Based on the Improved Particle Swarm for Portfolio Optimization Study

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Abstract—Portfolio is one of research focuses in modern finance field. There are some kinds of assets to choose. How should we define the ratio of the portfolio in order to minimum the risk in the level of the established income, or at the established risk makes the benefit maximum. This paper introduces an Improved Particle Swarm algorithm to solve the select problem of investment decision, and compare with the traditional PSO. The experimental results indicate that the improved PSO is an efficient and reliable algorithm.

Keywords-***portfolio; particle swarm optimization; Partial derivatives; punishment function***

I. INTRODUTION

Markowitz published a classical article named portfolio selection in 1952, it marks the beginning of modern portfolio theory. Investors always hope to find a set of investment, and achieve expected earning as soon as possible in the case of certain risk, so put forward an optimal portfolio investment model based on probability criteria. The model is a double objective function based on risk and realize the expected return, choose investment ratio, make the objective function is minimum, or make the risk is minimum in the case of achieve expected return, Investors determine the proportion of investment according to the principle of minimum risk. This article solve portfolio investment scheme by the traditional particle swarm optimization firstly. Particle swarm optimization is an evolutionary computation technique, it initialize a group of random particles as random solutions firstly, and then find the optimal solution through iteration. It makes the global extremum as the guide point in the process of iteration, each point constantly adjust their own direction in the direction of global extreme value point, if the global extremum stuck in a local extremum, it is difficult to jump out of local extremum automatically, aiming at this, many scholars increased the diversity of particles and particle search scope through introducing random factors, so that improve optimization ability; There is another method, it can find a potential optimal point, and make each particle do some adjustments in the direction of it, this method also can make the algorithm jump out of local extremum. This paper solve portfolio investment scheme by improved particle swarm optimization again, and compared with the traditional particle swarm optimization. The process of the algorithm use MATLAB.

II. THE BASIC PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO) is a global optimization algorithm that is proposed in recent years. Kennedy and Eberhart inspired from the group of animals foraging behavior, when the group search for the optimal target, each individual adjust the next step of search will refer to the best individual in the current group and their own optimal position that they had reached, the particle swarm optimization is designed by that. The iterative formula of the basic PSO are (1) and (2).

$$v_{id}(t+1) = \omega v_{id}(t) + c_1\phi_1(p_{id}(t) - x_{id}(t)) + c_2\phi_2(p_{gd}(t) - x_{id}(t)) \quad (1)$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \quad (2)$$

Among them, $\mathbf{v}$ and $\mathbf{x}$ are the speed and position of each particle respectively, i and d are the particle number and component number, t is the step of iterative calculation, ω , c_1 , c_2 are system control parameters, φ_1 are φ_2 are random number in(0,1), $\boldsymbol{p}_{id}$ is the optimal position that particle i had reached, $\boldsymbol{p}_{gd}$ is the optimal position that all particles had reached in the group. Thus, the basic idea of PSO is that each individual makes full use of the group and their own intelligence to adjust learning constantly, and achieve satisfied solution finally.

III. IMPROVED PARTICLE SWARM OPTIMIZATION

A. The analysis of particle swarm optimization algorithm

From the iteration formula of particle swarm optimization we can see that, it makes the global extremum as the guide point in the process of iteration, each point constantly adjust their own direction in the direction of global extreme value point, if the global extremum stuck in a local extremum, it is difficult to jump out of local extremum automatically, aiming at this, many scholars increased the diversity of particles and particle search scope through introducing random factors, so that improve optimization ability; There is another method, it can find a potential optimal point, and make each particle do some adjustments in the direction of it, this method also can make the algorithm jump out of local extremum.

B. The property of continuous functions on closed interval

For the optimization of function, that is solving the extremum of function. For a continuous function on a closed area, it must be exist extremum value, If the function exist partial derivatives to some extent within the considered scope, the extreme value point must be the point that the partial derivative is zero or the partial derivative does not exist. But for the general function, most of them can make function approximation that take advantage of the power series expansion or trigonometric function expansion, so their extreme value must be the point that it is non-differentiable points of the basis function or the partial derivative is zero. According to this analysis, it can seen that , if the non-differentiable point or the point that the partial derivative is zero can be find for an optimization function, this point is likely to be the extreme value point that is required by the optimization function. Therefore, the point that partial derivative is zero, the global extreme value point and local extremum point of particle swarm optimization adjust the movement direction of the particle together, it is possible to make the particle swarm optimization jump out of local extremum , thus obtains the optimal values of the questions.

C. The numerical algorithm of the points that partial derivative is zero

According to the definition of mathematical analysis，for a function $y = f(x)$, partial derivative is zero at the point $x = x_0$, that means the following limit exist and

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = 0 \qquad (3)$$

According to the above definition, find the points that partial derivative is zero for a function, there is only one way that is use every point verify whether the limit exist and the limit is equal to zero, this way is not feasible in practice obviously.

Every step of particle swarm optimization is adjusted according to the current best point that have obtained in the previous step and the best point of their own to find. Through this way, particle is adjusted step by step, the optimal value is found finally. Based on this idea, when using particle swarm optimization to solve a practical problem, as long as we construct an objective function artificially that in accordance with the problem and can evaluate the strengths and weaknesses fitness of a point. The point where make the partial derivative of function is zero have some certain properties, therefore, we can construct the performance of computing particles using the features of the point that partial derivative is zero, and make use of particle swarm optimization to solve the problem.

For the function $y = f(x), x \in [a,b]$, the key of finding the point where the partial derivative is zero by using particle swarm optimization is how to determine the overall extremum and local extremum of the particle swarm optimization. Giving different objective functions of calculating the fitness value of particle, then achieve different methods of solving partial derivative.

Through calculating the value of the following to determine whether the partial derivative is zero, the partial derivative is zero at this point when the value is small, otherwise the partial derivative is not zero.

$$p(x_0) = \left| \frac{f(x_0 + h) - f(x_0)}{h} \right| + \left| \frac{f(x_0) - f(x_0 + h)}{h} \right| \qquad (4)$$

Based on the analysis of the above, Whether the partial derivative is zero at a certain point as long as calculated according to (4), and this formula can be applied to judge obviously partial derivative of function of many variables

D. The searching of particle swarm optimization with the point that partial derivative is zero

Based on the above analysis，we can judge whether the partial derivative of one point is zero through (4). And the point where partial derivative is zero is a potential extremum point, thus adjust the movement of the particles by this point, it certainly can improve the ability to jump out of local extremum algorithm. The steps of the algorithm as following：

Step1 In the solution space, generate k particles randomly $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_k$; the corresponding initial velocity $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k$ and parameter $\mathbf{h}$ 。

Step2 Calculate the current global extremum $\boldsymbol{p}_g$ and local extremum $\boldsymbol{p}_i$, calculate $f(\boldsymbol{y}_1), f(\boldsymbol{y}_2), \cdots, f(\boldsymbol{y}_k)$ according to formula (5)

$$\begin{aligned} \mathbf{v}_i(t+1) &= \omega \mathbf{v}_i(t) + c_1 \varphi_1 (\mathbf{p}_i(t) - \mathbf{x}_i(t)) \\ &+ c_2 \varphi_2 (\mathbf{p}_g(t) - \mathbf{x}_i(t)) \end{aligned} \qquad (5)$$

$$\mathbf{y}_i = \mathbf{x}_i(t) + \mathbf{v}_i(t+1) . (i = 1,2,\cdots,k).$$

For each $\boldsymbol{x}_i$, calculate

$$p(x_0)=\left|\frac{f(x_0+h)-f(x_0)}{\|h\|}\right|+\left|\frac{f(x_0)-f(x_0+h)}{\|h\|}\right|$$

set the minimum of $p(\mathbf{x}_1), p(\mathbf{x}_2),\cdots, p(\mathbf{x}_k)$ as $p(\mathbf{x}^*)$, thus $\boldsymbol{x}^*$ is potential point where partial derivative is zero, and is also the potential optimal value point, therefore, make movement according

$\boldsymbol{yy}_i=\boldsymbol{x}_i(t)+\phi_3(\boldsymbol{x}_i^*-\boldsymbol{x}_i(t))\ (i=1,2,\cdots,k)$,

for each i, compare $f(\mathbf{y}_i)$ with $f(\boldsymbol{yy}_i)$.

if $f(\boldsymbol{y}_i)\le f(\boldsymbol{yy}_i)$,

then $\boldsymbol{x}_i(t+1)=\boldsymbol{y}_i$, otherwise, $\boldsymbol{x}_i(t+1)=\boldsymbol{yy}_i$.

Step3 Combined with global extremum $\boldsymbol{p}_g$ and local extremum $\boldsymbol{p}_i$ in the previous step and new $\boldsymbol{x}_1,\boldsymbol{x}_2,\cdots,\boldsymbol{x}_k$, find out the new global extremum $\boldsymbol{p}_g$ and local extremum $\boldsymbol{p}_i$.

Step4 Determine whether satisfy the conditions of the end，if meet the conditions, end the algorithm, otherwise go back to Step2, circulate this process until the end.

E. Test functions

In order to verify the superiority of the algorithm, the following four standard test functions are used for testing respectively.

$$(1)\min f_1(x)=100(x^2-y)^2+(1-x^2)^2$$

$x,y\in[-2.048,2.048]$, the global minimum value obtained by traditional particle swarm optimization is 0.0483, the result obtained by the improved particle swarm algorithm is 0.0028.

$$(2)\min f_2(x)=100\cdot(y-x^2)^2+\left[6.4(y-0.5)^2-x-0.6\right]^2$$

$x,y\in[-5,5]$, the global minimum value obtained by traditional particle swarm optimization is 0.0167, the result obtained by the improved particle swarm algorithm is 0.0034.

$$(3)\min f_3(x)=(|x|-5)^2+(|y|-5)^2$$

$x,y\in[-10,10]$, the global minimum value obtained by traditional particle swarm optimization is 0.0117, the result obtained by the improved particle swarm algorithm is 0.0014.

$$(4)\min f_4(x)=[1+(x+y+1)^2(19-14x+3x^2-14y+6xy+3y^2)]\times[30+(2x-3y)^2(18-32x+12x^2+48y-36xy+27y^2)]$$

$x,y\in[-2,2]$, the global minimum value obtained by traditional particle swarm optimization is 3.0277, the result obtained by the improved particle swarm algorithm is 3.0031.

IV. PORTFOLIO MODEL

A. The overview of model

There are n kinds of assets on the market $s_i\ (i=1,...,n)$ for investors to choose, such as stocks and bonds and so on. A company has a considerable amount of money can be used for a period of investment, assume the number is M, Financial analysts of the company have evaluated on the n kinds of assets, they estimate that the average yield of s_i is r_i during the period, and they predict the risk loss rate is q_i. Considering the more dispersed of the investment, the smaller the total risk, the company decided that, When purchase some kind of assets used these money, the overall risk can be measured with the biggest risk of the investment s_i. Moreover, assume that the bank deposit interest rate is $r_0(r_0=5\%)$ for a same period, and there is no risk. Financial analysts of the company have evaluated on the 15 kinds of assets, the results in table 1.

Table1 The result of asset appraisal

s_i	r%	q%
s_1	9.6	42
s_2	18.5	54
s_3	49.4	60
s_4	23.9	42
s_5	8.1	1.2
s_6	14	39
s_7	40.7	68
s_8	31.2	33.4
s_9	33.6	53.3
s_{10}	36.8	40
s_{11}	11.8	31
s_{12}	9	5.5
s_{13}	35	46
s_{14}	9.4	5.3
s_{15}	15	23

B. The establishment of the model

The referenced capital is s_0, the investment risk $q_0=0$, assume the funds proportion of investment in s_i is $\omega_i\ (i=0,...,n)$,

The weight of all kinds of assets： $\sum_{i=0}^{n}\omega_i=1$

Net income: $R = \sum_{i=0}^{n} r_i \omega_i$

The overall risk: $c = \max_{0 \le i \le n} (q_i M \omega_i)$

In order to show the risk tolerance of investors, introducing a parameter e that force investment dispersion, thus risk is reduced corresponding , Intuitively speaking, the probability of the various projects occurrence risk at the same time is small, In order to prevent the blind pursuit of the maximization of interests, and it is a kind of quantitative for risk, that is calculate the minimum risk under the circumstances of constraint condition $\max(q_i \omega_i) < e$.

We can receive the model based on analysis of above:

$$\begin{cases} \max \mathrm{Re} = \sum_{i=0}^{n} r_i \omega_i \\ \min Ri = M \cdot \max\left(q_i \omega_i\right) \\ s.t. \sum_{i=0}^{n} \omega_i = 1 and \omega_\mathrm{i} \ge 0. i = 0,1 \ldots n \end{cases}$$

C. The implementation process of the algorithm

(1) Parameter selection. The convergence of the algorithm is the fastest when the value range of inertial factor ω is $[0.8, 1.0]$, learning factor $c_1 = c_2 = 2$, the Value range of the maximum speed of each dimension $V_{\max}$ is $[0,1]$, the largest number of iterations is 600, the particle number is 50, e as constraint conditions were set to $(10, 15, 20)$ respectively.

(2) Initialize the particle. This paper bring to real number encoding, for each component x_i of the initial value of every particle $(x_1, x_2, ..., x_n)$, they all take the random number between (0, 1), then normalized processing based on $x_i = x_i \Big/ \sum_{i=0}^{n} x_i$ (i＝1， 2， ⋯， n). The x_i stand for the investment proportion of the ith kind of securities.

(3) The selection of fitness function. Because this model belongs to the multidimensional constrained optimization problem, we use penalty function method in this paper to solve such problems, the main purpose is to reduce or remove the constraint, the basic idea is to construct augmented function with parameters based on the objective function and constraint function according to certain way, it is concluded that the fitness function:

$$-\sum_{i=0}^{n} r_i \omega_i + 40 \left[\sum_{i=0}^{n} \omega_i - 1 \right]^2 + (-2)(5 - \max\ q_i \omega_i)$$

(4) The update of the velocity and position of particle. Update the particle velocity according to the formula (1), and update the location according the formula (2).

(5) Stopping criteria. It will stop when achieve the required number of iterations. The table 2 show the weight selection and the value of the proceeds between the improved particle swarm optimization and primary algorithm.

Table 2 Comparison of results between improved and primary algorithm

	Primary algorithm			Improved algorithm		
e	10	15	20	10	15	20
s_1	0.0349	0.0674	0.0012	0.0313	0.0171	0.0375
s_2	0.1096	0.0824	0.0294	0.0130	0.0762	0.0579
s_3	0.0280	0.0003	0.0566	0.0994	0.0395	0.0694
s_4	0.0913	0.0650	0.0473	0.0378	0.0730	0.0607
s_5	0.1014	0.0530	0.0927	0.1074	0.0961	0.0803
s_6	0.0078	0.0473	0.0354	0.0488	0.0357	0.0298
s_7	0.0212	0.0836	0.0258	0.0073	0.1226	0.0423
s_8	0.0811	0.0420	0.0487	0.0814	0.0549	0.0680
s_9	0.0928	0.0774	0.0809	0.0819	0.0867	0.0834
s_{10}	0.0963	0.0693	0.0381	0.0641	0.0454	0.0613
s_{11}	0.0034	0.0987	0.0595	0.1123	0.0390	0.0900
s_{12}	0.0210	0.0610	0.1198	0.0007	0.0204	0.0638
s_{13}	0.0342	0.0804	0.0858	0.1232	0.1239	0.0535
s_{14}	0.1132	0.0784	0.0620	0.1213	0.0998	0.0974
s_{15}	0.0595	0.0642	0.1102	0.0535	0.0246	0.0526
s_{16}	0.1042	0.0294	0.1066	0.0168	0.0450	0.0669
R	25.235	22.481	21.502	25.324	23.111	24.663

D. The experimental results

e as constraint conditions were set to （10,15,20） respectively， it can be seen from the data that the proceeds obtained by the improved particle swarm optimization is bigger than the primary algorithm, That is, under the condition of meeting a certain risk tolerance, the improved selection of weight can make the proceeds of investors become bigger. The experimental result is shown in figure 1, that is the contrast of both the convergence speed when the risk factor $e = 20$. The contrast test proved that the original particle swarm optimization has no iterative updates, the improved particle swarm optimization not only has updated, and the convergence speed is faster.

risk factor $e = 20$

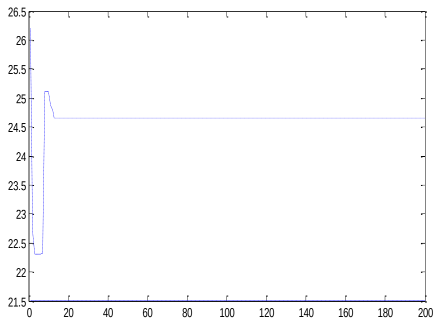

Figure1 The Curve evolution of average proceeds fitness value

Finally, according to the investment preferences of different investors, investors can choose different ways of investment to achieve maximum benefits, it can be seen from the above table that different e take corresponding R value, e represent different preferences of investors, R represent investment benefits of assets investment according to different investment preferences, Through the analysis we found that, the greater the risk does not mean the benefits also increase correspondingly when e within a certain range, so there must be a good investment strategy, the algorithm in this paper can give an ideal investment strategy basically aimed at investors' different investment preferences.

V. CONCLUSIONS

This paper specific to portfolio model, optimize the function combined with the property of the continuous functions on closed interval, that is the solving extremum problems of function. This paper present an improved particle swarm optimization, and calculate the choice weights of portfolio by using the new algorithm in order to make the investors profit maximization, the result is more ideal.

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