Adaptive Control and Application of Position Servo System

Jianqing Min

College of biological and environmental engineering, Zhejiang Shuren University, Hangzhou, 310015, China

email: Minjq@sina.com

Keywords: adaptive control; backstepping; nonlinear

Abstract. An adaptive backstepping neural network position controller design is presented for the electro-hydraulic servo system with mismatched uncertainties. By applying backstepping design strategy and online approaching uncertainties with RBF neural networks, a nonlinear controller for a hydraulic servo-system is developed based on Lyapunov stability theory. An adaptation law is also proposed to deal with uncertainties in hydraulic parameters. Theoretical analyses are provided to show the effectiveness of the proposed method.

Introduction

The electro-hydraulic servo systems provide many advantages over electric motors, including high durability and the ability to produce large force at high speeds. However, it is well know that they exhibit a significant nonlinear behavior. Nonlinear flow-pressure characteristics, variation in the trapped fluid volume due to piston motion, besides there may be other uncertain factors such as the parameters uncertainty, the unmodeled input dynamics and the external disturbance etc., the factors contributing to this mismatched nonlinear. The problem that external disturbance and parameter changes can cause the property decline of control system must be seriously considered when designing controller. In order to improve the control performance, many people have investigated widely and some results have been achieved [1].

In 1991, Kokotovic and his colleagues presented the theory of backstepping. As a nonlinear control method, it has aroused much attention of many scholars. It is an effective measure to solve the robust control of the mismatched and uncertainties system[4]. It has been extensively applied in industry control fields, such as aerospace, robot, hydraulic control, motor control etc.[2-4], applying backstepping technology in all kinds of servo system is one of hot topics of the present servo control study.

In this paper, we present an adaptive neural network method in servo controller design to increase system's robustness. The RBF neural network is used to approach uncertainties, and nonlinear tracking differentiator is introduced to deal with the problem of extreme expanded operation quantity of backstepping method. The paper gives the stability proof of the controller by Lyapunov theory.

Description of the Plant

Without loss of generality, consider an uncertain strict-feedback nonlinear system expressed by the following equations

$$\begin{cases} \dot{x}_i = f_i(\boldsymbol{X}_i) + g_i(\boldsymbol{X}_i)x_{i+1} + \Delta_i(\boldsymbol{X},t) \quad (1 \le i < n) \\ \dot{x}_n = f_n(\boldsymbol{X}) + g_n(\boldsymbol{X})u + \Delta_n(\boldsymbol{X},u,t) \\ y = x_1 \end{cases} \tag{1}$$

Where, $u,y \in \boldsymbol{R}$ are the control input and output respectively, $\boldsymbol{X} = [x_1, x_2 \cdots, x_n]^T \in \boldsymbol{R}^n$ is the state vector, $\boldsymbol{X}_i = [x_1, x_2 \cdots, x_i]^T \in \boldsymbol{R}^i$. $f_i(\boldsymbol{X}_i)$ and $g_i(\boldsymbol{X}_i)$ are known smooth function, $1 \le i \le n$, $g_i(\boldsymbol{X}_i)^{-1}$ exists in the whole definition domain, Δ_i denotes the mismatched uncertainties, including the external load disturbance and parameter uncertainty. If the nonlinear system doesn't possess strict-feedback form, the feedback terms which block the plant to be translated into strict-feedback form can be regard as the uncertainties.

Assumption 1: $|\Delta_i| \le D(\boldsymbol{X})$, $\forall t \ge 0$, $D(\boldsymbol{X})$ is nonnegative smooth nonlinear function.

Assumption 2: For a real continuous function $\Delta f : \Omega \mapsto \boldsymbol{R}$, Ω is a compact subset of $\boldsymbol{R}$. For any $\varepsilon > 0$, there always exists the Gauss function vector $\boldsymbol{h} \in \boldsymbol{R}^{l\times 1}$ and the optimum weight vector $\boldsymbol{w} \in \boldsymbol{R}^{l\times 1}$, which fulfils following equation

$$\Delta f = \boldsymbol{w}^T \boldsymbol{h}(\boldsymbol{X}) + \varepsilon \tag{2}$$

Where, the Gauss function $h_j = \exp(-0.5\|\boldsymbol{X} - \boldsymbol{c}_{ij}\|^2 / b_j^{\,2})$, ($i$=1,2,…, n; j=1,2,…, l), $\boldsymbol{X} \in \boldsymbol{R}^{n\times 1}$ is reference input vector; l is the number of hidden nodes, $\boldsymbol{c} \in \boldsymbol{R}^{m\times l}$ is the center matrix, $\boldsymbol{b} \in \boldsymbol{R}^{l\times 1}$ is the width vector, ε is the approximation error. Define $\hat{\boldsymbol{w}}$ as estimate value of $\boldsymbol{w}$, then the weight estimate error is $\tilde{\boldsymbol{w}} = \hat{\boldsymbol{w}} - \boldsymbol{w}$.

The control aim is to eliminate the effect of unexpected factors, and let the system output y=x_1 can track the desired control output x_d.

Adaptive Neural Network Controller Design

The Backstepping design, which aiming at the mismatch uncertainties system, is a kind of systematic synthetic technique to controller. It is a recursive procedure that combines the choice of a Lyapunov function with the design of feedback control. The Backstepping design technique starts from the differential equation in its smallest order of the system, it introduces the concept of virtual control and designs the satisfied virtual control step by step, and the real controller can be gotten in the end.

Define the state variable $\boldsymbol{Z}_i$

$$\boldsymbol{Z}_i = [z_{1,} \cdots, z_i], \quad z_i = x_i - a_{i-1}, \quad (1 \le i \le n), \quad a_0 = x_d \tag{3}$$

Where a_i is the fictitious controlling value of all orders, and defined by

$$a_i = g_i^{-1}(\boldsymbol{X}_i)[-f_i(\boldsymbol{X}_i) + \dot{\hat{a}}_{i\text{-}1} - z_{i-1}g_{i-1}(\boldsymbol{X}_{i-1}) - k_i z_i - \hat{\boldsymbol{w}}_i^T \boldsymbol{h}_i - \delta_{i-1}\mathrm{sgn}(z_i)] \qquad (1 \le i \le n-1) \tag{4}$$

Where $\dot{\hat{a}}_{i-1}$ is the output of nonlinear tracking differentiator whose input is $a_{i\text{-}1}$. The aim of introducing the nonlinear tracking differentiator is, by substituting $\dot{\hat{a}}_{i-1}$ for $\dot{a}_{i-1}$, to deal with the problem of extreme expanded operation quantity of computing $\dot{a}_{i-1}$ with the elevation of system order. Denote $e_{i-1} = \dot{\hat{a}}_{i-1} - \dot{a}_{i-1}$, where $\dot{a}_0 = \dot{\hat{a}}_0 = \dot{x}_d$, $k_i > 0$ and $\delta_{i-1} \ge |\varepsilon_i + e_{i-1}|$ are the design parameters.

Choose the backstepping control law as

$$u = g_n^{-1}(\boldsymbol{X})[-f_n(\boldsymbol{X}) + \dot{\hat{a}}_{n\text{-}1} - z_{n-1}g_{n-1}(\boldsymbol{X}_{n\text{-}1}) - k_n z_n - \hat{\boldsymbol{w}}_n^T \boldsymbol{h}_n - \delta_{n\text{-}1}\mathrm{sgn}(z_n)] \tag{5}$$

Define the parameters of neural networks control law as

$$\dot{\hat{\boldsymbol{w}}}_i = \Gamma_i \boldsymbol{h}_i z_i, \quad (1 \le i \le n) \tag{6}$$

Where Γ_i >0 is matrix of error weight.

Choose the Lyapunov function as

$$V_i(\boldsymbol{Z}_i) = \sum_{j=1}^{i} \frac{1}{2} z_j^{\,2} + \sum_{j=1}^{i} \frac{1}{2} tr[\tilde{\boldsymbol{w}}_j^{\,T} \Gamma_j^{-1} \tilde{\boldsymbol{w}}_j] \tag{7}$$

Lemma 1: For the uncertainties system described as Eq. (1), the RBF neural networks (2) is used to online approaching the system uncertainties, and its parameters adaptive law is (6), (4) and (5) are the fictitious control input and control law respectively, if $\delta_{i-1} > |\varepsilon_i + e_{i-1}|$ is satisfied, then the system output can asymptotically track the desired control output.

Proof:

Step 1 if i=1, and a_1 is virtual input, then the first-order subsystem of system (1) can be expressed as follows

$$\dot{x}_1 = f_1(\boldsymbol{X}_1) + g_1(\boldsymbol{X}_1)a_1 + \Delta_1 \tag{8}$$

Where, according to assumption 1 and 2, we have $\Delta_1 = \boldsymbol{w}_1^T \boldsymbol{h}_1 + \varepsilon_1$

Subsequently, we consider the stabilization of subsystem (8). Define the state variable as $z_1 = x_1 - x_d$, and choose the Lyapunov function candidate as

$$V_1(\boldsymbol{Z}_1) = \frac{1}{2} z_1^{\ 2} + \frac{1}{2} tr[\tilde{\boldsymbol{w}}_1^{\ T} \Gamma_1^{\ -1} \tilde{\boldsymbol{w}}_1]$$

Then the time derivative of V_1 is given by

$$\dot{V}_1(\boldsymbol{Z}_1) = z_1 \dot{z}_1 + \tilde{\boldsymbol{w}}_1^{\ T} \Gamma_1^{\ -1} \dot{\tilde{\boldsymbol{w}}}_1$$

$$= z_1[f_1(\boldsymbol{X}_1) + g_1(\boldsymbol{X}_1)a_1 + \boldsymbol{w}_1^{\ T}\boldsymbol{h}_1 + \varepsilon_1 - \dot{x}_d] + \tilde{\boldsymbol{w}}_1^{\ T} \Gamma_1^{\ -1} \dot{\tilde{\boldsymbol{w}}}_1$$

If the differential of Lyapunov function satisfies negative definite, we require that

$$\begin{cases} a_1 = g_1^{\ -1}(\boldsymbol{X}_1)[-f_1(\boldsymbol{X}_1) + \dot{x}_d - k_1 z_1 - \hat{\boldsymbol{w}}_1^{\ T}\boldsymbol{h}_1 - \delta_0 sgn(z_1)] \\ \dot{\hat{\boldsymbol{w}}}_1 = \Gamma_1 \boldsymbol{h}_1 z_1 \\ \delta_0 \geq |\varepsilon_1| \end{cases}$$

Then

$$\dot{V}_1(\boldsymbol{Z}_1) = -k_1 z_1^{\ 2} - \xi_1$$

Where $\xi_1 = z_1[\delta_0 sgn(z_1) - \varepsilon_1] > 0$

So, the subsystem (8) is consistent asymptotic stability.

Step *m* A similar procedure is employed recursively for each step m. we assume that a_m is virtual input, consider the stabilization of the *mth*-order subsystem expressed as follows

$$\begin{cases} \dot{x}_i = f_i(\boldsymbol{X}_i) + g_i(\boldsymbol{X}_i)x_{i+1} + \Delta_i, (1 \leq i \leq m-1) \\ \dot{x}_m = f_m(\boldsymbol{X}_m) + g_m(\boldsymbol{X}_m)a_m + \Delta_m \end{cases} \tag{9}$$

Where $\Delta_m = \boldsymbol{w}_m^{\ T}\boldsymbol{h}_m + \varepsilon_m$

Define the state variable as $z_1 = x_1 - x_d, \ldots\ldots, \ z_m = x_m - a_{m-1}$, Eq. (9) can be rewritten as

$$\begin{cases} \dot{z}_i = f_i(\boldsymbol{X}_i) + g_i(\boldsymbol{X}_i)x_{i+1} + \Delta_i - \dot{a}_{i-1}, (1 \leq i \leq m-1) \\ \dot{z}_m = f_m(\boldsymbol{X}_m) + g_m(\boldsymbol{X}_m)a_m + \boldsymbol{w}_m^{\ T}\boldsymbol{h}_m + \varepsilon_m - \dot{a}_{m-1} \end{cases}$$

The Lyapunov function candidate is chosen as

$$V_m(\boldsymbol{Z}_m) = V_{m-1}(\boldsymbol{Z}_{m-1}) + \frac{1}{2} z_m^{\ 2} + \frac{1}{2} tr[\tilde{\boldsymbol{w}}_m^{\ T} \Gamma_m^{\ -1} \tilde{\boldsymbol{w}}_m]$$

Then, being similar to Step 1, we obtain

$\dot{V}_m(\boldsymbol{Z}_m) = \dot{V}_{m-1}(\boldsymbol{Z}_{m-1}) + z_{m-1}g_{m-1}(\boldsymbol{X}_{m-1})z_m + z_m[f_m(\boldsymbol{X}_m) + g_m(\boldsymbol{X}_m)a_m + \boldsymbol{w}_m^{\ T}\boldsymbol{h}_m + \varepsilon_m - \dot{a}_{m-1}] + \tilde{\boldsymbol{w}}_m^{\ T} \Gamma_m^{\ -1} \dot{\tilde{\boldsymbol{w}}}_m$ We require that

$$\begin{cases} a_m = g_m^{\ -1}(\boldsymbol{X}_m)[-f_m(\boldsymbol{X}_m) + \dot{\hat{a}}_{m-1} - z_{m-1}g_{m-1}(\boldsymbol{X}_{m-1}) - k_m z_m - \hat{\boldsymbol{w}}_m^{\ T}\boldsymbol{h}_m - \delta_{m-1} sgn(z_m)] \\ \dot{\hat{\boldsymbol{w}}}_m = \Gamma_m \boldsymbol{h}_m z_m \\ \delta_{m-1} \geq |\varepsilon_m + e_{m-1}| \end{cases}$$

Where $e_{m-1} = \dot{\hat{a}}_{m-1} - \dot{a}_{m-1}$, $\dot{\hat{a}}_{m-1}$ is the output of nonlinear tracking differentiator whose input is $a_{m\text{-}1}$.

Then

$$\dot{V}_m(\boldsymbol{Z}_m) = \dot{V}_{m-1}(\boldsymbol{Z}_{m-1}) - k_m z_m^{\ 2} - \xi_m$$

Where $\xi_m = z_m[\delta_{m-1} sgn(z_m) - (\varepsilon_m + e_{m-1})] > 0$

So, the subsystem (9) is consistent asymptotic stability.

Step *n* select $V_n = V_{n-1} + \frac{1}{2} z_n^{\ 2} + \frac{1}{2} tr[\tilde{\boldsymbol{w}}_n^{\ T} \Gamma_n^{\ -1} \tilde{\boldsymbol{w}}_n]$, and define $z_1 = x_1 - x_d$,…, $z_n = x_n - a_{n-1}$, $z_{n+1} = u - a_n = 0$, we have

$$\begin{cases} \dot{\boldsymbol{z}}_1 = f_i(\boldsymbol{X}_i) + g_i(\boldsymbol{X}_i)x_{i+1} + \Delta_i - \dot{a}_{i-1} \quad (1 \leq i < n) \\ \dot{\boldsymbol{z}}_n = f_n(\boldsymbol{X}) + g_n(\boldsymbol{X})u + \Delta_n - \dot{a}_{n-1} \end{cases} \tag{10}$$

Where $\Delta_n = \boldsymbol{w}_n^{\ T}\boldsymbol{h}_n + \varepsilon_n$

The selected control law is

$u = g_n^{-1}(\boldsymbol{X})[-f_n(\boldsymbol{X}) + \dot{\hat{a}}_{n\text{-}1} - z_{n-1} g_{n-1}(\boldsymbol{X}_{n\text{-}1}) - k_n z_n - \hat{\boldsymbol{w}}_n^T \boldsymbol{h}_n - \delta_{n\text{-}1} sgn(z_n)]$

And $\dot{\hat{\boldsymbol{w}}}_{\boldsymbol{n}} = \Gamma_n \boldsymbol{h}_n z_n$

Given $\delta_{\mathrm{n}-1} \geq |\varepsilon_{\mathrm{n}} + e_{\mathrm{n}-1}|$, then we yield $\dot{V}_n = -\sum_{i=1}^{n} k_i z_i^2 - \sum_{i=1}^{n} \xi_i < 0$

Where $\xi_i = z_i[\delta_{i-1} \mathrm{s}gn(z_i) - (\varepsilon_i + e_{i-1})] > 0$

Now, we have completed the proof of Lemma 1.

Conclusion

We aim at a class of mismatch and uncertain control system, present an adaptive backstepping neural network control method based on Lyapunov stability theory, theoretical analysis results prove the effectiveness of the designed controller, and the advantages of the proposed scheme are as follows

(1) The nonlinear tracking differentiator is introduced to deal with the problem of extreme expanded operation quantity of backstepping method.

(2) If the nonlinear system doesn't possess strict-feedback form, the feedback terms which blocks the plant translate into strict-feedback form can be regard as the uncertainties. This strategy enlarges the applicable range and simplifies the system design.

References

[1] Li Wenlei, Jian Gangyi. Tracking Control of Electro-Hydraulic Position Servo System with Uncertainty Using Dynamic Surface Method[J], Opto-Electronic Engineering, 2007 34(2) 55-59.

[2] Dong Wenhan, Sun Xiu-xia, Lin Yan. Adaptive Backstepping Control: Development and Applications[J], Control and Decision, 2006 21(10) 1081-1086.

[3] Guan Cheng, Zhu Shanan. Backstepping-Based Multiple Cascade Adaptive Sliding Mode Control of an Electro-Hydraulic Servo System[J], Chinese Journal of Scientific Instrument, 2005 26(6) 569-573.

[4] Jia Qingzhong, Liu Yongshan. Backstepping Design for Nonlinear Control System with Uncertainties[J], Journal of System Simulation, 2007 19(21) 5002-5007.