Mathematical modelling analysis of the friction heating process of resin substance

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Abstract. The resin is `d for vehicle brake, in order to avoid the safety problems because of the significant temperature rise caused by the resin substance friction effect, the process of friction heating resin substance is made mathematical modelling analysis. Through the strengthen Lagrange algorithm obtain the accurate Lagrange multipliers, that is the contact force, to complete correction iterative operation of penalty function. Through the Hertz theory, it can obtain the pressure distribution of the contact spot bearing. The problem of resin substance friction heating is converted into a two-dimensional problem. According to the law of conservation of energy, the formula of resin substance friction heating can be obtained. The temperature distribution of resin substance in the start moment in the non-steady heat conduction process is studied, and the first boundary condition and third boundary condition are analyzed. Using direct coupling method solve the problem of resin substance friction heating, and the generated total flow rate and the heat flow rate of contact surface as well as the heat flow rate of one side of two resin substances because of friction can be calculated. The heat flow rate of flow into and out are effected in the micro element, and the computational domain is discretized. The temperature change is substituted into the equivalent integral equation of differential equation, obtaining integrated general equation in matrix form. Simulation results show that the proposed mathematical modelling process had high reliability.

Introduction

At present, with the operating speed of passenger train in our country continue to improve, the vehicles usually use disc brake as the basic brake form, and the brake pad usually uses resin substance [1, 2]. Therefore, studying on the resin substance friction heating process, avoiding the safety problem because of the significant temperature rise caused by the resin substance friction effect, is the key subject for research, and is attracted more and more attentions [3-5].

Mathematical modelling analysis of the resin substance friction heating process is made in this paper. Through the strengthen Lagrange algorithm obtain the accurate Lagrange multipliers, that is the contact force, to complete correction iterative operation of penalty function. Through the Hertz theory, it can obtain the pressure distribution of the contact spot bearing. The problem of resin substance friction heating is converted into a two-dimensional problem. According to the law of conservation of energy, the formula of resin substance friction heating can be obtained. The temperature distribution of resin substance in the start moment in the non-steady heat conduction process is studied, and the first boundary condition and third boundary condition are analyzed. Using direct coupling method solve the problem of resin substance friction heating, and the generated total flow rate and the heat flow rate of contact surface as well as the heat flow rate of one side of two resin substances because of friction can be calculated. The heat flow rate of flow into and out are effected in the micro element, and the computational domain is discretized. The temperature change is substituted into the equivalent integral equation of differential equation, obtaining integrated general equation in matrix form. Simulation results show that the proposed mathematical modelling process had high reliability.

Description of resin substance contact model

In this paper, through the strengthen Lagrange algorithm obtain the accurate Lagrange multipliers, that is the contact force, to complete correction iterative operation of penalty function. In order to make the calculation convergence to the exact solution of the model, let the specific neighborhood of contact surface to the refined grids.

The contact theory. Assuming the resin substance contact conditions meet the Hertz contact theory, at the same time, it can be processed according to the Hertz contact theory, thus, through the Hertz theory, it can obtain the pressure distribution of the contact spot bearing:

$$p(x) = p_0\sqrt{1-\left(x / a_0\right)^2} \quad (1)$$

The maximum contact pressure p_0 is in the center of contact spot, then:

$$p_0 = \frac{2W}{\pi a_0} \quad (2)$$

Wherein, $W = \frac{3F}{4b_0}$; b_0 is used to describe the radius of contact spot, its' typical value $b_0 = 6.25mm$; F is load; a_0 is 1/2 length of contact spot, $a_0 = \sqrt{\frac{2WR}{\pi E^*}}$.

$$\frac{1}{E^*} = \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \quad (3)$$

Where, E_1, E_2, v_1 and v_2 are used to describe elastic modulus and Poisson's ratio of the resin substance

Theory of transient heat transfer. On the basis of the above analysis, the problem of resin substance friction heating is converted into a two-dimensional problem, according to the law of conservation of energy, the resin substance friction heating can be described as:

$$[C]\left\{T^{\xi}\right\} + [K]\{T\} = \{Q\} \quad (4)$$

Among them, $[K]$ is used to describe the heat transfer matrix, including thermal conductivity, coefficient of convection, radiation rate and form coefficient. $[C]$ is used to describe the heat capacity matrix; $\left\{T^{\xi}\right\}$ is used to describe the temperature derivative of time; $\{Q\}$ is the node's heat flux vector; $\{T\}$ is the node's temperature vector.

If it is the non-steady heat conduction process, the temperature distribution of resin substance in the start moment, also is the initial conditions are described by formula (5):

$$T\big|_{t=0} = \varphi\left(x, y, z\right) \quad (5)$$

Among them, $\varphi(x, y, z)$ is the known functions, showed that initial temperature of resin substance is not uniform.

The process of resin substance friction heating is analyzed, it needs to consider the heat transfer boundary conditions, including the two boundary conditions:

first boundary condition

The temperature on the boundary surface of resin substances in any time can be described as:

$$T\big|_s = t_w \quad (6)$$

（2）third boundary condition

T_0 is used to describe the fluid medium temperature of which contact with resin substance; h is heat transfer coefficient, then have:

$$-k\frac{\partial T}{\partial n}\big|_{\tau} = h\left(T - T_0\right)\big|_{\tau} \quad (7)$$

Among them, n describes the external normal unit vector of free boundary; h is used to describe the heat transfer coefficient between the surface of resin and environment; T_0 is the environment temperature.

In the heat transfer process, the properties and boundary conditions of resin material will change with the temperature change, this kind of problem is non-linear thermal analysis. The material parameters of resin substances of this study are shown in table 1:

Table 1 material parameters

Parameters	numerical value
specific heat capacity c/(J/(kg·K))	1.2
heat conductivity k/(W/(m·K))	0.2
Density ρ/(kg/m^3)	1.25
Poisson's ratio v	0.38
Modulus of Elasticity E/Pa	1.02×10^{6}
environment temperature T_0/℃	20
coefficient of surface heat transfer h/(W/(m^2·K))	20
coefficient of thermal expansion α/(1/k)	1.2×10^{-5}

Mathematical modelling of resin substance friction heating process

Calculation of the resin body friction heating process belongs to the thermal stress analysis of coupling field calculation. Calculation of coupling field includes indirect coupling method and the direct coupling method. For the resin substance friction heating problem, because the friction stress directly leads to the emergence of heat flux and thermal strain on the structure of the resin substance friction surface, it is not suitable for using the indirect coupling method, and needs to adopt the direct coupling method.

The total generated heat rate q of two resin substances because friction can be calculated by formula (8):

$$q = FHTG \times \tau \times v \quad (8)$$

Among them, $FHTG$ is used to describe the energy conversion factor of resin substance friction heating; τ is used to describe the equivalent friction stress; v is used to describe the relative sliding speed of resin substance.

Heat flow rate q_c of resin substance friction contact surface can be calculated by formula (9):

$$q_c = FWGT \times FHTG \times \tau \times v \quad (9)$$

Among them, $FWGT$ is used to describe the weight factor of heat distribution in the process of resin substance friction factor.

The heat flow rate of one side of resin substance can be calculated by the formula (10):

$$q_t = (1 - FWGT) \times FHTG \times \tau \times v \quad (10)$$

In resin substance, due to temperature gradient will appear internal energy exchange phenomenon, i.e. the heat conduction is consistent with the Fourier's theorem, as formula (11) showed. Wherein, the minus sign represents direction of heat quantity of resin substance friction heating along the decreasing temperature.

$$q^* = -Knn \times \partial T / \partial n \quad (11)$$

Among them, q^* is used to describe the heat flux, the unit is W / m2; Knn is the thermal conductivity, the unit is W / （m·℃）; $\partial T/\partial n$ is used to describe the temperature gradient.

On the basis of the first law of thermodynamics, the resin substance friction heating process is:

$$Q - W = \Delta U + \Delta KE + \Delta PE \quad (12)$$

Wherein, Q is quantity of heat; W is power; ΔU internal energy of system; ΔKE is kinetic energy of system; ΔPE is potential energy of system

In the process of friction heating analysis of resin substance, usually $\Delta KE = \Delta PE = 0$, and then $q = dU / dT$, that is the rate of heat flow q of flow into and flow out is same to the internal energy in the whole process. Making its effect on element, can obtain the control differential equation of heat conduction (13), (14):

$$\frac{\partial}{\partial x}\left(\kappa_{xx}\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\kappa_{yy}\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\kappa_{zz}\frac{\partial T}{\partial x}\right)+q=\rho c\frac{dT}{dt} \quad (13)$$

$$\frac{dT}{dt}=\frac{\partial T}{\partial x}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z} \quad (14)$$

Among them, V_x, V_y and V_z are the conduction velocity of medium.

The computational domain is discretized, to get the finite element,that it can access the temperature vector per unit node T which is the is unknown polynomial:

$$T=\{N\}^T\{T_e\} \quad (15)$$

Where, $\{N\}^T$ is used to describe unit shape function; $\{T_e\}$ is used to describe the temperature vector per unit node.

According to the unit node temperature it can get each unit temperature gradient and heat flow density:

$$\{\alpha\}=\{L\}^T=[B]\{T_e\} \quad (16)$$

$$[B]=\{L\}^T N \quad (17)$$

Wherein, $\{\alpha\}$ is the thermal gradient vector; $\{L\}^T$ is temperature gradient; $[B]$ is to describe heat flux; N is used to describe the unit volume.

Heat flux can be determined by formula (18):

$$\{q\}=(D)\{L\}^T=(D)[B]\{T_e\} \quad (18)$$

Where, (D) is used to describe the heat conduction property matrix of resin material.

The temperature changes of the above analysis is substituted into an equivalent integral equation to differential equation, to obtain integrated general equation of matrix form, by using the formula (19) ~ (22) to describe:

$$(C)\{T\}+(K)\{T\}=\{Q\} \quad (19)$$

$$(C)=\sum_{i=1}^{n}(C) \quad (20)$$

$$(K)=\sum_{i=1}^{n}\left(K^{m,d,c}\right) \quad (21)$$

$$(Q)=\sum_{i=1}^{n}\left(Q^{f,c,g}\right)+\{Q_0\} \quad (22)$$

Analysis of simulation experiments

In order to verify the authenticity and effectiveness of mathematical modelling of resin substance friction heating process, it requires related experiments analysis. Experiments is performed on Windows XP, MATLAB 7 environment, a resin substance is taken as the experimental object to study.

In this paper, the proposed mathematical model and traditional mathematical model are applied respectively for the resin substance friction heating process modelling, there are a total of eight times simulation experiments. Wherein, the steady-state temperature values from five experiments are statistically compared, to obtain results in Table 2.

Table 2 comparison results of the steady temperature value using the proposed model and traditional model

The number of experiments	Actual value	Results of the proposed model	Results of the traditional model
1	32.675	30.8	27.2
2	38.426	36.1	31.6
3	44.479	43.8	40.5
4	38.855	37.9	33.9
5	40.236	39.1	34.7
6	34.698	32,4	29.1
7	36.539	35.2	30.4
8	44.938	43.8	39.2

We can see from the analysis of Table 2 that, compared with the traditional model, resin's value of steady-state temperature obtained through the proposed model is more consistent with the actual value, explained the model with real and high reliability.

In order to validate the reliability of the model in this paper, on the basis of above results, error of this model and the traditional model are made statistics, to obtain the results shown in Figure 1.

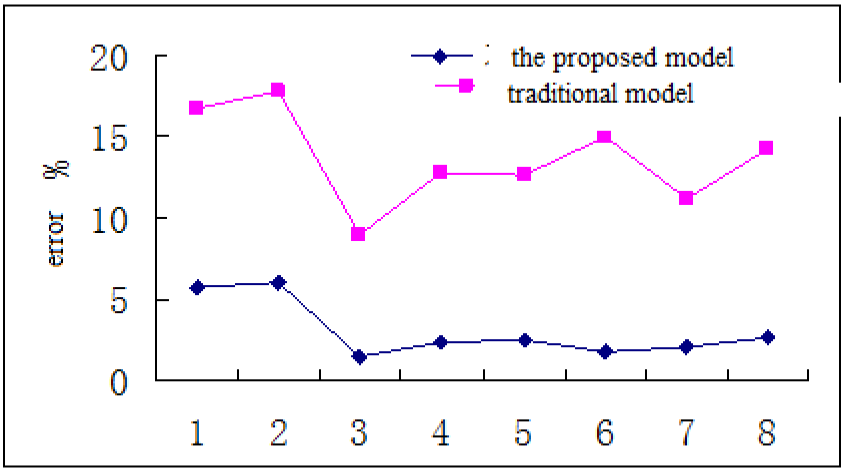

Fig. 1 Comparison of error using the proposed model and traditional model

It can be seen from Figure 1, the error curve obtained by using the proposed model is less than that of traditional model, and it has been lower than the traditional model. In addition, the error curve of the model in this paper is more stable than that of traditional model, to verify the effectiveness of the proposed model.

Conclusions

The process of friction heating resin substance is made mathematical modelling analysis in this paper. Through the strengthen Lagrange algorithm obtain the accurate Lagrange multipliers, that is the contact force, to complete correction iterative operation of penalty function. Through the Hertz theory, it can obtain the pressure distribution of the contact spot bearing. The problem of resin substance friction heating is converted into a two-dimensional problem. According to the law of conservation of energy, the formula of resin substance friction heating can be obtained. The temperature distribution of resin substance in the start moment in the non-steady heat conduction process is studied, and the first boundary condition and third boundary condition are analyzed. Using direct coupling method solve the problem of resin substance friction heating, and the generated total flow rate and the heat flow rate of contact surface as well as the heat flow rate of one side of two resin substances because of friction can be calculated. The heat flow rate of flow into and out are effected in the micro element, and the computational domain is discretized. The temperature change is substituted into the equivalent integral equation of differential equation, obtaining integrated general equation in matrix form. Simulation results show that the proposed mathematical modelling process had high reliability.

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