

# A Study on the Characteristics of Lamb Wave Based on the Finite Element Method

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**Abstract**— Electromagnetic ultrasonic Lamb wave, with the advantage of quickly detecting the defect, is widely used in nondestructive testing of thin plate. However, it is related to the characteristics of Lamb waves that selecting mode of guided waves and estimating the detecting range. Therefore, this paper studies the propagation characteristics of Lamb waves, including directivity of sound field, multi-mode and dispersion. This paper mainly studied on the Lamb wave characteristics of the 1mm-thick aluminum plate based on the finite element method. Results show that: A<sub>0</sub> and S<sub>0</sub> modes, which are only detected in the plate, have a strong directivity of sound field when the excitation frequency is in the range of 0.08MHz to 1MHz. There is only A<sub>0</sub> mode in aluminum plate when the excitation frequency is lesser than 0.188MHz. When the excitation frequency is in the range of 0.45MHz to 0.64MHz, the displacement amplitude of the S<sub>0</sub> mode is gradually increasing. The displacement amplitude of the A<sub>0</sub> mode is minimum, while it is larger in the S<sub>0</sub> mode when the excitation frequency is 0.64MHz. The results of this paper provide a basis for selecting the best excitation frequency and mode of guided waves, which have the promising application prospect in quickly and accurately nondestructive testing.

**Keywords**-nondestructive testing; EMAT; Lamb; dispersion; finite element

## I. INTRODUCTION

With the quickly development of modern industry, the aluminum plate is widely used for the industry of aerospace and shipbuilding due to the virtues of easy processing, lower density, anti-corrosion and high hardness<sup>[1]</sup>. The aluminum plate which thickness is less than 6mm is the most widely used. However, it is inevitable the thin plate have some defects such as impurity, crack, oxide flim and so on. Therefore, the thin plate should be detected. The traditional nondestructive testing includes magnetic particle testing, penetrant testing, eddy current testing, radiographic testing and piezoelectric ultrasonic testing. Dou to the reason that the traditional nondestructive testing is susceptible to interference from the outside environment, pollute the environment and over-dependence on couplant, the application range is

small. However, EMAT is widely used for detecting the defect of thin plate with the virtues of quickly detecting the defect and being freed of couplant.

In the practical application of EMAT, the parameter design of EMAT, selecting mode of guided waves and estimating the detecting range of lamb waves is related to the directivity of the sound field, multi-mode and dispersion nature of lamb waves .It has important significance for electromagnetic ultrasonic nondestructive testing to study the characteristics of mode and the directivity of the sound field. The domestic and international scholars have a few studies in the aspects of increasing the detection efficiency of EMAT and the dispersion nature of lamb waves. Professor P.Cawley had transformed the receiving record of amplitudes-times into the record of amplitudes-waves by the two-dimensional Fourier transform .This method was used for calculating the amplitudes and phase velocity of lamb waves in the thin steel plate<sup>[2]</sup>. Professor Sachse studies lamb wave dispersion curve in fiber-reinforced composite material and in thin wires. He calculated the phase and group velocities of lamb waves in low the frequency band in solids through the phase spectral analysis of a broadband pulse. This technique is expected to be applicable to measurements of acoustic or electromagnetic wave speeds in other dispersive media<sup>[3]</sup>. Professor Yao Jun in the china iron & steel research institute group, keeping the distance between the receiving coil and the transmitting coil the same, studied the changing trends of acoustic pressure when the angle is changing through transforming the angle of receiving coil<sup>[4]</sup>. Zhen Qing Liu and De An Ta in Tongji University's acoustic laboratory study the characteristic of lamb waves in plate. They mainly studied the principle of stimulate of lamb waves and the dispersion curves through the numerical computation method. At the same time, they also introduced the dispersion nature of lamb waves. This study provided a theoretical basis for the electromagnetic ultrasonic nondestructive testing<sup>[5]</sup>. In addition, Zhen Qing Liu studied a phase spectrum. He accurately calculated the phase velocity of lamb waves according to this method. A detailed dispersion of the theory is given<sup>[6]</sup>. Chong Fu Ying in Chinese academy of sciences' acoustic laboratory

directly observed the process of generation, propagation and scattering of lamb waves by the technology of dynamic photoelasticity. The principle of scattering was studied in this paper<sup>[7]</sup>. Xiang Ming Zheng in Beijing University of Technology analyzed the ultrasonic signals in the aluminum plate by the time-frequency analysis. By combining the pseudo Wigner-Ville distribution with reassignment technique and feature extraction, portions of dispersion curves of Lamb waves can be obtained accurately<sup>[8]</sup>. Professor Ludwig study the working process of piezoelectric ultrasonic sensor and eddy current sensor based on the 2-D finite element method<sup>[9-11]</sup>. Professor Shapoorabadi established a EMAT modeling based on the finite element method, however, the experimental materials is ferromagnetic materials<sup>[12-14]</sup>.

This paper studied the dispersion nature of lamb waves and the directivity of the sound field from the point of view the numerical computation method and experiments. The numerical computation method provided a theoretical basis for the study of lamb waves. However, this result had skew deviation comparing with experiments duo to the idealized parameter. Experiments acquired accurately data; however, this method is blind. In additional, experiments and numerical computation can't reveal the process of generation, propagation and scattering of lamb waves in the aluminum plate. It is more important to directly reveal the process of generation, propagation and the power distribution, because stimulating lamb waves is a special and complicated process in the aluminum plate. The finite element model, owning higher accuracy, can directly reveal the process of generation, propagation and the changing of modes. At the same time, it is flexible compared with experiments in the aspect of the modeling and the selection of model. Some domestic and international scholars study the EMAT based on the finite element method; however, most of them established the EMAT modeling or optimize the structure of EMAT. Few papers study the characteristics of Lamb wave based on the 3-D finite element method.

Therefore, this paper mainly reveals the process of generation, propagation and the changing of modes using the software called ANSYS. In additional, it is analyzed that the directivity of the sound field of different modes and the changing trends of the group velocity, phase velocity and displacement amplitudes of the different modes, Which provide theoretical foundation for the parameter design of EMAT, selecting mode of guided waves and estimating the detecting range of lamb waves.

## II. OPERATING PRINCIPLE OF EMAT

Guided waves contain surface wave, lamb wave and SH wave. Lamb wave is stimulated when the thick of plate is less than the wave length and the bias field is perpendicular to the plate. EMAT consist of a meander coil and a static permanent magnet. The meander coil determines the propagation directions and the concentration of the energy of lamb wave. The EMATs used to generate Lamb waves include the Lorentz force EMAT and the magnetostrictive EMAT. In the non-ferromagnetic material, the Lorentz force is the major force to generate Lamb wave<sup>[15]</sup>. The resultant force of the Lorentz force, magnetostrictive force and magnetic force generate Lamb wave in the ferromagnetic material.

Nevertheless, the magnetostrictive force is largest. It is 5times as large as the Lorentz force. The aluminum plate, the non-ferromagnetic material, is studied in this paper. Therefore, this paper studies the lamb waves generated by the Lorentz force.

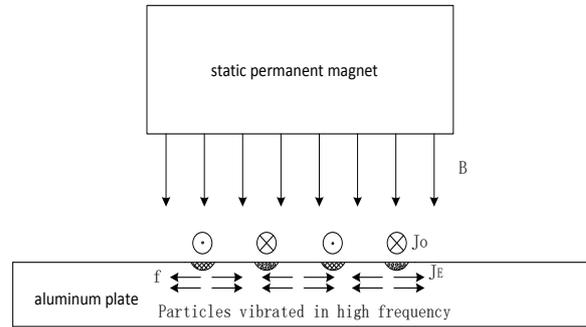


Figure 1. Configuration of a Lorentz force EMAT for the generation of Lamb waves

The configuration of a Lorentz force EMAT is shown in Fig .1. The meander coil passes an alternating current  $I_0$ . High-frequency alternating current creates eddy currents  $I_E$  that the directions of the currents is contrary to  $I_0$  and the same frequency, which then create alternating Lorentz force in the plate by the bias field  $B$  interaction with the alternating field generated by  $I_0$ . The particle in the plate will vibrate due to the alternating Lorentz force, which will generate Lamb waves in the plate when the thick of plate is less than the wave length. The propagation directions are same as the vibration directions.

## III. THE FINITE ELEMENT MODELING OF EMAT

### A. The electromagnetic field modeling of EMAT

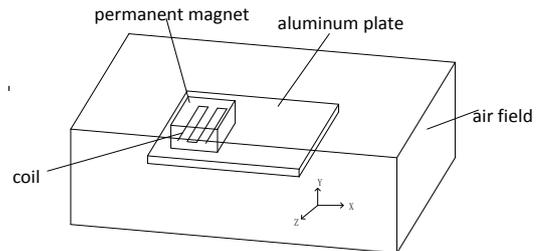


Figure 2. Physical model of EMAT

According to the designing principle of EMAT, the electromagnetic field modeling of EMAT was established in the ANSYS software. The model is shown in Fig .2. The configuration of EMAT includes air field, aluminum plate, permanent magnet and meander coil. The meander coil is under the permanent magnet and the bias field is perpendicular to the plate. The wavelength of Lamb wave is  $\lambda$ . According to the phase matching condition ,When the spacing  $D$  between connecting fingers is  $a=\lambda/2$ , the efficiency is greatest<sup>[16]</sup>. The size of the magnet is  $20*30*15\text{mm}^3$  and the coercivity is  $880\text{ kA/m}$ . The size of aluminum plate is  $684\text{mm}*1\text{mm}*50\text{mm}$ . The left-off value is  $1\text{mm}$ . The alternating current is a tone burst signal, which signal is shown in Fig .3.

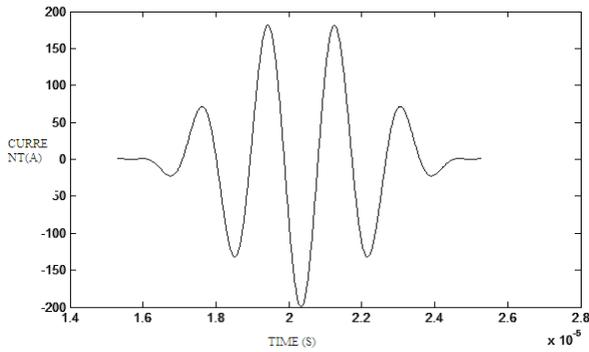


Figure 3. Tone burst signal

The simulation flow of the physical model of EMAT based on the finite element method is shown in Fig .4. The solid97 element is selected in the software. Due to the 1-mm-thick plate, the plate in the direction of Y should have refined grid. The plate divided by the method of sweep meshing. The permanent magnet can't be accurately divided. The permanent magnet and air field meshed in the method of free meshing to decrease the calculated quantity. Due to avoid the disturbance of the static magnetic field, the transient solution proceed when the static magnetic field is stable. Therefore, in the solution stage, the static solution is first proceeding, and the transient solution is second proceeding when the static magnetic field is stable.

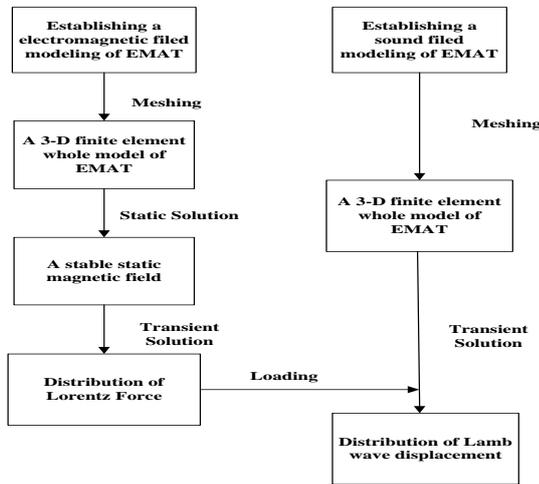


Figure 4. Simulation flow of the physical model of EMAT

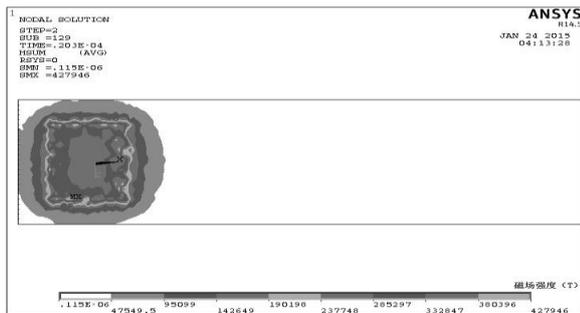


Figure 5. Contour plot of the electromagnetic field intensity in the plate

The contour plot of the magnetic field intensity on the plate is shown in Fig .5. The magnetic field intensity is highest on the edge of permanent magnet; however, it is

low far away from the edge of the permanent magnet. Therefore, the coil should be distributed on the edge of the permanent magnet as soon as possible. The vector plot of eddy density is shown in Fig .6. The shape of eddy on the plate is same as the coil (length L, spaces D, width C). The largest eddy density is  $2.16 \times 10^8 \text{ A/m}^2$ .

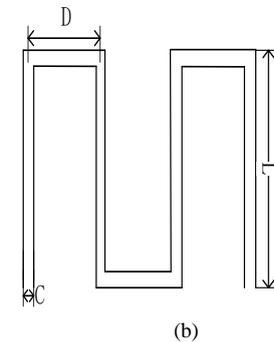
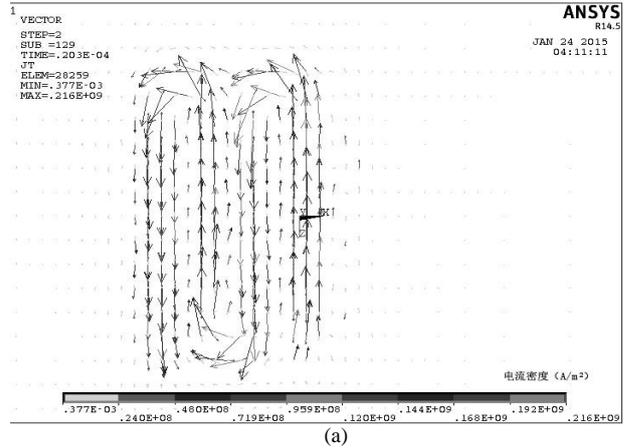


Figure 6. Vector plot of the eddy current density in the plate(a) and the meander coil(b)

### B. The sound field modeling of EMAT

In the sound field modeling, only the aluminum plate was established in ANSYS. The solid45 element is selected in the sound field. The displacement of eight acmes of aluminum plate should be restrained in the direction of X, Y, Z. The Lorentz Force regarded as loading, acquired from the electromagnetic field, is loaded to the aluminum plate. The propagation process of lamb wave in the plate was analyzed by the transient solution.

## IV. ANALYZING THE PROPAGATION F LAMB WAVE BASED ON THE FINITE ELEMENT METHOD

Lamb wave contain symmetry and antisymmetry, and the group and phase velocity of the every mode lamb wave is changing with the excitation frequency. The multi-mode and dispersion nature effect the resolution ratio of the echo signal. Therefore, to enhance the resolution ratio of the echo signal, it is needed to study the multi-mode and dispersion nature. In order to study them, this paper analyzed the changing trends of the group velocity, phase velocity and displacement amplitudes of the particle based on the finite element model when the distance of the fingers, the excitation current is unchanged and the excitation frequency is changing, and the directivity of the sound field is studied.

### A. Studying the directivity of the sound field

The directivity of the sound field should be study due to the multi-mode, which serves as a good foundation for estimating the detecting range.

In order to study the directivity of the sound field, the change of the particle displacement in the plate is simulated by the ANSYS, and the result is shown in the Fig .7. When the excitation frequency is 0.5MHz. The left on the picture is S0 mode and the right on the picture is A0 mode. The direction of propagation of Lamb wave is same as the X. In the direction of X, the displacement of Lamb wave is stronger. It is weaker in the direction of Z. Consequently, lamb wave is more sensitive to the defect on the direction of propagation and insensitivity to the defect perpendicular to the direction of propagation.  $\alpha$  is the half of the radiation angle of S0 mode, and  $\beta$  is half of the radiation angle of A0 mode.  $\beta$  is smaller than  $\alpha$ , therefore, the scope of detection of A0 mode is smaller than S0 mode. Near the dotted line, the displacement of A0 mode is large than S0 mode .Therefore, the ability of detect of A0 mode is larger than S0 mode near the dotted line.

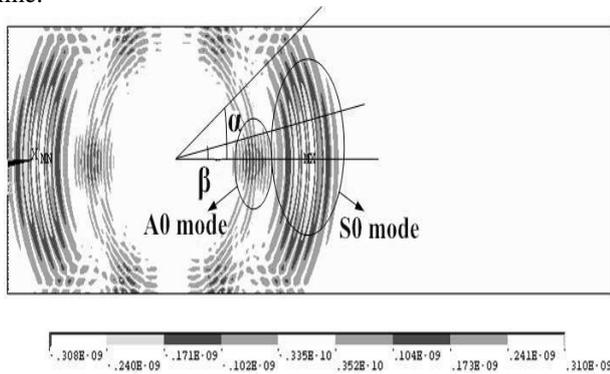


Figure 7. Contour plot of the particle displacement

### B. Studying the multi-mode

Usually, the low excitation frequency is used to generate lamb wave in the engineering. Therefore, the excitation frequency is in the range of 0.08MHz to 1MHz in this paper. There are only A0 and S0 modes in this range. The excitation signal was load on the plate, after a moment, lamb wave generate multi-mode phenomenon that S0 mode with large propagation speed and A0 mode with small propagation speed exist in the plate. As shown in Fig .8, this is a process lamb wave generated multi-mode phenomenon when the excitation frequency is 0.5MHz. After the time of  $1 \times 10^{-5}$ s, the load is zero. There is group wave in each of the positive and negative direction of Y. This group wave is gradually divided at the time of  $1.5 \times 10^{-5}$ s. It is thorough divided into A0 and S0 modes at the time of  $2.0 \times 10^{-5}$ s.

In order to study the particle displacement for each mode in the plate, a particle should be selected. According to the directivity of the sound field, the displacement amplitudes is larger. Therefore, this paper selects a particle on the X axes. At the same time, it should be considered that whether the A0 mode and S0 mode are separated. Therefore , this paper study the changing curve of Uy and Ux at the point (0.19,0.75E-3,-0.048) . As shown in Fig .9, the displacement amplitudes of S0 mode in the

direction of X is six times as large as the direction of Y. Therefore, the Lorentz force in the direction of X is the major force to generate S0 mode. For A0 mode, the displacement amplitudes in the direction of Y is 1.67 times as large as the direction of X, consequently, the Lorentz force in the direction of Y is the major force to generate A0 mode. The displacement amplitudes of S0 mode can be increased by increasing the Lorentz force in the direction of X, similarly, the displacement amplitudes of A0 mode can be increased by increasing the Lorentz force in the direction of Y, which provide a basis for selecting a single mode.

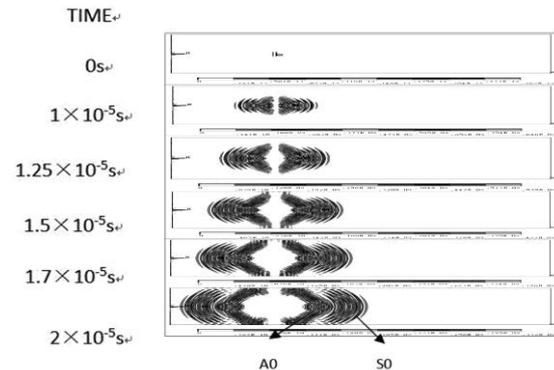


Figure 8. Propagation process of lamb wave

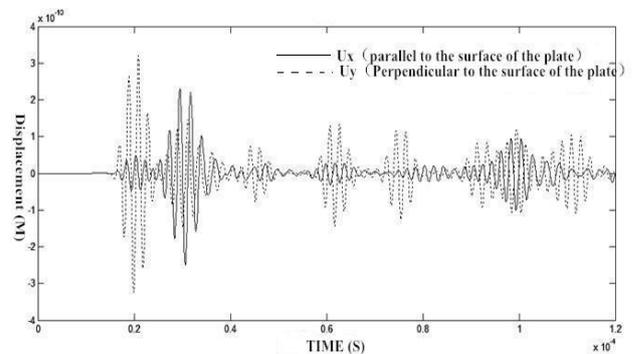


Figure 9. The displacement amplitudes curve that varied with time at a particle

With the purpose of studying the excitation frequency when lamb wave generated multi-mode phenomenon and the lamb wave mode existed in the plate when the excitation frequency is different, this paper studies the lamb wave modes when the excitation frequency is in the range of 0.08MHz to 1MHz and the displacement amplitude. As shown in Fig .10, it is a displacement amplitude curve of a particle about 83mm from the acoustic source. There is only A0 mode in the plate when the excitation frequency is less than 0.188MHz, the displacement amplitude of A0 mode being lager in the direction of X and Y. When the excitation frequency is range of 0.188MHz to 0.45MHz, there are A0 mode and S0 mode and the displacement amplitude of A0 is larger than the S0 mode. When the excitation frequency is more than 0.45MHz, the displacement amplitude of S0 mode gradually increased and more than the displacement amplitude of A0 mode in the direction of X and Y, and the displacement amplitude of A0 mode gradually decreased. When the excitation frequency is 0.64MHz, the

displacement amplitude of A0 mode is minimum, and the displacement amplitudes of S0 mode is larger. The displacement amplitudes of S0 mode gradually decreased and the displacement amplitudes of A0 mode gradually increased when the excitation frequency is more than 0.64MHz. If only the displacement amplitude is considered, the excitation frequency of A0 mode should be less than 0.188MHz, and the excitation frequency of S0 mode should be 0.64MHz .

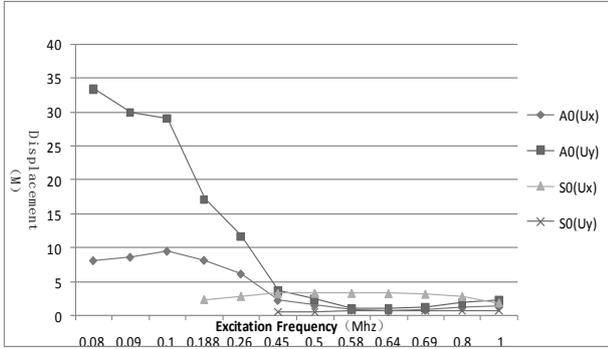


Figure 10. The changing curve of the displacement amplitudes with the change of excitation frequency

### C. Studying the dispersion

It is need to study that the changing of group and phase velocity with the excitation frequency due to the dispersion. Therefore, this paper mainly studies the dispersion of every mode in the plate when the excitation frequency is in the range of 0.08MHz to 1MHz. As shown in Fig .11, it is the dispersion curve of the group velocity and phase velocity for every mode when the excitation frequency is in the range of 0.08MHz to 1MHz. The group velocity and phase velocity of A0 mode is quickly varied with the variation of the excitation frequency, when the excitation frequency is in the range of 0.08MHz to 0.45MHz. It is slowly varied when the excitation frequency is more than 0.45MHz. The phase velocity of A0 mode is about 2200m/s, and the group velocity of A0 mode is about 3300m/s. So mode is generated when the excitation frequency is more than 0.188MHz.

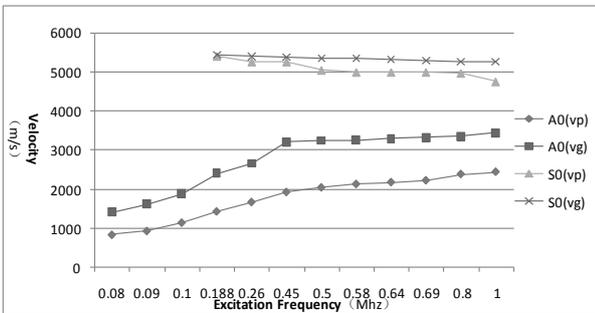


Figure 11. The dispersion curve of the group velocity and phase velocity

In the engineering application, different modes have different sensitivity for the various defects, which is reason that the mode with only single mode and smaller dispersion should be generated according to the actual need. When the thickness of plate is 1mm, if A0 mode is selected to detect the defect on the plate, the single A0 mode should be excited and the displacement amplitudes

of A0 mode is larger. In addition, if the group velocity of A0 mode is slow, the speed of detection is slow. Taking into account all factors, the excitation frequency should be in the range of 0.09MHz to 0.1MHz. If S0 mode is selected to detect the defect on the plate, the displacement amplitudes of S0 mode is larger and the displacement amplitudes of A0 mode is minimum, therefore, the excitation frequency should be in the range of 0.58MHz to 0.69MHz, which decreased the multi-modes and dispersion influence.

## V. EXPERIMENTAL RESULTS

The experimental setup for exciting and receiving Lamb waves is shown in Fig .11. This system include burst generator, transmitter amplifier, T-EMAT, R-EMAT, Receiver pre-amplifier, and digital oscilloscope. The thickness of aluminums plate is 1mm. In order to avoid receiving complex single, the T-EMAT is located on the left side of the plate. The distance between the T-EMAT and R-EMAT is 50mm. This system is used to excite the Lamb wave. The result shows that the S0 mode is excited when the excitation frequency is 0.64MHz. In addition, the single of S0 mode is the strongest and the single of A0 mode is the weakest. As shown in Fig .13, the experimental results verify the result of simulation analysis.

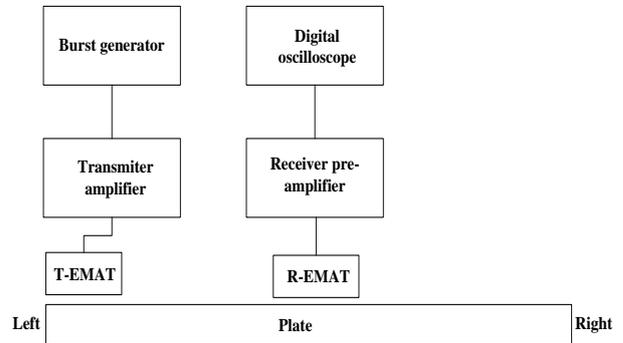


Figure 12. Experimental setup for exciting and receiving Lamb waves.

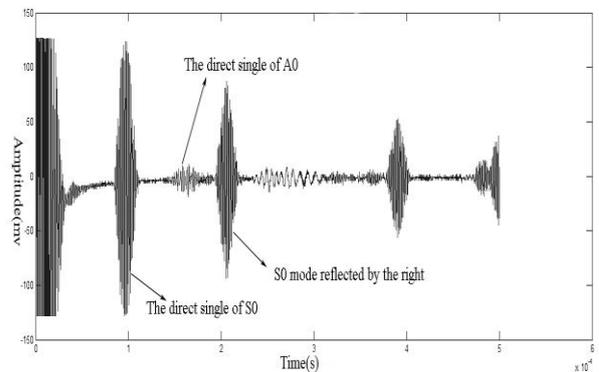


Figure 13. Ultrasonic signal when the excitation frequency is 0.64MHz

## VI. CONCLUSION

Simulating the propagation process of lamb wave in the 1mm thick plate by the ANSYS, this paper mainly studies the changing trends of the group velocity, phase velocity and displacement amplitudes of the different modes when excitation frequency is different. The main conclusions are as follows:

a) The displacement amplitudes of lamb wave is stronger in the direction of propagation, and it is weaker in the direction that is perpendicular to the direction of propagation.  $\beta$  is smaller than  $\alpha$ , Therefore, the ability of detect of A0 mode is larger than S0 mode .

b) As shown in the displacement amplitudes curve, the Lorentz force in the direction of X is the major force to generate S0 mode, and the Lorentz force in the direction of Y is the major force to generate A0 mode. Therefore, increasing the Lorentz force in the direction of X can increase the displacement amplitudes of S0 mode, and increasing the Lorentz force in the direction of Y can enhance the displacement amplitudes of A0 mode, which serves as a good foundation for the optimal design technique of EMATs.

c) When the excitation frequency is less than 0.188MHz, only the A0 mode is exist in the plate. It is more suitable to excite A0 mode used for detecting many defects when the excitation frequency is in the range of 0.09MHz to 1MHz. In the range of 0.58MHz to 0.69MHz, the dispersion of S0 mode is better. According to the experimental results, the S0 mode is excited when the excitation frequency is 0.64MHz. In addition, the single of S0 mode is the strongest and the single of A0 mode is the weakest. The experimental results are consistent with the result of simulation analysis. The result provides a reference for selecting the optimal excitation frequency to detect defects in the 1mm thick plate.

This paper simulated the electromagnetic field and acoustic field for EMAT by the ANYSY, which visually reveal the propagation process of Lamb wave in the thin plate and the group velocity, phase velocity and displacement amplitudes of the different modes. Meanwhile, it is a reference for the parameter design of EMAT, selecting mode of guided waves and estimating the detecting range of lamb waves that studying the directivity of the sound field, multi-mode and dispersion nature, which have a certain guiding significance for the Electromagnetic ultrasonic nondestructive testing.

#### ACKNOWLEDGMENT

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