A General Dynamics Model under Timing-Sequence Geometry Principle

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Abstract—This paper presents a universal model for dynamic system based on geometric modeling methods. The model uses time series that implicates dynamic characteristics as study object, uses geometric modeling methods as basic ideas, and uses embedding time element in NURBS model as the key, for representing arbitrary dynamic system and obtaining dynamic characteristics, called Series-Geometry NURBS (SNURBS for short). Then, we provided two methods based on the principle of S-NURBS in detail. One is Direct Time NURBS method that is appropriate for the research of analytic properties of dynamic system state space. The other is Tangent Vector NURBS method for the research of differential properties of system. Then we proposed a new algorithm for Maximum Lyapunov Exponent of chaotic time series on the foundation of our methods. In the end, we used simple physical projectile system and classical complex system Lorenz chaos to verify validity of these methods. S-NURBS is of specific advantages compared with traditional models for dynamic time series within acceptable error limits according to the experimental results. This is an original idea for dynamics.

Keywords-Timing sequence; Geometry; NURBS; dynamics

I. INTRODUCTION

Geometric modeling method, NURBS, can be used to express any curve track in high precision. It can be used to express exquisitely N-dimensional state space track of nonlinear dynamic system by extending NURBS [1, 2]. The inadequacy is that the new NURBS model can not express dynamic properties from dynamic track along with time changing, the key reason is no time element in NURBS equation.

This research used a mean of time embedding in NURBS parameter to describe time series in high precision. It connects series geometry and dynamic system, describes time series' form with geometry method, studies inner evolution properties of dynamic system with time-form transformation, and describes dynamic rules with time parameter embedded. The NURBS model build by that is a subversive method, of academic significance. Reconstruct non-linear time series of dynamic system by NURBS dynamics model, we can get a strict uniform mathematical equation. On the premise that guaranteeing rapid calculation and high precision, that makes the effect of model more approach to real system behavior. This is an original thought for simulation complex system.

The related research as follows: Weiss discussed a kind of dynamics of geometrically non-linear rods [3], which has its limits (not suitable for any systems, the same as [4-7]). With respect to embedding time in NURBS, Terzopoulos presented a kind of dynamic NURBS (D-NURBS)[8], it aimed at solving geometric modeling by physical concept (Lagrange analytical mechanics), but our S-NURBS principle aims at solving nonlinear system issues by geometric modeling. The two of them have fundamental difference. Our research group have published some paper, Shao firstly presented the concept of S-NURBS, and proved the feasibility and effectiveness that S-NURBS expresses chaotic system [9, 10].

II. TIMING GEOMETRY PRINCIPLE AND MODEL

For most of systems, there does not exist or has not found deterministic equation to express. However, we can obtain discrete space changing with time to express properties through a series of means such as observation and record. In order to apply to arbitrary dynamic system, this work positions research object as nonlinear time series. See Eq. 1, where t represents time set, p represents corresponding state variables set.

$$\begin{cases} t: t_1, t_2, \cdots, t_n \\ p: p_1, p_2, \cdots, p_n \end{cases} \quad (1)$$

Dynamic of NURBS in study has two key principles: one is that retaining feature of NURBS as soon as possible for using geometry matching algorithm conveniently and making research easily. The other is that do not introduce new element, and the sole time parameter t of dynamic system is contained in the equation.

On the basis of above principles, substitute u to map function of time $u(t)$, Eq. 2. It not only tremendously reserves the unique advantage of NURBS, but also meets the requirement of S-NURBS.

$$C(u(t)) = \frac{\sum_{i=0}^{n} N_{i,p}\,(u(t)) w_i P_i}{\sum_{i=0}^{n} N_{i,p}\,(u(t)) w_i} \quad (2)$$

The advantages of S-NURBS are mainly reflected as the following: Firstly, do not need know specific, deterministic analytical or differential equation of nonlinear dynamic system, just need corresponding time series. So this method is for arbitrary nonlinear dynamic system to reconstruct dynamic behavior. Secondly, the amount of errors is only concerned with the amount of properties contained in time series. That is, we can reconstruct the system more accurate by the time series with modest step and length.

According to the fundamental of S-NURBS, the key is dynamic of NURBS, embedding of time parameter and map function $u(t)$. The main body of this research is the design of map function, two kinds of methods will be discussed, and they are respectively Direct Time NURBS (DT-NURBS for short) and Tangent Vector NURBS (TV-NURBS for short).

The DT-NURBS method directly constructs knot vector of NURBS from time element of time series. In view of physics, the distance of two position is related to time span. The particle with changing time draws a track in space, which is a NURBS curve cross certain position point. If the same time span corresponds to the different distance, that means the larger distance, the faster speed; the shorter distance, the slower speed. It is in good coincidence with real situation. According to that, attempt to construct knot vector of NURBS by time set T.

The key of the method is that dividing T by scale, set scale values as u, whose parameter is t, Eq. 3. If the same time span corresponds to a very large distance, which means an accelerating process.

$$u(t)=\frac{t-t_0}{t_n-t_0}\rightarrow u_i=\frac{t_i-t_0}{t_n-t_0},\quad (i=0,1,\cdots,n)$$
$$\Rightarrow U=\{\underbrace{0,0,\cdots,0}_{k},u_0,u_1,\cdots,u_n,\underbrace{1,1,\cdots,1}_{k}\} \tag{3}$$

This method uniformly divides time, the map between parameter u and time t is simple linear relationship. The advantage is relatively simple calculations and not bad performance. It reflects some nonlinear system property, is of big application meaning.

In order to get the analysis equation of dynamic system, or study analysis property, Direct Time NURBS method in previous section mainly meet requirements. However, we are more concerned about the differential properties for some systems, such as velocity, acceleration of kinematics. The previous method is not good at dealing these issues. So, on basis of DT-NURBS method, the section presents an improvement method to get a differential equations, see Eq. 4, called Tangent Vector NURBS (TV-NURBS for short).

$$C(t)=\begin{cases}x_1'=f_1(x_1,x_2,\cdots,x_n)\\ x_2'=f_2(x_1,x_2,\cdots,x_n)\\ \cdots\\ x_n'=f_n(x_1,x_2,\cdots,x_n)\end{cases} \tag{4}$$

As follows specifically: we can get tangent vector of points as goal of NURBS by tangent algorithm through data points of time series. The NURBS curve is formed by the differential points of system which is differential equation. This method is based of DT-NURBS method, along with Tangent algorithm, borrowing from Fmill and Bessel Algorithm [11]. The two kinds of algorithm is of some certain geometry properties like affine, rotation and translation invariance.

III. A NEW ALGORITHM FOR LYAPUNOV EXPONENT BASE ON SNURBS

In the above part, we discuss S-NURBS method for time series of nonlinear dynamic system to study system properties. This next will use the presented method to design an algorithm for Maximum Lyapunov Exponent of chaotic system (MLE for short). The meaning is as follows: One is that this is a new idea for solving chaotic system (different from traditional phase space reconstruction [12-14]). The other is that the new S-NURBS model can deal with intrinsic features (like Lyapunov Exponent) of nonlinear system, and as a proof for the validity of S-NURBS.

Set the distance between the two adjoining points as $\varepsilon(t)=x(t+t_1)-x(t+t_2)$.

And t_1,t_2 is the initial selected time. If system is of period feature, $t\rightarrow\infty$ means the limit value in a period. And that $\varepsilon(0)\rightarrow 0$ refers to the two closest points between two periods.

The explanation of mathematical symbol: average period of time series: $period$; the number of periods: n; current time: t; evolutionary time in a period: T; the formula for S-NURBS: $C(t)$.

Step 1: Divide periods for time series, get the values of $period$ and n; transform frequency domain to time domain by FFT for time series; compute energy spectrum, get frequency list $frequency=([0,1,\cdots,N-1])/N$, and get the frequency value corresponding to the max value of spectrum; the average period is the reciprocal of the frequency.

Step 2: Use S-NURBS model method to get S-NURBS expression $C(t)$.

Step 3: Take any moment in the first period as the first initial t_1. Select the second initial t_2 in the adjacent second period, which meet the shortest distace between the two times as possible. Set $MinDistance=C(t_2)-C(t_1)$.

Step 4: Select an adaptive time span T(not greater than the length of period), compute the distance of initial time t_1,t_2 corresponding points T time later. Set $Distance=C(T+t_2)-C(T+t_1)$.

The value of T should be larger as possible on the premise that $T+t_1$ and $T+t_2$ are not greater than $period$. See Eq. 5 which meets our requirement.

$$\mathrm{T}=\mathrm{MIN}(\mathrm{period},\mathrm{period}-(\mathrm{t2}-(\mathrm{t1}+\mathrm{period}))) \tag{5}$$

Step 5: Compute MLE in the two period, Eq. 6:

$$\lambda=\frac{1}{T}\ln\frac{C(T+t_2)-C(T+t_1)}{C(t_2)-C(t_1)} \tag{6}$$

Step 6: Loop processes the n periods of time series, and then the average is MLE, Eq. 7.

$$\lambda=\frac{1}{nT}\sum_{i=1}^{n}\ln\frac{C(iT+t_2)-C(iT+t_1)}{C(t_2)-C(t_1)} \tag{7}$$

IV. Experiment and Discussion

In order to measure the validity of model what the above presents, we use three indices in experiment. There are root mean square error (RMSE for short), Pearson correlation coefficient (Pearson for short) and a user-defined combined index: similarity-index of dynamic system (SI for short).

Similarity-Index means approximation ratio that S-NURBS model can express nonlinear system. In order to be convenient for experiment, we select only one index to measure the similarity between two systems. RMSE and Pearson are one-sided. Nevertheless, if Pearson is closer to one and RMSE is smaller, then the two systems are more similarity. So set SI as the ratio between RMSE and Pearson (Eq. 8).

$$SI = \frac{RMES}{Pearson} \qquad (8)$$

In experiment for verification, we used two systems, Projectile system and Lorenz system.

Projectile system (Eq. 9), is a simple physical system, is non-chaotic and analytical system. Parameter Values $v_0 = 39.2$, $\alpha = \frac{\pi}{6}$, $g = 9.8067$.

$$\begin{cases} x(t) = v_0 t \cos \alpha \\ y(t) = v_0 t \sin \alpha - \frac{1}{2} g t^2 \end{cases} \qquad (9)$$

Time range $[0,10]$, step range $[0.50,0.01]$, interval 0.02. Get the new system expression through S-NURBS model, and compared with the original, get the values of SI. Table 1 (The values of RMSE and Pearson) is the experimental results ($step = 0.3$) of two methods. As can be seen from Table 1, the values of Pearson is around 0.99, the explanation is that the state variables of two systems is linear correlation; the values of RMSE is approximate to zero, the explanation is that linear coefficient is around 1. So, that seems tell us the two systems approach equivalent.

TABLE I. RMSE-Pearson

DT-M	RMSEx	RMSEy	Pearson x	Pearson y
Step=0.3	4.207e-03	2.440e-03	0.999999	0.999999
TV-M	RMSE-x	RMSE-y	Pearson x	Pearson y
Step=0.3	1.118e-14	1.215e-03	0.999140	0.999999

Next, we got the values of SI through RMSE and Pearson, shown in Fig .1. It can be seen from the figure that the two systems are more equivalent with the decreasing of step, and converges very fast. That shows remarkable results that S-NURBS expresses dynamic system.

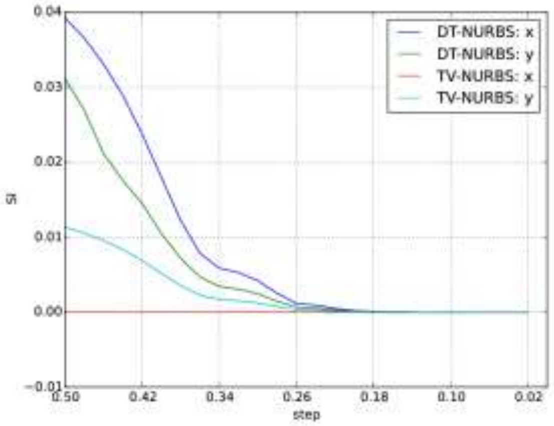

Figure 1. Similarity Tendency of Projectile System

Object just goes simply by gravity in Projectile system. So the acceleration is gravitational acceleration g , its value can be deducted from analysisical equation. The aim of this experiment is verifying the acceleration deducted from S-NURBS compared with standard g . For convenience, we used horizontal projection (set $\alpha = 0$), and the two-derivative in y-axis is g . Select step $0.5, 0.3, 0.1$, and get gravitational acceleration by S-NURBS model, the results is in Table 2. Intuitively, the result is close to $g = 9.8067$ (standard value).

TABLE II. Gravity Acceleration

Step	0.50	0.30	0.10
DT-g	9.78542379082	9.81548997236	9.80669244144
TV-g	9.80542112754	9.80544601408	9.80669996402

Through the above experiments, we can say S-NURBS system is of highly approximation expression for the original system.

Lorenz system (Eq. 10) is classical chaotic system [15]. The system has simple expression but complex motion.

$$\begin{cases} \dot{x} = -\sigma(x - y) \\ \dot{y} = -xz + rx - y, \sigma = -10, b = \frac{8}{3}, r = 30 \\ \dot{z} = xy - bz \end{cases} \qquad (10)$$

On account of complexity of chaotic system, we should select step below 0.02 . So the range of step is $[0.02, 0.001]$, $interval: -0.001$ the result is shown in Fig .2.

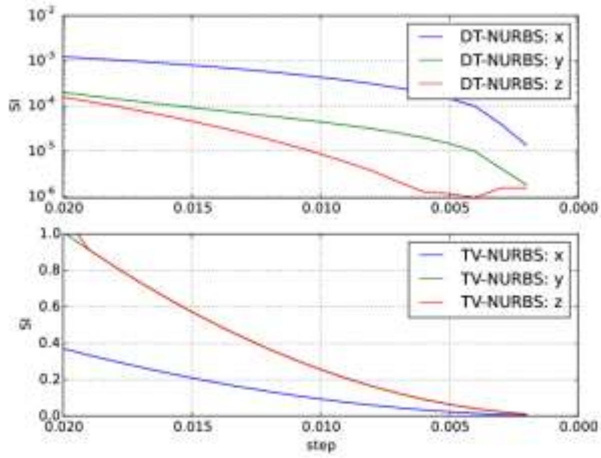

Figure 2. The Lorenz-Similarity Tendency of Lorenz System

From Fig .2 the value of SI starts with around one, but the speed of convergence to zero is very fast with the reducing of step. It is convictive to say the system expressed by S-NURBS is equivalent to the original system. The properties of Lorenz chaos show in Eq. 11 [16], when the parameter values are set as follows $a = 10, b = 8/3$, initial values $[0,1,0]$, step length 0.01.

$$\begin{cases} r = [1,500] \\ 1 \leq r < 24, steady\ state, \lambda \leq 0 \\ r \geq 24, chaotic\ state, \lambda > 0 \\ r \geq 220, \lambda \to 0 \end{cases} \quad (11)$$

Fig .3 is the results of MLE through the above S-NURBS algorithm. X-coordinate refers to the parameter r, Y-coordinate refers to the values of MLE. It shows clearly the same as Eq. 11.

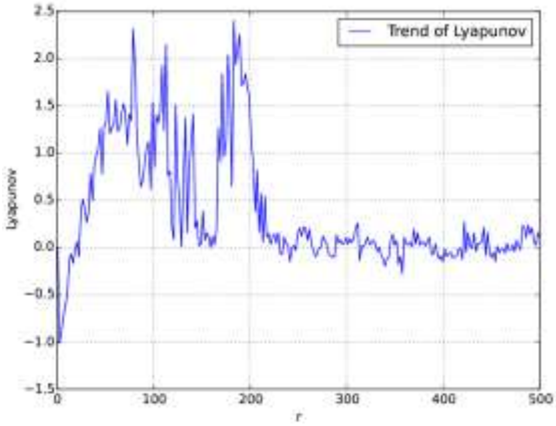

Figure 3. The Lyapunov Exponent Tendency of Lorenz System

Select a special parameter values $a = 10, b = 8/3, r = 28$. Compared with the value of MLE in different methods, the results are shown in Table 3.

TABLE III. LYAPUNOV COMPARISON

Methods	Define	Wolf	Small-Data	S-NURBS
λ max	1.3671	0.0229	0.024	0.5278

The above results are full proof that S-NURBS can express chaos and reconstruct its properties.

V. CONCLUSION

This article presents a fundamental of S-NURBS based on geometric modeling tool NURBS for dynamic system. The S-NURBS corresponding methods apply to time series (especially nonlinear). This means they apply to arbitrary dynamic systems whether the system with special equation or only time series, the handling method is the same. There is uniform algorithm which is very simple and clear. Also it is stable, it not need select some subjective parameters values. In experiment, S-NURBS model not only expresses dynamic system, but also reconstructs analytical and differential properties in an acceptable error. What's more, verify inherent property of system, for example, acceleration of gravity of projectile motion, periodicity of pendulum, and MLE of chaos. The experiments results explain that the theory of the method proposed in this study can be used to study dynamic systems and reconstruct the dynamic behavior. Provides a new idea for reconstruction and prediction of chaotic properties of chaotic systems and expect to solve difficult issues.

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