The Determination of Air Materiel Repairable Parts Consumption Quota Based on GM (1, 1)

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Abstract. Aiming the problem of air materiel repairable parts consumption quota, the paper creates a calculation model, gives the methods and processes solving the model, and explain the solving process with examples specifically, for the development of air material consumption quota provides an effective approach for air force air materiel system.

Introduction

Air Material Repairable Parts are the parts that can be repaired technically and the cost of repairs is lower than the cost of buying new parts. Air Material Repairable Parts are generally expensive and important. So, the determination of air material repairable parts consumption quota is one of the main content of the determination of air material consumption quota.

Generally, there is few consumption data because of the low consumption of air material repairable parts. Especially the main aircraft, most of them are equipped a short time and the fewer consumption data can be obtained. The law of demand is more difficult to master, and thus more difficult to determine their consumption quota. Gray theory, is known with "some of the information is known, some information is unknown," the "small sample", "poor information" uncertainty system as the research object, provides an effective way to determine the air material consumption quota [1]. $GM(1,1)$ Model is one of the most commonly used model of gray theory. It can solve the problem of air material consumption forecast [2-3]. The paper further researches the determination of air material consumption quota based on $GM(1,1)$ model.

Air material repairable parts consumption quota based on $GM(1,1)$ model

The original data series $X^{(0)}$ is non-negative sequence.

$$X^{(0)}=\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\} \tag{1}$$

In the equation, $X^{(0)}(k)\geq 0, k=1,2,\cdots,n$, The corresponding generate data sequence is $X^{(1)}$

$$X^{(1)}=\{x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\} \tag{2}$$

In the equation, $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i), k=1,2,\cdots,n$, $Z^{(1)}$ is closed to the mean of generating sequence of $X^{(1)}$

$$Z^{(1)}=\{z^{(1)}(2),z^{(1)}(3),\cdots,z^{(1)}(n)\} \tag{3}$$

In the equation, $Z^{(1)}(k)=0.5x^{(1)}(k)+0.5x^{(1)}(k-1), k=1,2,\cdots,n$.

$x^{(0)}(k)+az^{(1)}(k)=b$ is the model of $GM(1,1)$. Among, a, b is the parameters that through modeling and solving. If $\hat{a}=(a,b)^T$ is parameter column, and

$$Y=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B=\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{4}$$

Seeking differential equations $x^{(0)}(k)+az^{(1)}(k)=b$ least squares estimation coefficient column, meet the

$$\hat{a}=\left(B^T B\right)^{-1} B^T Y \tag{5}$$

The parameter column can be obtained

$$\begin{cases} b=\dfrac{1}{n-1}\left[\sum_{k=2}^{n} x^{(0)}(k)+a\sum_{k=2}^{n} z^{(1)}(k)\right] \\ a=\dfrac{\dfrac{1}{n-1}\sum_{k=2}^{n} x^{(0)}(k)\sum_{k=2}^{n} z^{(1)}(k)-\sum_{k=2}^{n} x^{(0)}(k)z^{(1)}(k)}{\sum_{k=2}^{n}\left(x^{(0)}(k)\right)^2-\dfrac{1}{n-1}\left(\sum_{k=2}^{n} z^{(1)}(k)\right)^2} \end{cases} \tag{6}$$

$\dfrac{dx^{(1)}}{dt}+ax^{(1)}=b$ is the albino equation of differential equation $x^{(0)}(k)+az^{(1)}(k)=b$, so the solution of albino equation $\dfrac{dx^{(1)}}{dt}+ax^{(1)}=b$ or called temporal response function is

$$\hat{x}^{(1)}(t)=\left(x^{(1)}(0)-\frac{b}{a}\right)e^{-at}+\frac{b}{a}$$

(7)

The time response sequence of $GM(1,1)$ gray differential equation $x^{(0)}(k)+az^{(1)}(k)=b$ is

$$\hat{x}^{(1)}(k+1)=\left(x^{(1)}(0)-\frac{b}{a}\right)e^{-at}+\frac{b}{a}, k=1,2,\cdots,n \tag{8}$$

The restore value is

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k), k=1,2,\cdots,n \tag{9}$$

The predictive value of $n+1$ period is $\hat{x}^{(0)}(n+1)$, as number of air material failures next year, if T is the average maintenance cycle of an aircraft materials(monthly count), so the air material consumption quota is

$$S=\frac{\hat{x}^{(0)}(n+1)}{12/T} \tag{10}$$

The annual average consumption of equipment which don't satisfy $GM(1,1)$ quasi-exponential is N, so the air material consumption quota is

$$S=\frac{N}{12/T} \tag{11}$$

Case study

The annual average consumption of some kind air materiel repairable items in stock the last six years as shown in table 1, th average repair intervals is three months.

Table 1 the annual average consumption of some kind air materiel repairable items in stock the last six years

2008	2009	2010	2011	2012	2013
11	12	12	15	14	17

Use $GM(1,1)$ model, $x^{(0)}(k)+az^{(1)}(k)=b$ establish model. Simulated the air material last six years of annual demand and compare their simulation accuracy.

Assume original sequence is $X^{(0)}=\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(6)\}=\{11,12,12,15,14,17\}$,

Do accumulate to $X^{(0)}$, then get $X^{(1)}=\{x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(6)\}=\{11,23,35,50,64,81\}$,

Do quasi-smooth test to $X^{(0)}$. From $\rho(k)=\dfrac{x^{(0)}(k)}{x^{(1)}(k-1)}$, we can get $\rho(3)\approx 0.52$, $\rho(4)\approx 0.42$, $\rho(5)\approx 0.28$, $\rho(6)\approx 0.27$.

When $k>3$, it meets the quasi-smooth conditions. Verify $X^{(1)}$ whether quasi-exponential.

From $\sigma^{(1)}(k)=\dfrac{x^{(1)}(k)}{x^{(1)}(k-1)}$ we can get $\sigma^{(1)}(3)\approx 1.52$, $\sigma^{(1)}(4)\approx 1.42$, $\sigma^{(1)}(5)\approx 1.28$, $\sigma^{(1)}(6)\approx 1.26$.

When $k>3$, $\sigma^{(1)}(k)\in[1,1.5]$, it meet the quasi-exponential. So we can establish $GM(1,1)$ model of $X^{(1)}$.

$X^{(1)}$ as close to the average generation. Order $z^{(1)}(k)=0.5x^{(1)}(k)+0.5x^{(1)}(k-1)$, we can get $Z^{(1)}=\{z^{(1)}(2),z^{(1)}(3),\cdots,z^{(1)}(6)\}=\{17,29,42.5,57,72.5\}$, then

$$Y=\begin{bmatrix}x^{(0)}(2)\\x^{(0)}(3)\\x^{(0)}(4)\\x^{(0)}(5)\\x^{(0)}(6)\end{bmatrix}=\begin{bmatrix}12\\12\\15\\14\\17\end{bmatrix},\quad B=\begin{bmatrix}-z^{(1)}(2)&1\\-z^{(1)}(3)&1\\-z^{(1)}(4)&1\\-z^{(1)}(5)&1\\-z^{(1)}(6)&1\end{bmatrix}=\begin{bmatrix}-17&1\\-29&1\\-42.5&1\\-57&1\\-72.5&1\end{bmatrix}$$

Do least squares estimation to parameter column $a=(a,b)^T$, calculate and then get

$a=-0.0867, b=10.2179$

And get the model

$x^{(0)}(k)-0.0867z^{(1)}(k)=10.2179$

Through restore obtains analog values of $X^{(1)}$

$\hat{X}^{(1)}=\{\hat{x}^{(1)}(1),\hat{x}^{(1)}(2),\cdots,\hat{x}^{(1)}(6)\}=\{11.0000,22.6710,35.3996,49.2816,64.4217,80.9337\}$

Through restore obtains analog values of $X^{(0)}$. From $\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)$ we can get

$\hat{X}^{(0)}=\{\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\cdots,\hat{x}^{(0)}(6)\}=\{11.0000,11.6710,12.7286,13.8820,15.1400,16.5120\}$

Test errors. From following table we can calculates sum of squares of residuals

$$s=\varepsilon^T\varepsilon=[\varepsilon(2)\ \varepsilon(3)\cdots\varepsilon(6)]\begin{bmatrix}\varepsilon(2)\\\varepsilon(3)\\\varepsilon(4)\\\vdots\\\varepsilon(6)\end{bmatrix}=[0.3290\ 1.5908\ 1.1180\ 1.1400\ 0.8132]\begin{bmatrix}.03290\\1.5908\\1.1180\\1.1400\\0.8132\end{bmatrix}=3.4267$$

$GM(1,1)$ Model fitting results are shown in Figure1. In order to further investigate the fitting results of $GM(1,1)$ model, the paper also use the method of linear equation regression, the error sum of squares is 3.77, the fitting results are shown in Figure2. It can be shown from Figure1 and Figure2, the fitting results of $GM(1,1)$ model is obviously better than the fitting results of linear regression.

The number of failures predicted values is 18 in 2013 basing on model, the average service intervals is three months, the calculating consumption quota is 4.5 every year, and the actual consumption quota is 5 every year.

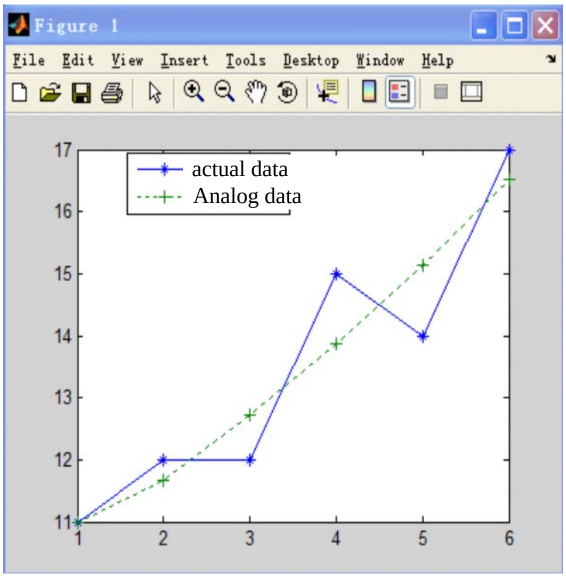

Figure 1 the fitting results of $GM(1,1)$ model

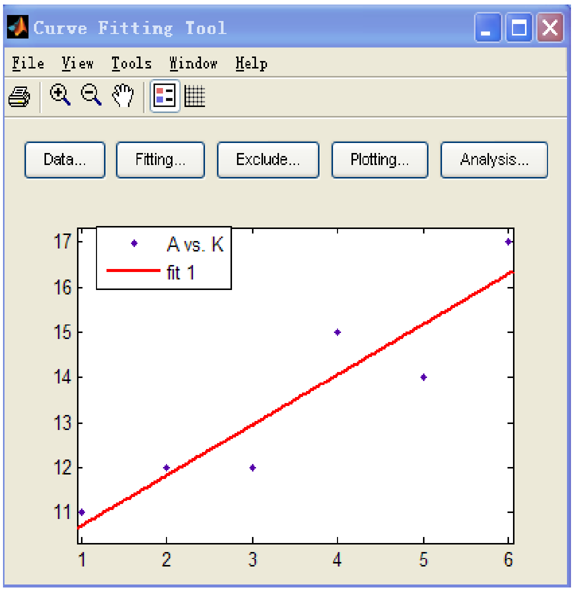

Figure 2 the fitting results of linear regression

Conclusion

The consume of air material repairable parts is not much, so, the acquisition amount of historical data is also not many, especially the main aircraft, most of them are equipped a short time and the fewer consumption data can be obtained. Using $GM(1,1)$ model of the gray theory can effectively aim to the "small sample" air material, getting the consumption quota, providing effective method for air materiel decision departments to determine air materiel consumption quota.

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