*y=15*sin(2*t)';
cond='y(-0.001)=0,Dy(-0.001)=0,D2y(-0.001)=0';
yzs=dsolve(eq,cond) %zero state response
eq='D3y+6*D2y+11*Dy+6*y=15*sin(2*t)';
cond='y(-0.001)=1,Dy(-0.001)=0,D2y(-0.001)=0';
y=dsolve(eq,cond) %complete response
y1=yzi+yzs %zero input response+zero state
response
subplot(141);ezplot(yzi,[0,8]);xlabel('time(sec));title('ze
ro input response');grid on;
subplot(142);ezplot(yzs,[0,8]);xlabel('time(sec));title('ze
ro state response');grid on;
subplot(143);ezplot(y,[0,8]);xlabel('time(sec));title('com
plete response');grid on;
subplot(144);ezplot(y1,[0,8]);xlabel('time(sec));title('z-i
response+z-s response');grid on;
The result:
yzi = -3*exp(-2*t)+3*exp(-t)+exp(-3*t)
yzs = -27/52*sin(2*t)-21/52*cos(2*t)+3*exp(-t)-
3.75*exp(-2*t)+1.15*exp(-3*t)
y = -27/52*sin(2*t)-21/52*cos(2*t)+6*exp(-t)-
6.75*exp(-2*t)+2.15*exp(-3*t)
y1 = -27/52*sin(2*t)-21/52*cos(2*t)+6*exp(-t)-
6.75*exp(-2*t)+2.15*exp(-3*t)
After abbreviated:
```

$$u_{Ozi}(t) = yzi = (3e^{-t} - 3e^{-2t} + e^{-3t})\varepsilon(t)$$

$$u_{Ozs}(t) = yzs = [3e^{-t} - 3.75e^{-2t} + 1.15e^{-3t} + 0.66\sin(2t - 142^\circ)]\varepsilon(t)$$

$$u_O(t) = y = y1 = [6e^{-t} - 6.75e^{-2t} + 2.15e^{-3t} + 0.66\sin(2t - 142^\circ)]\varepsilon(t)$$

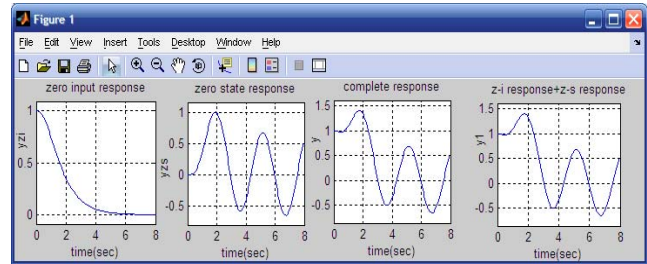


Fig.3 lyy2 The response waveform figure

2) The system frequency response method

If you only require dynamic sine steady-state response of the circuit analysis, the system frequency response function may be used to solve it[5].

Write a Matlab source program:

```
%lyy3 use freqs function to reach frequency response
and solve steady state response
b=[3];a=[1,6,11,6];w=0:0.1:2*pi;H=freqs(b,a,w);HM=ab
s(H);HP=angle(H);
a=2/0.1+1;uos=5*HM(a)*sin(2*t+HP(a))
The result:
uos=3/52*130^(1/2)*sin(2*t-27928504337
/112589990684 )
```

After abbreviated:

$$u_{Os}(t) = 0.66\sin(2t - 142^\circ)$$

IV. CONCLUSION

Traditional dynamic analysis method of the circuit has many shortcomings: strong theoretical demand and a large amount of mathematical deduction, multifarious calculation, poor intuitive, tough analysis, or it even can not carry out high order system analysis. The high quality numerical kit comes with the function, makes the dynamic analysis of the circuit more simpler and quicker and easier to understand. There are also many methods of using Matlab analyzing dynamic circuit, such as the use of Matlab Simulink toolbox, we can simulate the input and output mathematical model and the dynamic circuit[3]. Anyway, the more and the better method is looking forward us to continually research, explore and share.

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