

The Research Of Mapping Laws For Image Histogram Matching

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Keywords. Histogram Matching; Digital image; mapping law; histogram equalization

Abstract: This paper analysis the principles of histogram matching for digital image processing, and shows that what's the different between "Group Mapping Law" and "Single Mapping Law" and their advantage and shortcoming. This paper also looks into the histogram equalization method and implement the algorithm for the different mapping laws. The purpose is to eliminate the unfavorable influence when the picture is imaging.

Introduction

There are many methods to process the digital images, we choose the appropriate method for the different purpose. Histogram matching is a practical method for the image processing. Histogram is a process where a time series, image, or in different condition such that its histogram matches that of another dataset. It is often need to show the difference when we process the two pictures which in the different phase but the same area. The method of relative radiation correction contains Calibration coefficient and Image statistic. Histogram Matching is a method for enhance the effect of the image to make the gray histogram to become the corresponding shape. The detail of the picture could become more clear when the image is processed by histogram matching as it can make the grayscale spacing wide and gray distributed uniform.

SML(single mapping law) and GML (group mapping law) can used to histogram matching. A common application of this is to match the images from two sensors with slightly different responses, or from a sensor whose response changes over time.

Histogram matching principle

The purpose of Histogram Matching is make the histogram of original image match the histogram expectations. In this paper, we process the two pictures which in the different phase but the same area by the histogram matching and to eliminate the bad effect because of different condition.

Let the value of the function for the probability density of the original Image is $p_r(r)$ and $p_z(z)$ is the value of the function for the probability density of the target Image.

A. Histogram Equalization

First, the example given here should be processed by histogram equalization :

$$s = T(r) = \int_0^r p_r(r) dr \quad (1)$$

Then the grayscale of the target image should be processed by histogram equalization :

$$v = G(z) = \int_0^z p_z(z) dz \quad (2)$$

The formula transform of (2) is

$$Z = G^{-1}(v) \quad (3)$$

The grayscale of the target image could be calculated by the grayscale of the image which is equalization. The images which has equalized should have the same function for the probability density as the original image and the target image have equalized, so $p_r(r)$ is equal to $p_z(z)$, so we can use the grayscale s which comes from the original image was equalized replace the value v which in the formula (3) above.

$$Z = G^{-1}(v) = G^{-1}(s) \quad (4)$$

The example given here is a gray bit map picture which grayscale is 8 and the original picture like the followed *Fig. 1* and the histogram is given,for example *Fig. 2*

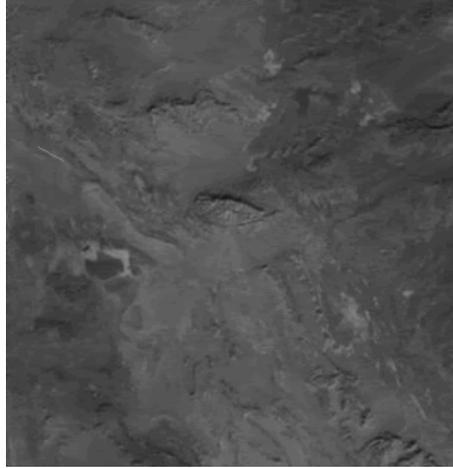


Figure 1. The Original Image

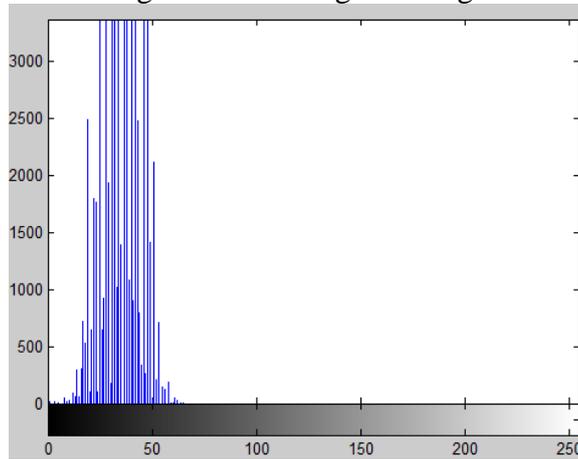


Figure 2. The Histogram

And the image which equalized and its histogram is shown in *Fig. 3* and *Fig. 4*.

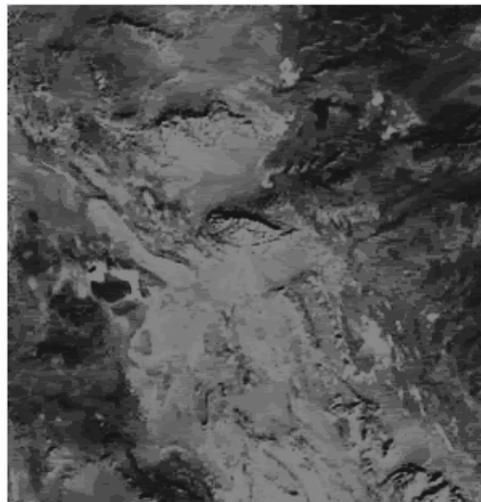


Figure 3. The target image

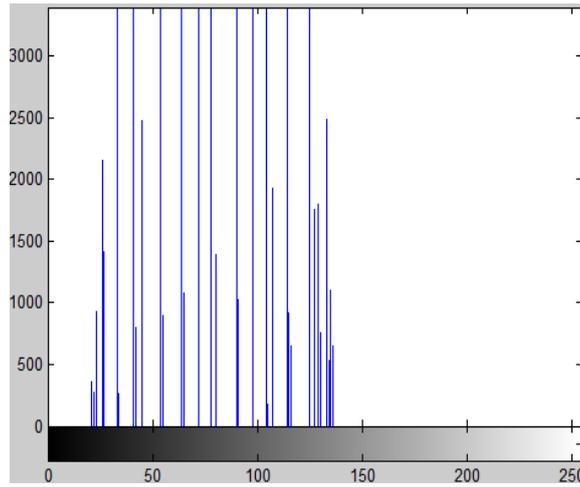


Figure 4. The histogram of the target image

The experimental material is to adjust the image's hue more bright which showed in *Fig. 1*, and the image which would be matched and its histogram is shown in *Fig. 5* and *Fig. 6*.

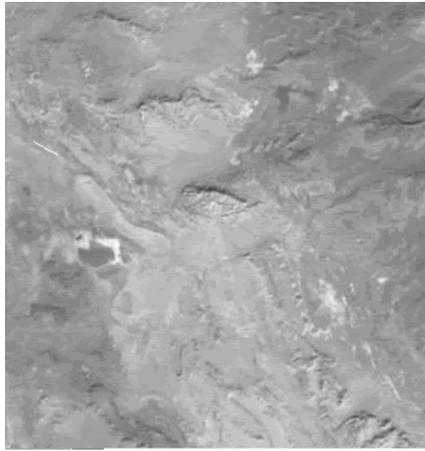


Figure 5. The Histogram

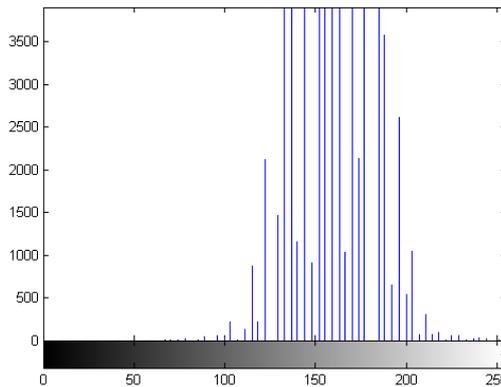


Figure 6. The Histogram

The single mapping law

The single mapping law in histogram is a mapping from original cumulative histogram to specified cumulative histogram.

First, if the value of grayscale of original image is M and the value of grayscale of specification image is N ($M \geq N$). Then the original image should be equalized.

$$x_k = \sum_{S_i} P_{S_i} (S_i) \quad (k=0, 1, 2, \dots, M-1)$$

The value of i is range from 0-k, then the transform formula of histogram equalization of the specification image like that.

$$y_i = \sum P_x(u_i) \quad (i=0, 1, 2, \dots, N-1)$$

the value of j is range from 0-1. And at the end, the original histogram should mapping to the target one.

Histogram normalisation is a similar process where the reference histogram is a uniform distribution (the cumulative distribution is a constant slope). The effect is to spread the data values over the available dynamic range. Single mapping law is a process to find the suitable point from the original histogram to the target one, and get the value of k and l to make sure the value of the formula is the smallest which is $|\sum P_y(s_i) - \sum P_x(u_i)|$, then put the value of $P_y(s_i)$ mapping to the value of $P_x(u_j)$. This method is simple and useful but the accuracy is weakness.

The group mapping law

Group Mapping Law (GML) is a process to make the value of the following formula is smallest .

$$|\sum_{i=0}^{I(l)} P_y(s_i) - \sum_{j=0}^l P_x(u_j)| \quad l=0, 1, 2, \dots, N-1$$

The integer function $I(l)$ ($l=0, 1, \dots, N-1$), should satisfy $0 \leq I(0) \leq \dots \leq I(l) \leq \dots \leq I(N-1) \leq M-1$.

$P_y(s_i)$ which i is between 0 and $I(0)$ should be mapping to $P_x(u_0)$ if the value of l equals 0. Then $P_y(s_i)$ which i is between $I(l-1)+1$ and $I(l)$ should be mapping to $P_x(u_j)$ if the value of l greater than or equal 1.

Comparison of two mapping laws

The following example is order to comparison of two mapping laws. Assume that the values of grayscale of the original image and the normal image and the probability of grayscale is know. The following pictures are an examples using single mapping law and group mapping law.

TABLE I. MAPPING VALUE BETWEEN SML AND GML

Num	Title	Procedure							
		0	1	2	3	4	5	6	7
1	Grayscale Of Original Image								
2	Probability Of Grayscale Of Original Image	0.172	0.086	0.088	0.03	0.061	0.056	0.061	0.446
3	Cumulative Histogram Of Original Image	0.172	0.258	0.346	0.376	0.437	0.493	0.554	1.00
4	Probability Of Grayscale Of Matching Image	0	0.4	0	0	0.2	0	0	0.4
5	Matching Cumulative Histogram Calculate	0	0.4	0.4	0.4	0.6	0.6	0.6	1.00
6a	SML Mapping	1	1	1	1	1	1	4	7
7a	Mapping Relationship	0, 1, 2, 3, 4, 5 → 1						6 → 4	7 → 7
8a	Transform Histogram	0	0.496	0	0	0.12	0	0	0.384

6b	GML Mapping	1	1	1	1	4	4	4	7
7b	Mapping Relationship	0, 1, 2, 3→1				4, 5, 6→4			7→7
8b	Transform Histogram	0	0.428	0	0	0.188	0	0	0.384

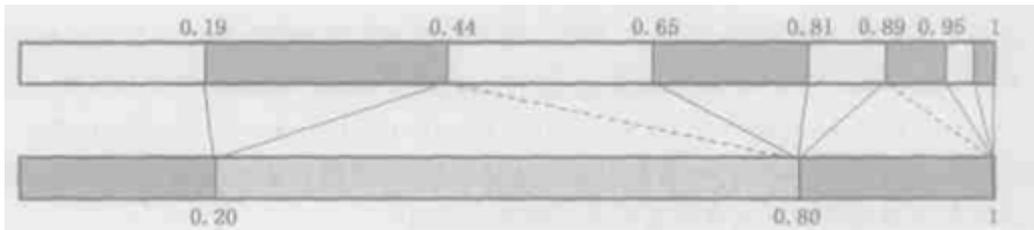


Figure 7 Single Mapping Law

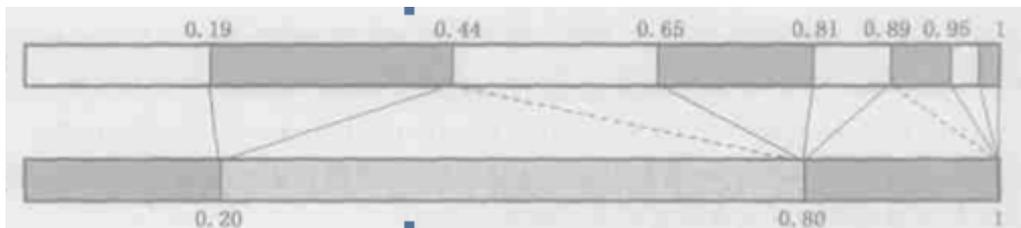


Figure 8 Group Mapping Law

- a) In the discussion here the two datasets assumed to have the same range of values.lines.
- b) As can be seen in the examples above ,the matching introduces gaps in the different mapping laws.
- c) Margin of error can be .calculated by the sum of the difference between the value of different mapping laws. The smaller the better. And the value is zero at ideal conditions.
- d) Margin of error is $|0.4-0.496| + |0.2-0.12| + |0.4-0.384| = 0.192$ if the principle is single mapping law and the other is $|0.4-0.428| + |0.2-0.188| + |0.4-0.384| = 0.056$ if the principle is group mapping law. The effect of GML is better than SML clearly.

From the above analysis,the single mapping law is a margin of error principle,some grayscale was mapping the similar start calculate value. However,group mapping law is a more accurate mapping law so the margin of error is more smaller .

The image matches one image to another and is based upon algorithm of GML is showed at Fig. 9.

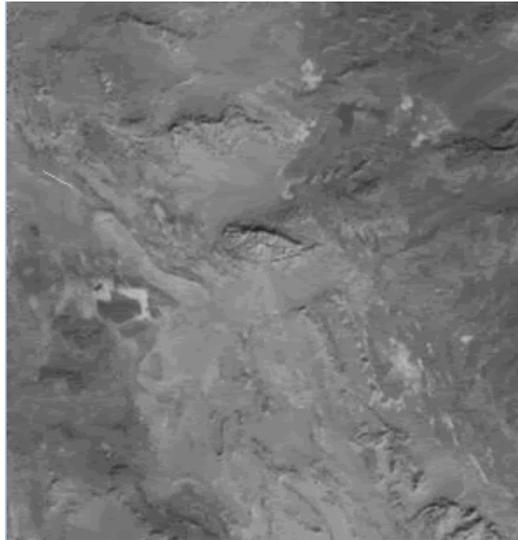


Figure 9 Matched in Group Mapping Law

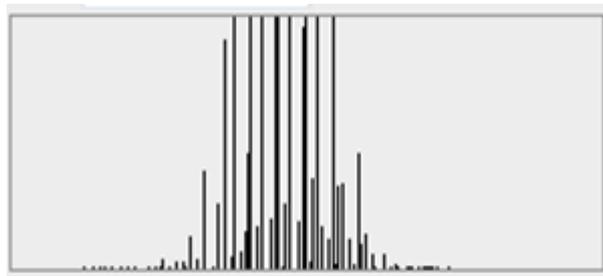


Figure 10 histogram

Conclusions

The examples above, the matching introduces gaps in the different mapping laws. And the accuracy of the group mapping law is more better than the single mapping law at the same condition.

Acknowledgment

Many thanks to the classmates of the group at College of computer science for encouragement and discussions. The author would also like to thank Kong Dexing for his feedback on the algorithm implementation..

References

- [1]Liuyang Fang and Mi Wang,"CPU/GPU near real-time preprocessing for ZY-3 satellite images: Relative radiometric correction," MTF compensation, and geocorrection,January 2014.
- [2]Histogram Matching , Paul Bourke,January 2014.
- [3]XUE Lixia, LIU Huang and WANG Zuocheng. "Moving object detection algorithm based on matching of brightness histogram." Computer Engineering and Applications, 2013,
- [4]Wang Xiaoyan and Long Xiaoxiang,"Analysis of Relative Radiometric Correction Method of CBERS - 02B Camera. Spacecraft Recovery & Remote Sensing," , 2008.06
- [5]JIAO Bin, LV Xia-fu, CHEN Yong andLI Yuan, Improved algorithm of adaptive Gaussian mixture model for real-time moving object detection