

Medical Image Segmentation Based on Fractional-Order Derivative

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Abstract—Objective: In the traditional edge detection differential operators, the first-order derivative masks are easy to loss image details information, and the second-order derivative masks are more sensitive to noise. As for these problems, this paper proposes a fractional-order mask for medical image segmentation. **Methods:** Combining the frequency characteristic and the memorability of fractional differential, the classical first-order Sobel operator is generalized to fractional-order mode; A fractional-order differential mask is constructed for extracting the edge feature of medical images. **Results:** The experiment results show that compared with the integer order differential mask, the fractional-order differential mask can detect more edge details feature of the medical images, and is more robust to noise. **Conclusion:** Based on the global characteristic of the fractional differential, the proposed fractional-order Sobel mask can extract more image edge feature details. Experiment results show that the proposed fractional-order mask yields good visual effects for brain MRI image segmentation.

Keywords- *medical image; image segmentation; fractional-order; frequency characteristic; Sobel operator*

I. INTRODUCTION

Medical image segmentation has important applications in medical image visualization, localization of pathology, and computer-aided diagnosis. Edge detection[1-3] is one of the most frequently used techniques in image segmentation. It primarily can be classified into two categories: first-order derivative and second-order derivative methods. However, the first-order derivative methods generally produce thicker edges, which results in the loss of image details. The second-order derivative methods have a stronger response to fine detail, but they are more sensitive to noise. To solve these problems, the fractional-order derivative has been introduced to the edge detection methods. Since the frequency characteristic determines it can preserve more low-frequency contour feature, keep high-frequency marginal feature and also enhance medium-frequency

texture details. Considering this superiority, Pu et al. [4] proposed a class of fractional differential masks for image texture enhancement. Bai and Feng [5] proposed a class of fractional-order anisotropic diffusion models for noise removal. In this paper, we generalize the classical Sobel operator[6] to constitute a fractional-order Sobel operator for edge detection. The proposed operator is tested in brain MRI image segmentation. The results are promising compared to those of the first-order Sobel operator.

II. DIFFERENTIAL MASK

A. Sobel mask

Edges occur where there is a discontinuity in the intensity function or a very steep intensity gradient in the image. The gradient of an image $f(x,y)$ at (x,y) can be defined as a vector

$$\nabla f(x,y) = [G_x, G_y]^T \quad (1)$$

where

$$\begin{cases} G_x = f(x+1,y) - f(x,y) \\ G_y = f(x,y+1) - f(x,y) \end{cases} \quad (2)$$

Sobel detection is a classical first-order derivative detection method, which brings out the horizontal and vertical edges individually based on a pair of 3×3 convolution operator shown in Fig .1.

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Figure 1. Sobel mask

The operator can be applied separately to a 3×3 neighbourhood of the center point pixel. The gradient components along the x- and y- directions can be found using the following approximation

$$G_x = -f(x-1, y-1) + f(x+1, y-1) - 2f(x-1, y) + 2f(x+1, y) - f(x-1, y+1) + f(x+1, y+1) \quad (3)$$

$$G_y = -f(x-1, y-1) + f(x-1, y+1) - 2f(x, y-1) + 2f(x, y+1) - f(x+1, y-1) + f(x+1, y+1) \quad (4)$$

In discrete images, we can consider Δx and Δy in terms of numbers of pixel between two pixel points. Let $\Delta x = \Delta y = 2$, then the differential form of the gradient components can be given by

$$G_x = \frac{1}{2} \left(\frac{\partial f(x+1, y-1)}{\partial x} + 2 \frac{\partial f(x+1, y)}{\partial x} + \frac{\partial f(x+1, y+1)}{\partial x} \right) \quad (5)$$

$$G_y = \frac{1}{2} \left(\frac{\partial f(x-1, y+1)}{\partial y} + 2 \frac{\partial f(x, y+1)}{\partial y} + \frac{\partial f(x+1, y+1)}{\partial y} \right) \quad (6)$$

B. Fractional-order Sobel mask

Fractional-order derivative has been applied in various scientific fields including image processing [7-9]. But the definition of the fractional-order derivative is not unified until now. In this paper, the Grünwald-Letnikov definition [10] is used. Assume the size of image f is $M \times N$, then the discrete form of $\nabla^\alpha f$ can be represented as

$$(\nabla^\alpha f)_{i,j} = ((\Delta_1^\alpha f)_{i,j}, (\Delta_2^\alpha f)_{i,j}) \quad 1 \leq i \leq M, 1 \leq j \leq N \quad (7)$$

with

$$\begin{cases} (\Delta_1^\alpha f)_{i,j} = \sum_{k=0}^{K-1} (-1)^k C_k^\alpha f_{i-k,j} \\ (\Delta_2^\alpha f)_{i,j} = \sum_{k=0}^{K-1} (-1)^k C_k^\alpha f_{i,j-k} \end{cases} \quad (8)$$

where $K \geq 3$ is an integer constant, $C_k^\alpha = \frac{\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)}$, Γ is the gamma function.

By generalizing the order from integer to fraction, a fractional-order differential form of the gradient components transformed from (5) and (6) can be obtained as

$$G_x^\alpha = \frac{1}{2} \left(\frac{\partial^\alpha f(x+1, y-1)}{\partial x^\alpha} + 2 \frac{\partial^\alpha f(x+1, y)}{\partial x^\alpha} + \frac{\partial^\alpha f(x+1, y+1)}{\partial x^\alpha} \right) \quad (9)$$

$$G_y^\alpha = \frac{1}{2} \left(\frac{\partial^\alpha f(x-1, y+1)}{\partial y^\alpha} + 2 \frac{\partial^\alpha f(x, y+1)}{\partial y^\alpha} + \frac{\partial^\alpha f(x+1, y+1)}{\partial y^\alpha} \right) \quad (10)$$

Then the fractional-order gradient components along the x- and y- directions can be found using the following approximation

$$\begin{aligned} G_x^\alpha = & \frac{1}{2} [f(x+1, y-1) - \alpha f(x, y-1) + \frac{\alpha^2 - \alpha}{2} f(x-1, y-1) \\ & + \dots + (-1)^k C_k^\alpha f(x+1-k, y-1) + 2f(x+1, y) - 2\alpha f(x, y) + \\ & (\alpha^2 - \alpha)f(x-1, y) + \dots + 2^* (-1)^k C_k^\alpha f(x+1-k, y) + \\ & f(x+1, y+1) - \alpha f(x, y+1) + \frac{\alpha^2 - \alpha}{2} f(x-1, y+1) + \dots + \\ & (-1)^k C_k^\alpha f(x+1-k, y+1)] \end{aligned} \quad (11)$$

$$\begin{aligned} G_y^\alpha = & \frac{1}{2} [f(x-1, y+1) - \alpha f(x-1, y) + \frac{\alpha^2 - \alpha}{2} f(x-1, y-1) \\ & + \dots + (-1)^k C_k^\alpha f(x-1, y+1-k) + 2f(x, y+1) - \\ & 2\alpha f(x, y) + (\alpha^2 - \alpha)f(x, y-1) + \dots + \\ & 2^* (-1)^k C_k^\alpha f(x, y+1-k) + f(x+1, y+1) - \\ & \alpha f(x+1, y) + \frac{\alpha^2 - \alpha}{2} f(x+1, y-1) + \dots + \\ & (-1)^k C_k^\alpha f(x+1, y+1-k)] \end{aligned} \quad (12)$$

According to the former approximation, we propose a fractional-order Sobel mask described as follows.

$(-1)^k C_k^\alpha / 2$	$(-1)^k C_k^\alpha$	$(-1)^k C_k^\alpha / 2$		
\vdots	\vdots	\vdots		
$(\alpha^2 - \alpha) / 4$	$(\alpha^2 - \alpha) / 2$	$(\alpha^2 - \alpha) / 4$		
$-\alpha / 2$	$-\alpha$	$-\alpha / 2$		
$1 / 2$	1	$1 / 2$		
$\frac{(-1)^k C_k^\alpha}{2}$...	$\frac{(\alpha^2 - \alpha)}{4}$	$\frac{-\alpha}{2}$	$\frac{1}{2}$
$(-1)^k C_k^\alpha$...	$\frac{(\alpha^2 - \alpha)}{2}$	$-\alpha$	1
$\frac{(-1)^k C_k^\alpha}{2}$...	$\frac{(\alpha^2 - \alpha)}{4}$	$\frac{-\alpha}{2}$	$\frac{1}{2}$

Figure 2. Fractional-order Sobel mask

From the definition, an important property can be observed. The integer derivative just involves finite terms; However, the fractional derivative involves infinite number of terms. Therefore, the integer derivative is a local operator while the fractional derivative is a global operator. The fractional mask can consider more neighboring pixels information, extracting more image details.

III. SEGMENTATION THRESHOLDING

An important quantity in edge detection is the gradient magnitude of the image, as for the fractional-order gradient, the magnitude measure can be denoted as

$$\nabla^\alpha F = \text{mag}(\nabla^\alpha f) = [(G_x^\alpha)^2 + (G_y^\alpha)^2]^{1/2} \quad (13)$$

This quantity gives the maximum rate of increase of $f(x, y)$ per unit distance in the direction of $\nabla^\alpha f$. In this paper, the thresholding is selected based on the average gradient,

$$T = \tau \sum_{i=1}^M \sum_{j=1}^N \nabla^\alpha F / (MN) \quad (14)$$

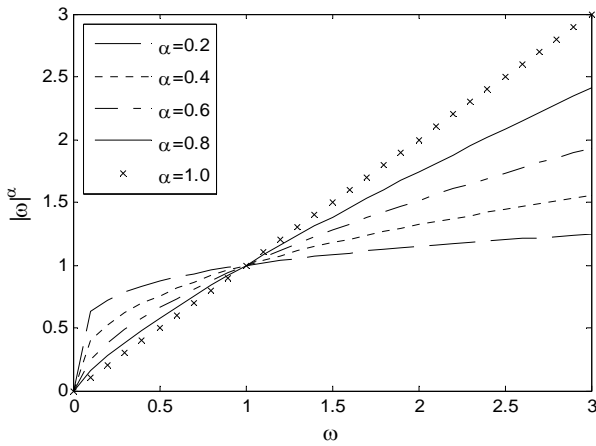
where T denotes the thresholding, $\tau \geq 1$ is a pre-determined parameter. Then the pixel can be marked as edge points if $\nabla^\alpha F > T$.

IV. NUMERICAL RESULTS

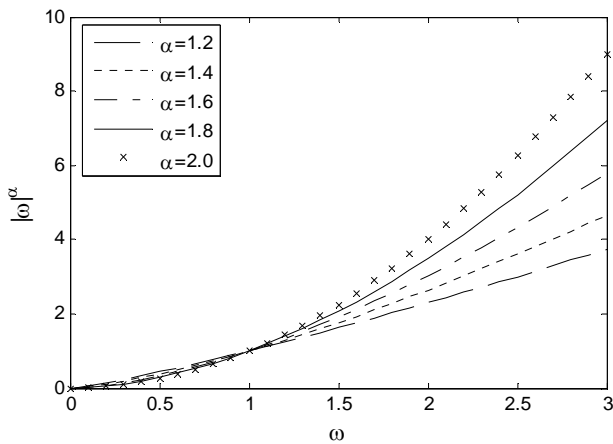
In this section, we provide numerical results to show the behavior of the proposed fractional-order mask with respect to its edge extraction capability.

A. The selection of fractional order

In the following, we will analysis the probelm of order selection in the fractional edge detection operator based on its frequency characteristic. Fig.3 shows the amplitude-frequency response of differential, when $\alpha \in (0, 2.0]$.



(a) $\alpha \in (0, 1.0]$



(b) $\alpha \in (1.0, 2.0]$

Figure 3. Amplitude-frequency response

When $\alpha \in (0, 1.0]$, in the extremely low frequency section, corresponding to images' smooth area, the attenuation degree of image contour resulted by fractional differential is better than the first order differential. In the medium frequency section, corresponding to images' texture area, the fractional order can protect more image details than the first order. In the high frequency section, corresponding to images' edge and noise area, the fractional order has obvious reduction function than the first order. When $\alpha \in (1.0, 2.0]$, in the extremely low frequency section, the attenuation degree of image contour resulted by fractional differential is similar to the one obtained by the first order differential. In the medium frequency and the high frequency sections, the enhancement degree of image edge and noise with fractional differential is much better than the first order differential. Considering the details protection ability and the noise roustness, we select differential order $\alpha \in (0, 1.0)$

B. The selection of fractional differential items number

In the following, we will analysis the influence of the fractional differential items number on image segmentation effect. Select a simulated brain MRI image as test image, the size is 258*258, noise is 5%, and intensity inhomogeneity is 20%, shown in Fig.4 Let $\alpha = 0.2$, $\tau = 3$, Fig.5 shows the segmentation results of the fractional Sobel operator in 3*3, 4*4, 5*5, 6*6, and 7*7 template.

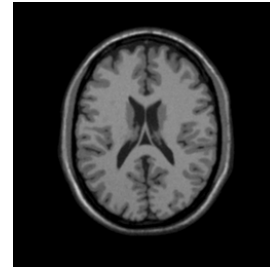


Figure 4. Test image

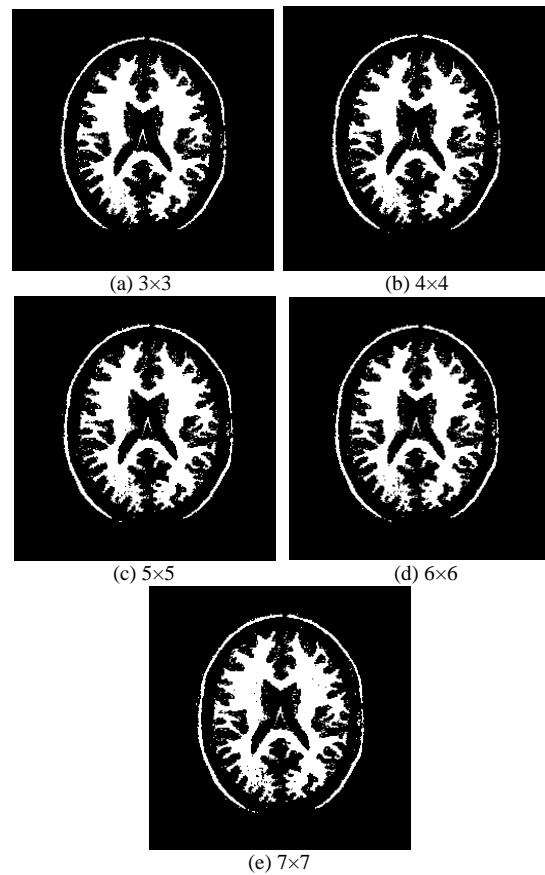


Figure 5. Segmentation results of fractional-order Sobel operator

Simulation results show that the fractional Sobel operator can segment image effectively, when it effect on the narrowband neighborhood of the image pixel. When the template is too large, the segmentation results can remain too much noise, and the segmentation processing need too long time.

C. Performance analysis

In the following, we provide numerical results to show the behavior of the proposed fractional- order Sobel

operator. Let $k=3$, α be selected from 0 to 1, take 0.1 as interval. We select Fig.4 as test image. Let $\tau=3$. Fig.6 lists the extraction results under different fractional order.

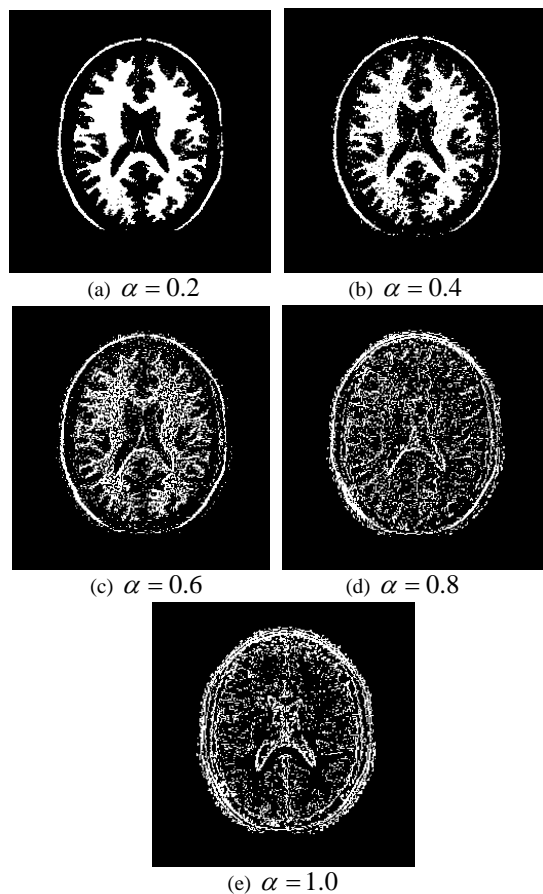


Figure 6. Comparison of segmentation results under different orders on a brain MRI image.

Experiment results show that the fractional-order Sobel operator is robust to noise and can restrain the influence of irrelevant detail compared with the Sobel operator. And the larger α is, the better image feature is preserved, but meanwhile the more noise is residual too.

V. CONCLUSION

From the definition, an important property can be observed that the integer derivative is a local operator

while the fractional derivative is a global operator. The fractional operator can consider more neighboring information. Considering this characteristic, we generalize the traditional first-order Sobel edge detection operator to fractional-order for extracting image structure features. The novel operator has been validated on brain MR image with desirable performance.

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