A Method for the Determination of Utility Harmonic Responsibility Using Improved Least Squares Estimation

Yue Na
School of Electrical and Electronic Engineering
North China Electric Power University
Baoding, China
e-mail: 809225940@qq.com

Jia Xiufang
School of Electrical and Electronic Engineering
North China Electric Power University
Baoding, China
e-mail: xiufangjia@163.com

Abstract—Harmonic voltage of the concern bus in the power grid is the combinations of voltage contributions of each harmonic source. When the main harmonic sources are known to the system, how to distinguish the responsibility of these harmonic sources has important significance. This paper firstly established the multiple linear regression model and defined the harmonic responsibility. When the harmonic current is linearly related, using the least squares regression will get the wrong solution. Therefore, this paper proposed a method called improved least squares. Simulation results show that no matter the coefficient matrix is sick or not, this method can get a better result than the least squares method.

Keywords-power system; power quality; multiple harmonic sources; harmonic responsibility; least squares estimation.

I. INTRODUCTION

With the rapid development of power electronics technology and the increasing using of non-linear loads, harmonic currents was injected into the power system networks, harmonic pollution has become an serious problem[1-3]. To solve the problem of harmonic pollution, Bureau of Technical Supervision regulated the permissible value of the utility grid harmonic [4]. But this can not change the situation that “Everyone should be responsible, no irresponsible”. In order to control the harmonic pollution, international organizations proposed a program of incentive or punishment [5]. After the proposed scheme, the domestic and foreign scholars have done a lot of research, the main research focus on the use of "interference" or "non-invasive" method to assess the harmonic emission level of a single harmonic [6], but there are many harmonic sources actually. The harmonic voltage of the concerned bus is caused by all the harmonic sources. It is necessary to divide multi-harmonic source responsibility.

The literature [7] proposed a method to divide multi-harmonic responsibility using least squares estimation firstly. On the basis of the least squares estimation, the literature [8] proposed robust regression method. The literature [8] discussed centralized multi-harmonic source harmonic responsibility.

The least squares estimation was applied in the harmonic responsibility commonly, but when the model is sick, we can not get the exactly result. To solve this problem, we proposed a modified method of least squares. Simulation examples show the method is effective on solving ill-posed problem, also in the non-pathological cases.

II. MATHEMATICAL MODEL AND HARMONIC RESPONSIBILITY DEFINITION

There are n harmonic source, the problem of harmonic of the concern bus is made by all the harmonic sources.

In the Figure.1, the load joined harmonic current $\dot{I}_i^h$ to the system. The harmonic voltage U_X^h of concerned X node is equal to the following equation:

$$\dot{U}_X^h = \sum_{i=1}^{n} Z_{Xi}^h \dot{I}_i^h + \dot{U}_0^h \tag{1}$$

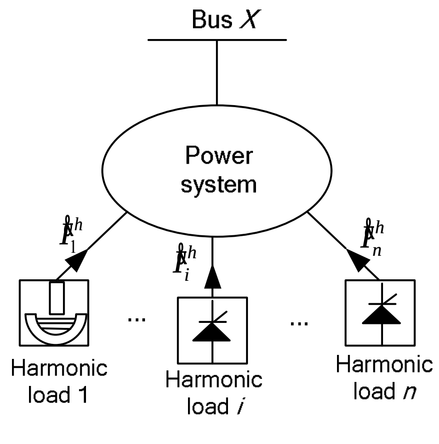


Fig.1 A typical multi-harmonic source grid

Where Z_{Xi}^h is the hth harmonic Impedance between the node of harmonic source and the concerned bus. $\dot{U}_0^h$ is equal to the background harmonic voltage. The sampling value of harmonic current and harmonic voltage is plural. The formula (1) is written in Cartesian coordinates as follows:

© 2015. The authors - Published by Atlantis Press

$$\begin{cases} \dot{U}_X^h = U_{X,\mathrm{r}}^h + jU_{X,\mathrm{x}}^h \\ Z_{Xi}^h = Z_{Xi,\mathrm{r}}^h + jZ_{Xi,\mathrm{x}}^h \\ \dot{I}_i^h = I_{i,\mathrm{r}}^h + jI_{i,\mathrm{x}}^h \\ \dot{U}_0^h = U_{0,\mathrm{r}}^h + jU_{0,\mathrm{x}}^h \end{cases} \tag{2}$$

Where, r and x denote the real and imaginary part of the plural. The formula (2) is applied to the formula (1), and organized as follow:

$$\begin{cases} U_{X,\mathrm{r}}^h = \sum_{i=1}^{n} (Z_{Xi,\mathrm{r}}^h I_{i,\mathrm{r}}^h - Z_{Xi,\mathrm{x}}^h I_{i,x}^h) + U_{0,\mathrm{r}}^h \\ U_{X,\mathrm{x}}^h = \sum_{i=1}^{n} (Z_{Xi,\mathrm{r}}^h I_{i,\mathrm{x}}^h + Z_{Xi,\mathrm{x}}^h I_{i,\mathrm{r}}^h) + U_{0,\mathrm{x}}^h \end{cases} \tag{3}$$

A Case Study in the real, m samples are as follows:

$$\begin{cases} U_{X,\mathrm{r}}^h(1) = \sum_{i=1}^{n} (Z_{Xi,\mathrm{r}}^h I_{i,\mathrm{r}}^h(1) - Z_{Xi,\mathrm{x}}^h I_{i,\mathrm{x}}^h(1)) + U_{0,\mathrm{r}}^h(1) \\ U_{X,\mathrm{r}}^h(2) = \sum_{i=1}^{n} (Z_{Xi,\mathrm{r}}^h I_{i,\mathrm{r}}^h(2) - Z_{Xi,\mathrm{x}}^h I_{i,\mathrm{x}}^h(2)) + U_{0,\mathrm{r}}^h(2) \\ \vdots \\ U_{X,\mathrm{r}}^h(m) = \sum_{i=1}^{n} (Z_{Xi,\mathrm{r}}^h I_{i,\mathrm{r}}^h(m) - Z_{Xi,\mathrm{x}}^h I_{i,\mathrm{x}}^h(m)) + U_{0,\mathrm{r}}^h(m) \end{cases} \tag{4}$$

Written in matrix form is as follows:

$$\boldsymbol{U} = \boldsymbol{I}\boldsymbol{Z} + \boldsymbol{U}_0 \tag{5}$$

Where

$$\boldsymbol{U} = \begin{bmatrix} U_{X,\mathrm{r}}^h(1) & U_{X,\mathrm{r}}^h(2) & \cdots & U_{X,\mathrm{r}}^h(m) \end{bmatrix}_{m\times 1}^{\mathrm{T}}$$

$$\boldsymbol{I} = \begin{bmatrix} I_{1,\mathrm{r}}^h(1) & I_{1,\mathrm{x}}^h(1) & I_{2,\mathrm{r}}^h(1) & I_{2,\mathrm{x}}^h(1) & \cdots & I_{n,\mathrm{r}}^h(1) & I_{n,\mathrm{x}}^h(1) \\ I_{1,\mathrm{r}}^h(2) & I_{1,\mathrm{x}}^h(2) & I_{2,\mathrm{r}}^h(2) & I_{2,\mathrm{x}}^h(2) & \cdots & I_{n,\mathrm{r}}^h(2) & I_{n,\mathrm{x}}^h(2) \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ I_{1,r}^h(m) & I_{1,\mathrm{x}}^h(m) & I_{2,\mathrm{r}}^h(m) & I_{2,\mathrm{x}}^h(m) & \cdots & I_{n,\mathrm{r}}^h(m) & I_{n,\mathrm{x}}^h(m) \end{bmatrix}_{m\times 2n}$$

$$\boldsymbol{U}_0 = \begin{bmatrix} U_{0,\mathrm{r}}^h(1) & U_{0,\mathrm{r}}^h(2) & \cdots & U_{0,\mathrm{r}}^h(m) \end{bmatrix}_{m\times 1}^{\mathrm{T}}$$

$$\boldsymbol{Z} = \begin{bmatrix} Z_{X1,\mathrm{r}}^h & -Z_{X1,\mathrm{x}}^h & Z_{X2,\mathrm{r}}^h & -Z_{X2,\mathrm{x}}^h & \cdots & Z_{Xn,\mathrm{r}}^h & -Z_{Xn,\mathrm{x}}^h \end{bmatrix}_{2n\times 1}^{\mathrm{T}}$$

Harmonic current of each harmonic source multiplied by harmonic impedance constitutes the harmonic voltage component of the concerned node. All the components made up of the harmonic voltage of the concerned bus. Therefore the harmonic responsibility should be defined as follows:

$$m_i^h = \frac{\left|Z_{Xi}^h\right| \cdot \left|\dot{I}_i^h\right|}{\left|\dot{U}_X^h\right|} \cos a \times 100\% \tag{6}$$

Where $\cos a$ is the cosine between the component of the harmonic source and the harmonic voltage of the concerned node.

III. IMPROVED METHOD OF LEAST SQUARES

Multiple linear regression model in matrix form as follows:

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \qquad \backslash \tag{7}$$

Where $\boldsymbol{y}$ is n-dimensional measurements, $\boldsymbol{X}$ is $n\times t$ order coefficient matrix, $\boldsymbol{\beta}$ is t-dimensional parameters to be estimated, $\boldsymbol{\varepsilon}$ is n-dimensional error vector.

Least-squares solution of the formula (7) is $\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{y}$, when the coefficient matrix was ill, $\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}$'s inversion is unstable. This may lead to great deviation between the true value and the calculated value, even sometimes has the erroneous result. Compared with the least squares estimation, the least squares method to improve the essence is in inverse partial least squares add an item $k\boldsymbol{I}$.

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X} + k\boldsymbol{I})^{-1}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{y} \tag{8}$$

Where $\hat{\boldsymbol{\beta}}$ is the estimation of $\boldsymbol{\beta}$; k is correction parameter; $\boldsymbol{I}$ is matrix; $\|\cdot\|$ is Euclidean norm -2; $\Omega(\hat{\boldsymbol{\beta}})$ is stable function.

Formula (8) is only related to *k*, select different parameter k can get different results. Ridge trace method is the most commonly used method to select the parameters k, $\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X} + k\boldsymbol{I})^{-1}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{y}$ varies with the change of k, $\hat{\boldsymbol{\beta}}_i(k)$ is the i-th component of $\hat{\boldsymbol{\beta}}(\boldsymbol{k})$. When k changes between $[0,+\infty)$, the figure of $\hat{\boldsymbol{\beta}}(\boldsymbol{k})$ is called ridge trace. The method to choose k is painting the ridge trace of $\hat{\boldsymbol{\beta}}_1(k), \cdots, \hat{\boldsymbol{\beta}}_p(k)$ on the same graph, then select the *k* value based on trend of ridge track, so that each regression coefficient estimates generally stable, and the regression coefficient estimates for each sign of the value is reasonable. .

IV. CASE STUDY

In this paper, take the example of the fifth harmonic. We use IEEE14 nodes using standard test system shown in figure 2 to verify, in which HL1, HL2 represent non-linear loads, L3 represent linear load.

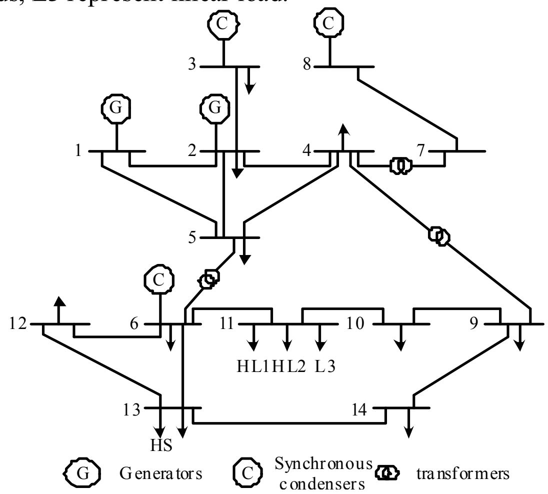
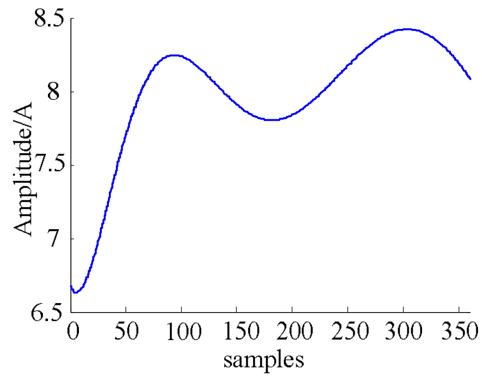
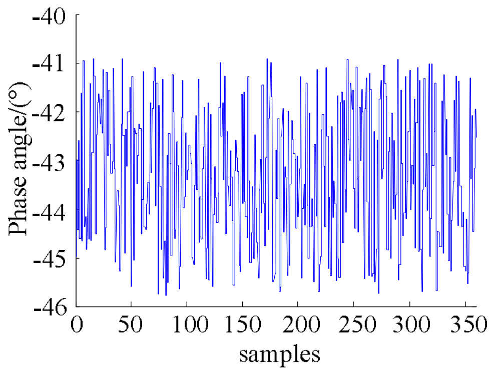


Fig. 2 IEEE14-bus system

According to the formula of harmonic responsibility, to accurately calculate the harmonic responsibility, we must first accurately calculated impedance values, but in multiple regressions, morbid problems will seriously affect the estimated impedance. Use the centralized case to illustrate the feasibility of this method. 11th bus is the concerned bus, the harmonic source HL1, HL2's 5th harmonic current injection waveform shown in figure 3(a) and (b). When the HL2 was concerned only, other parts of the system became system, so as to HL2. The system impedance true value is 0.5023+j2.8228 Ω and 0.51+j2.82Ω.



(a) 5th harmonic current amplitude



(b) 5th harmonic current phase angles

Fig. 3 Nonlinear harmonic source's 5th harmonic current

When HL1 and HL2 joint action and there is no linear correlation, using least squares estimation and improvement have the same result, k=0. The two impedance values were calculated for 0.426+j2.873Ω, 0.44+j2.883Ω, ridge trace is shown in Figure 4.

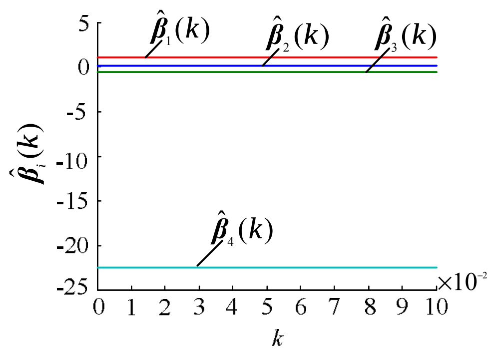


Fig. 4 Ridge track of non-pathological

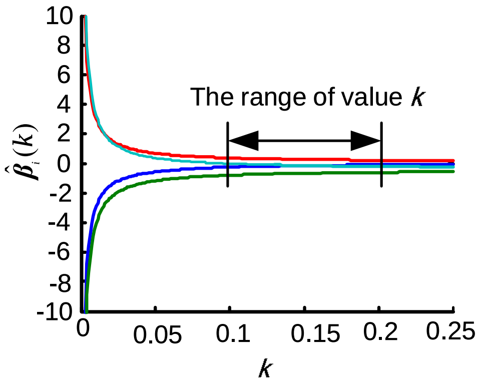


Fig. 5 Ridge track of pathological

HL1 and HL2 are linear correlation as shown in figure 3(a). The results of using least squares estimation are(-0.001-j2.6219)×10^6 Ω and (-2.5577-j0.0045) ×10^6Ω. At the same time, the results of improved least squares estimation are 0.3964+j2.9231Ω, 0.5005+j2.8401Ω, ridge trace is shown in figure 5.

There are three harmonic sources of linearly related. Table 1 shows the least squares estimation, improved least squares estimation and method of fluctuation to calculate the harmonic responsibility of these three harmonic sources.

Table1 Harmonic responsibility calculated by three method

Method	*Harmonic responsibility*		
	HL1	*HL2*	*HL3*
Exact value	*17.01%*	*66.30%*	*66.30%*
Method 1	*17.06%*	*67.67%*	*-970.51%*
Method 2	*16.56%*	*67.79%*	*15.02%*
Method 3	*92.22%*	*64.26%*	*207.52%*

V. Conclusions

There are n harmonic source, the problem of harmonic of the concern bus is made by all the harmonic sources. When the main harmonic sources are known to the system, how to distinguish the responsibility of these harmonic sources has important significance. Least-squares regression analysis estimated is a basis method. When the coefficient matrix sick, we can not get right results using this method. So we proposed an improved least-squares method to improve the results of sickness. The principle is simple, easy to calculate.

References

[1] Gong Hualin , Xiao Xianyong , Liu Yamei , et al. Harmonic emission level assessment method for screening based on the principle of the dominant fluctuation [J]. Proceedings of the CSEE, 2010, 30(4): 22-27.

[2] Xiangning Xiao. Analysis and control of power quality, China Electric Power Press, Beijing, 2010.

[3] Wu Jingchang . Power system harmonics[M]. China Electric Power Press, Beijing, 1998.

[4] National voltage and current levels and frequency Standardization Technical Committee. GB / T 14549-93 utility grid power quality harmonic[S]. 1994.

[5] McEachern A, Grady W M, Moncrieff W A, et al. Revenue and harmonics: an evaluation of some proposed rate structures[J].IEEE Trans. on Power Delivery,1995,11(2): 123-128.

[6] Sumner M, Palethorpe B,Thomas D,et al. A technique for power supply harmonic impedance estimation using a controlled voltage distribance[J]. IEEE Transactions on Power Electronics,2002,17(2): 207-215.

[7] Mazin H E, Xu W, Huang B. Determining the harmonic impacts of multiple harmonic-producing loads[J].IEEE trans. On Power Delivery, 2011, 26(2): 1187-1195.

[8] SunYuanyuan, Yin Zhiming. Multi-harmonic source harmonic responsibility based on M robust regression estimate [J]. Proceedings of the CSEE, 2012,32(31): 166-173.

[9] SunYuanyuan, Li Jiaqi, Yin Zhiming. Centralized multi-harmonic source responsibility to assess based on the measured [J]. Proceedings of the CSEE, 2014,34(13): 2164-2171.

[10] He Xiaoqun, Liu Wenqing. Regression analysis[M]. China Renmin University Press, Beijing, 2011.