Interaction of Two Anomalous Hollow Gaussian Beams in SNNM

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Abstract. The interaction of two in-phase and out-phase anomalous hollow Gaussian beams in strongly nonlocal nonlinear media (SNNM) is investigated. The analytical expression describing the two beams co-propagating in SNNM is derived. It is found that when two anomalous hollow Gaussian beams separated by a distance at the source plane propagate together in SNNM, the evolution of two beams varies periodically. The two beams always attract each other, whatever the relative phase of the two beams is in-phase or out-phase. At the interactional region, many interference fringes appear, and the central intensity is a prominent peak for two in-phase beams; however, it is equal to zero for two out-phase beams.

Introduction

Spatial nonlocality can be encountered in many physical systems, such as nematic liquid crystals, lead glasses, photorefractive crystals, atomic vapors, Bose–Einstein condensates, and so on. For the strongly nonlocal case, Snyder and Mitchell simplified the nonlocal nonlinear Schrodinger equation into a linear model, i.e. the Snyder Mitchell model (SMM) [1, 2]. Based on SMM, various laser beams propagating in strongly nonlocal nonlinear media (SNNM) are investigated, and many novel solitons and breathers are found, for instance Gaussian and higher-order Gaussian solitons [1, 2], multipole solitons [3], elliptical solitons [4], nematicons [5], and these solitons do not exist in the Kerr local nonlinear media. The interaction between nonlocal solitons has also attracted considerable interest [1, 6-9]. Typically, two nolocal solitons always attract each other, whatever the relative phase between the two solitons is [1, 6], and the nonlocal dark solitons are no exception [7, 8], which is much different from the local solitons.

The anomalous hollow Gaussian beam is firstly observed in experiments, and it can be used as a powerful tool to study the linear and nonlinear particle dynamics in the storage ring [10]. Subsequently, a mathematical model is constructed to describe this beam [11]. Up to now, an anomalous hollow Gaussian beam through some optical systems has been studied, for examples a turbulent atmosphere, uniaxial crystals, fractional Fourier system, and a misaligned first order optical system etc. Recently, the anomalous hollow Gaussian beam propagating in SNNM is also investigated, and the transversal reverse transformation is found [12]. However, to the best of our knowledge, the interaction between anomalous hollow Gaussian beams in SNNM remains unexploited. In this paper, the interaction of anomalous hollow Gaussian beams in SNNM is investigated and an analytical expression describing the interaction between two anomalous hollow Gaussian beams is obtained. The characteristics of interaction are discussed, and some numerical simulations are carried out to exhibit these characteristics.

Analytical theory

The optical field of an anomalous hollow Gaussian beam at the source plane can be expressed as follows [10]

$$E\left(x_0,y_0,0\right)=C_0\left(-2+\frac{8x_0^2}{w_{0x}^2}+\frac{8y_0^2}{w_{0y}^2}\right)\exp\left(-\frac{x_0^2}{w_{0x}^2}-\frac{y_0^2}{w_{0y}^2}\right), \tag{1}$$

where C_0 is a constant associated with the input power $P_0=\iint E\left(x_0,y_0,0\right)dx_0dy_0$, w_{0x} and w_{0y} are the beam waist width of an astigmatic Gaussian beam in x and y directions, respectively. In this paper, we consider two anomalous hollow Gaussian beams are separated by a distance d in x direction, then the combined optical field of the two anomalous hollow Gaussian beams at the source plane can be expressed as,

$$\begin{aligned}E_C\left(x_0,y_0,0\right)=C_0&\left[-2+\frac{8\left(x_0-d/2\right)^2}{w_{0x}^2}+\frac{8y_0^2}{w_{0y}^2}\right]\exp\left[-\frac{\left(x_0-d/2\right)^2}{w_{0x}^2}-\frac{y_0^2}{w_{0y}^2}\right]\\&\pm C_0\left[-2+\frac{8\left(x_0+d/2\right)^2}{w_{0x}^2}+\frac{8y_0^2}{w_{0y}^2}\right]\exp\left[-\frac{\left(x_0+d/2\right)^2}{w_{0x}^2}-\frac{y_0^2}{w_{0y}^2}\right]\end{aligned}, \tag{2}$$

where $\pm$ represents in-phase and out-phase, respectively.

An optical beam propagating in SNNM is governated by the Snyder–Mitchell linear model [1, 2], and it can be expressed as follows in the rectangular coordinates,

$$2ik\frac{\partial E}{\partial z}+\Delta_\perp E-k^2\gamma^2P_0\left(x^2+y^2\right)E=0, \tag{3}$$

where $\Delta_\perp=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the two dimensional transverse Laplacian operator, k is the wave number in the media without nonlinearity, and γ is a material constant associated with the nonlocal response function of the media.

A single anomalous hollow Gaussian beam propagating in SNNM has been investigated, and the analytical evolution expression is obtained [12]. In addition, it has proved that an optical beam propagating in SNNM can be regarded as a self-induced fractional Fourier transform [13]. Therefore, based on the previous results and the linear rule and the shift rule of the fractional Fourier transform, one can obtained the analytical evolution expression for two anomalous hollow Gaussian beams propagating in SNNM, and it is given as

$$\begin{aligned}E_C\left(x,y,z\right)=&A\exp\left[-id\sin\alpha\left(x-\frac{d\cos\alpha}{2}\right)\right]\times\exp\left\{-\frac{iz_R\cos\alpha}{z_p\sin\alpha}\left[\frac{\left(x-d\cos\alpha/2\right)^2}{w_{0x}^2}+\frac{y^2}{p^2w_{0x}^2}\right]\right\}\\&\times\exp\left[-\frac{z_R^2}{z_p^2\sin^2\alpha+iz_pz_R\sin\alpha\cos\alpha}\cdot\frac{\left(x-d\cos\alpha/2\right)^2}{w_{0x}^2}-\frac{p^2z_R^2}{z_p^2\sin^2\alpha+ip^2z_pz_R\sin\alpha\cos\alpha}\cdot\frac{y^2}{w_{0x}^2}\right]\\&\pm A\exp\left[-id\sin\alpha\left(x+\frac{d\cos\alpha}{2}\right)\right]\exp\left\{-\frac{iz_R\cos\alpha}{z_p\sin\alpha}\left[\frac{\left(x+d\cos\alpha/2\right)^2}{w_{0x}^2}+\frac{y^2}{p^2w_{0x}^2}\right]\right\}\\&\times\exp\left[-\frac{z_R^2}{z_p^2\sin^2\alpha+iz_pz_R\sin\alpha\cos\alpha}\cdot\frac{\left(x+d\cos\alpha/2\right)^2}{w_{0x}^2}-\frac{p^2z_R^2}{z_p^2\sin^2\alpha+ip^2z_pz_R\sin\alpha\cos\alpha}\cdot\frac{y^2}{w_{0x}^2}\right]\end{aligned}, \tag{4}$$

where

$$A=\frac{iC_0}{D}\left(\rho_1+\rho_2x^2+\rho_3y^2\right),$$

$$\begin{aligned}D=&8z_pz_R^3\sin\alpha\left(\cos\alpha-i\frac{z_p}{z_R}\sin\alpha\right)^2\left(ip^2\cos\alpha+\frac{z_p}{z_R}\sin\alpha\right)^2\\&\times\left(\frac{iz_R\cos\alpha}{z_p\sin\alpha}+1\right)^{\frac{1}{2}}\left(\frac{iz_R\cos\alpha}{z_p\sin\alpha}+\frac{1}{p^2}\right)^{\frac{1}{2}}\end{aligned},$$

$$\rho_1 = -48 z_p^4 \sin^4\alpha - i64 z_p^3 z_R \left(1+p^2\right)\sin^3\alpha\cos\alpha + 16 p^4 z_R^4 \cos^4\alpha + 16 z_p^2 z_R^2 \left(1+4p^2+p^4\right)\sin^2\alpha\cos^2\alpha,$$

$$\rho_2 = 64\frac{z_R^4}{w_{0x}^2}\left(\frac{z_p}{z_R}\sin\alpha + ip^2\cos\alpha\right)^2,$$

$$\rho_3 = 64\frac{p^2 z_R^4}{w_{0x}^2}\left(\frac{z_p}{z_R}\sin\alpha + i\cos\alpha\right)^2,$$

$p = w_{0y}/w_{0x}$, $\alpha = z/z_p$, $z_p = \sqrt{P_0/P_{gc}}\,z_R$, $P_{gc} = 1/\gamma^2 z_R^2$ is the critical power of Gaussian beam [2].

Characteristics of interaction

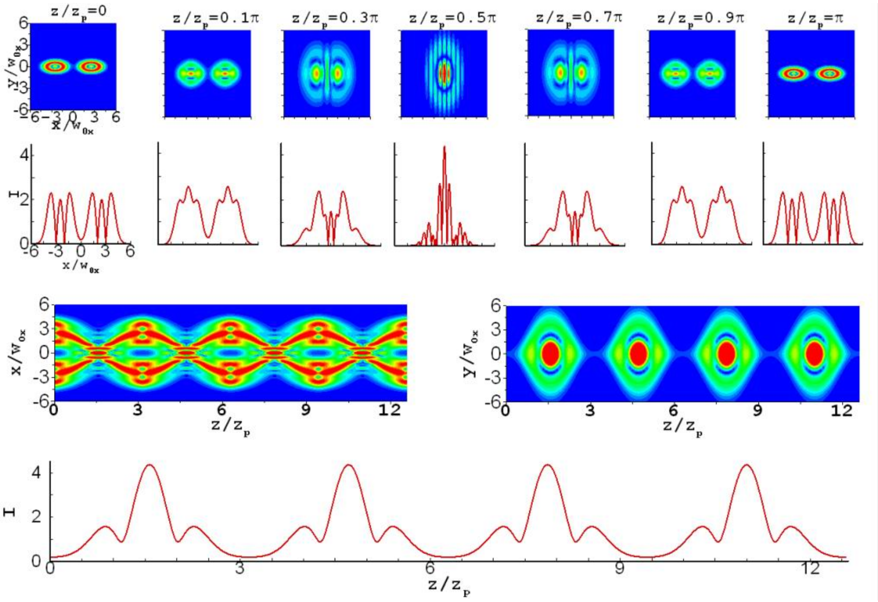

Fig.1. The intensity evolution of two interactional in-phase anomalous hollow Gaussian beams in SNNM. Row 1: the transverse intensity distributions at different propagation distances; row 2: the intensity distributions in x direction corresponding to row 1; row 3: the intensity evolutions in x direction (left) and in y direction (right); row 4: the on-axis intensity evolution. Parameters: $d = 5w_{0x}$, $p = 0.5$, $P_0 = 7P_{gc}/19$.

From Eq. (4), one can find that the evolution of two interactional in-phase or out-phase anomalous hollow Gaussian beams is periodical and the period is πz_p, which is the same as that of a single anomalous hollow Gaussian beam [12]. Fig. 1 gives the interaction of two in-phase anomalous hollow Gaussian beams and Fig. 2 shows the same as Fig. 1 except that the two beams are out-phase. One can find that at the source plane, the two beams is separated by a distance of $d = 5w_{0x}$, and the optical fields are not superposed. Before the optical fields begin to lap over, they propagate solely similar to a single beam, for example at $z/z_p = 0.1\pi$ in Figs. 1 and 2. It is also found from Figs. 1 and 2, they are always attracted whatever they are in-phase or out-phase. Subsequently they begin to collide and the optical fields are beginning to superpose. In the interactional region, many interference fringes appear, especially at $z/z_p = 0.5\pi$. The evolution from $z/z_p = 0.5\pi$ to $z/z_p = \pi$ is an inverse process from $z/z_p = 0$ to $z/z_p = 0.5\pi$, which is similar to the evolution of a single beam. From Figs. 1 and 2, one can see that the evolutions of two in-phase and out-phase beams are similar. The difference between two in-phase and out-phase beams is at the center of the two beams, there exists an intensity peak for the in-phase case, and the intensity in y direction is always zero for the out-phase case. Therefore, the intensity evolutions in y direction and on axis are not given in Fig. 2.

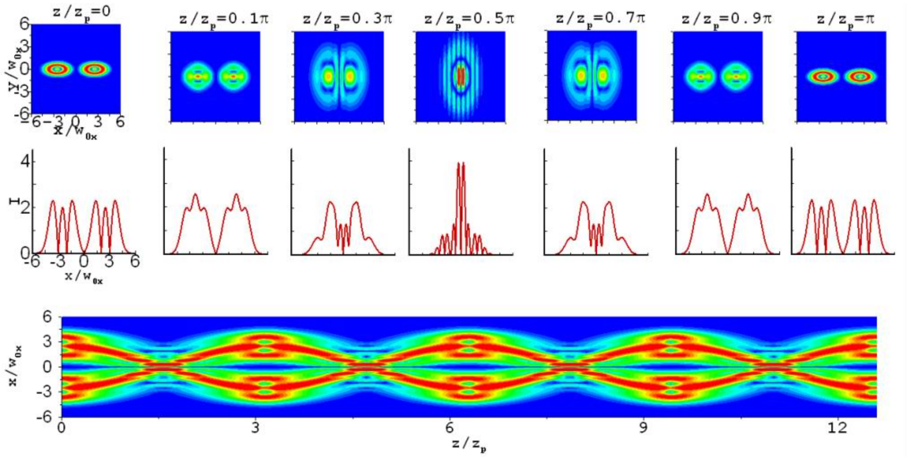

Fig. 2. The intensity evolution of two interactional out-phase anomalous hollow Gaussian beams in SNNM. Rows 1-3 are the same as rows 1-3 in Fig. 1, respectively, except that the two beams are out-phase.

Summary

In summary, the interaction of two anomalous Gaussian beams in SNNM has been investigated, and the analytical expression describing the interaction is obtained. We focus on the interaction of two in-phase and out-phase anomalous Gaussian beams in this paper and find that they always attract each other whatever the relative phase of two anomalous Gaussian beams is in-phase or out-phase. For the in-phase case, the center of interactional region exists an intensity peak, however, it is always zero for the out-phase case.

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