# Global Motion Estimation Based on Fourier Mellin and Phase Correlation 

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#### Abstract

Global motion estimation aims at estimating the whole image changes causing by camera moving. This paper presents a global motion estimation method based on Fourier-Mellin and Phase-Correlation. Rotational angle and scaling factor are acquired by Phase-Correlation between Fourier-Mellin transform images of the reference and current image. Log-Polar coordinates are used to the Flourier amplitude spectrum. Translational parameter is obtained using sub-block based on Phase- Correlation. The proposed algorithm is robust to camera vibration or unwanted movement regardless of foreground object's movement. Experimental results show that the proposed algorithm can efficiently and accurately estimate the parameter between two consecutive frames.


Keywords-fourier-mellin; phase-correlation; global motion estimation; log-polar coordinate

## I. INTRODUCTION

Digital image stabilization systems aim to remove irregular global motion effects from an image sequence in order to obtain a compensated sequence that displays smooth camera movements only [1].Hand-held video capturing devices such as digital video cameras and cell phones are becoming more and more accessible due to their lowered price and reduced size. While the annoying jitter due to unstable camera motion caused by unintentional shake of human hands are hardly to be avoided. Therefore image stabilization technique is needed to remove this inevitable and undesirable fluctuation motion during the image capturing process in order to acquire high quality videos.

The digital image stabilization system can be divided into motion estimation and motion compensation [2]. The motion compensation system computes inter-frame global motion vector, which is the most important part of the system. The motion compensation system stabilizes the image sequence according to the motion vectors. Various digital image stabilizing system have been developed to minimize motion degradation such as block matching [3], representative point matching [4], gray scale projection algorithm [5], bit-plane matching [6] and PhaseCorrelation algorithm [7].

In this paper, we proposed a novel method based on Fourier-Mellin domain and Phase-Correlation. This algorithm can efficiently deal with rotational and scaling motions in addition to translational motion, even when the sequence has both large rotation motion and intended panning motion, proposed algorithm can work well.

## II. PHASE-CORRELATION THEORY

Phase-Correlation was proposed by Kugin and Hines in 1975 [8], which based on Fourier domain. It only uses phase information and significantly reduces the reliance on the image content. It is also robust to illumination and noise.

Assume $f_{2}(x, y)$ is the result of $f_{1}(x, y)$ translated by $\left(x_{0}, y_{0}\right)$ and it can be described in Eq.1.

$$
\begin{equation*}
f_{2}(x, y)=f_{1}\left(x-x_{0}, y-y_{0}\right) \tag{1}
\end{equation*}
$$

Their corresponding Fourier transforms $f_{1}$ and $f_{2}$ will be related by Eq. 2

$$
\begin{equation*}
F_{2}(\xi, \eta)=e^{-j 2 \pi\left(\xi_{0}+\eta y_{0}\right)} * F_{1}(\xi, \eta) \tag{2}
\end{equation*}
$$

The cross-power spectrum of two images $f_{1}$ and $f_{2}$ with Fourier transform $F_{1}, F_{2}$ is defined as Eq. 3

$$
\begin{equation*}
\frac{F_{1}(\xi, \eta) F_{2}^{*}(\xi, \eta)}{\left|F_{1}(\xi, \eta) F_{2}^{*}(\xi, \eta)\right|}=e^{j 2 \pi\left(\xi x_{0}+\eta y_{0}\right)} \tag{3}
\end{equation*}
$$

Where $F_{2}{ }^{*}(\xi, \eta)$ is the complex conjugate of $F_{2}$, the inverse Fourier transformation of $e^{j 2 \pi\left(\left\{x_{0}+\eta y_{0}\right)\right.}$ is a Dirac $\delta$-function define as Eq. 4

$$
\begin{equation*}
c p s=\delta\left(x-x_{0}, y-y_{0}\right) \tag{4}
\end{equation*}
$$

It is approximately zero everywhere except at the displacement $\left(x_{0}, y_{0}\right)$.

## III. FOURIER-MELLIN THEORY

Fourier-Mellin Transform is a technique which turns the rotation and scaling in spatial domain into phase shifts [9].

Considering the current image $s(x, y)$ is rotated, scaled and translated from the reference image $f(x, y)$. Supposing the rotation angle is $\varphi$, the scaling coefficient is $\alpha$ and the shift is $\left(x_{0}, y_{0}\right)$.

In the spatial domain $s(x, y)$ can be described by Eq. 5

$$
\begin{equation*}
s(x, y)=f\left(a(x \cos \varphi+y \sin \varphi)+x_{0}, a(-x \sin \varphi+y \cos \varphi)+y_{0}\right) \tag{5}
\end{equation*}
$$

After Fourier transform, in the frequency domain the relation Eq. 5 is described by Eq.6.

$$
\begin{equation*}
S(u, v)=a^{-2} e^{-j 2 \pi\left(x_{0} u+y_{0} v\right)} \cdot F\left(a^{-1}(u \cos \varphi+v \sin \varphi), a^{-1}(-u \sin \varphi+v \cos \varphi)\right) \tag{6}
\end{equation*}
$$

Considering the magnitude of $S(u, v), F(u, v)$ we have Eq. 7

$$
\begin{equation*}
\left|F_{1}(u, v)\right|=a^{-2}\left|F\left(a^{-1}(u \cos \varphi+v \sin \varphi), a^{-1}(-u \sin \varphi+v \cos \varphi)\right)\right| \tag{7}
\end{equation*}
$$

From Eq. 7 we can see that if we use Log-Polar coordinate [10], the rotation and scaling will change into translation.

Consider the Log-Polar transform Eq. 8

$$
\left\{\begin{array}{l}
u=e^{\rho} \cos \theta  \tag{8}\\
v=e^{\rho} \sin \theta
\end{array}\right.
$$

From Eq. 7 and Eq.8, the Log-Polar of spectral magnitude is shown as Eq.9. In this Log-Polar domain presentation, both rotation and scaling are turned into translations in parameter domain.

$$
\begin{gather*}
\left|F 1\left(e^{\rho} \cos \theta, e^{\rho} \sin \theta\right)\right|=\mid F_{1}\left(\alpha^{-1}\left(e^{\rho} \cos (\theta-\phi)\right), \alpha^{-1}\left(e^{\rho} \sin (\theta-\phi)\right) \mid\right. \\
=\alpha^{-2} \mid F_{1}\left(\left(e^{(\rho-\log \alpha)} \cos (\theta-\phi)\right),\left(e^{(\rho-\log \alpha)} \sin (\theta-\phi)\right) \mid\right. \tag{9}
\end{gather*}
$$

## IV. PROPOSED METHOD

Phase-Correlation can gain good result if there is only translation between two images. But it is sensitive to scale and rotate. Experiment has shown when the rotation exceeds 1.50.The estimation error cannot be accepted.

Considering existing Phase-Correlation can't handle scale, rotation between frames. We extend the PhaseCorrelation using Fourier-Mellin. The overview of our algorithm is shown in Fig1.


Figure 1. Overview of the proposed method

## V. COMPUTE THE ROTATIONAL, SCALE, TRANSLATIONAL FACTOR

## A. Compute the rotational and scale factor

The process is shown in Fig2. According to FourierMellin theory, we need to extract the amplitude spectrum from the two consecutive images, and directly to logarithm amplitude spectrum of polar coordinates transformation. The process describes in detail as follows:

Step1: we use Fourier transform to (a) ~ (b) image, and obtain the magnitude spectrum (c) ~ (d) image.

Step2: we apply log-polar transform to (c) ~ (d) image and obtain (e) $\sim(f)$ image. From image (e) $\sim(f)$ we can see that the rotation and scale in spatial domain convert to transition in Fourier-Mellin domain.

Step3: we apply phase-correlation to (e) ~ (f) image and get a Dirac $\delta$-function as is shown in image (g).

According to Eq.4, we can get the peak's position $\left(x_{0}, y_{0}\right)$. The peak should be 1 in theory, but in fact it is less than 1 , as for the noise. Considering the images of size $256 \times 256$, and in the log-polar plane representation a size of $256 \times 256$. The conversion formulate need for angle of rotation and scale are as shown in Eq. 10

$$
\left\{\begin{array}{l}
\text { scale }=\log \left(x_{0}\right)  \tag{10}\\
\text { angle }=\left(180^{*} y_{0}\right) / 256
\end{array}\right.
$$



Figure 2. The processing of computing rotational angle and scale factor
Image (a) the original image. Image (b) rotation and translation from image (a).Image (c) the amplitude spectrum of image (a). Image (d) the amplitude spectrum of image (b).Image (e) the result of log-polar transform to image (c). Image (f) the result of log-polar transformation of image (d). Image (g) the result of phase correlation of image (e) and image (f)

## B. Compute the translatonal factor

Using rotational and scale factor to inverse transform the current frame, the influence caused by rotational and scale factor will be eliminated. As most image sequences may contain foreground objects moving which may mislead the global motion vector, we divide the image into sub-blocks (usually $64 \times 64$ ) .and then we choose the vector which has the largest probability as the final transitional factor. The overview of the process results is shown in Fig3.

Step1: Dividing the images to many sub-blocks (usually $64 \times 64$ ).Then compute the displacements between correspondence sub-blocks using phase-correlation. If the
sub-blocks are too small, similarity between blocks area is low. It may mislead the result of the computed translational factor.

Step2: Statistics the computed motion vectors, and finding the vector of the most probability as the final transitional result.

## VI. EXPERIMENT AND ANALYSIS

Table1 describes the experimental data in details. It was found in Table1 that the estimated transformation parameter values were nearly equal to the real parameter values. Using Fourier-Mellin and log-polar transformation, we successfully isolated rotation, translation and rotation factor. The peak value ought to be 1 in theory, but it is always less than 1 in practice. The noise and the similarity between two images influence the peak value. By experimental verification, the overlap of the two images if larger than $1 / 3$, we can obtain right results.

TABLE1.EXPERIMENTAL DATA IN DETAIL

| Image transform | Real <br> Parameter | Estimate Parameter | error | peak |
| :---: | :---: | :---: | :---: | :---: |
| Rotate | 50 | 5.040 | 0.040 | 0.2 |
|  |  |  |  | 8 |
| Translate | [8,10] | [8,10] | 0 | 0.7 |
|  |  |  |  | 6 |
| Scale | 1.5 | 1.59 | 0.09 | 0.1 |
|  |  |  |  | 6 |
| Rotate | 50, [8,10] | 5.040,[9,10] | 0.040,[10] | 0.2 |
| Translate |  |  |  | 7 |
| Scale | 1.5, [8,10] | 1.59, [9,10] | 0.09,[1 0] | 0.1 |
| Translate |  |  |  | 1 |
| Rotate ,Scal | 50, $[8,10]$ | 5.040,[9,10] | 0.04,[1 0] | 0.0 |
| e |  |  |  | 5 |
| Rotate |  |  |  |  |
| Translate | 50,[8,10],1. | 1.59,5.040,[9,1 | 0.09,[1,0],0.04 | 0.0 |
| Scale | 5 | $0]$ | 0 | 5 |





Figure 3. Processing of computing transition parameter
Image (a) original image .Image (b) the inverse transformed image. Image (c) the statistics of computed vector. Image (d) the statistics of computed vector

## VII. CONCLUSIONS

In this paper, we propose an algorithm based on Fourier-Mellin and Phase-Correlation. The main advantage of the proposed method is that it can compute the rotation, scale, and transition factor at the same time. Using log-polar is invariant to rotation and scale. Using sub-block and data clustering can effectively eliminate the error caused by the foreground moving objects.

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