The Quantum Teleportation Fidelity of the Squeezed Vacuum State

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Abstract. We use two-mode squeezed vacuum state as a quantum channel to teleportate the quantum squeezed vacuum state and analyze the fidelity of the teleportation by the Wigner function. The result shows that the squeeze factor of the quantum squeezed vacuum state is smaller, the quantum features of the teleportation state is more easily preserved.

Introduction

Quantum teleportation is a basic process of quantum communication and its unique role in the field of communication has attracted wide attention. In 1993, Bennett et al first proposed the quantum teleportation scheme[1]. In 1994, Vaidman et al extended this scheme to a continuous-variable (CV) system[2]. In 1998, Braunstein and Kimble proposed a protocol to teleport the coherent state with high fidelity using squeezed-state entanglement, which is a viable option for the teleportation of continuous variables[3]. Then there are more and more studies on coherent state teleportation [4-6], but other quantum state teleportation is rarely. The fidelity, which is a measure that quantifies the overlap between the input and the output states, is an important physical quantity in quantum communication. In order to get higher fidelity and preserve the information characteristics of the input state better, many studies about this area are doing recent [7-10]. In this paper, we will calculate the fidelity of the quantum squeezed vacuum state teleportation by the Wigner function.

The Fidelity Represented by Wigner Function

For a input quantum state $|\psi_{in}\rangle$, the fidelity $F = \langle \psi_{in} | \hat{\rho}_{out} | \psi_{in} \rangle$ shows how close between the input state and the output quantum state. When $\hat{\rho}_{out} = |\psi_{in}\rangle\langle\psi_{in}|$, $F = 1$ is the ideal fidelity. When the input and output state are orthogonal, $F = 0$。 Fidelity of quantum information science is an important physical quantities, it can measure the performance of quantum teleportation.

Using the well-known representation of the density operator in terms of the coherent displacement operator $\hat{D}(\xi)$ [5,11]

$$\hat{\rho} = \frac{1}{\pi}\int d^2\xi \chi(\xi) \hat{D}^{+}(\xi), \tag{1}$$

where $\chi(\xi)$ is Characteristic function of the Wigner function

$$\chi(\xi) = \langle \psi | \hat{D}(\xi) | \psi \rangle = \int W(\alpha) e^{\xi\alpha^* - \xi^*\alpha} d^2\alpha . \tag{2}$$

According to Eqs. (1) and (2), the fidelity can be written as

$$\begin{aligned} F &= \frac{1}{\pi}\int d^2\xi \chi_{out}(\xi) \langle \psi_{in} | \hat{D}^{+}(\xi) | \psi_{in} \rangle \\ &= \frac{1}{\pi}\int d^2\xi \chi_{out}(\xi) \chi_{in}^{*}(\xi) \end{aligned}, \tag{3}$$

By the relationship between the Characteristics function and Wigner function, the fidelity can also be expressed with only the Wigner function

$$F = \pi \int d^2\beta W_{out}(\beta) W_{in}(\beta) \, . \tag{4}$$

Calculation Methods for Fidelity in Quantum Teleportation

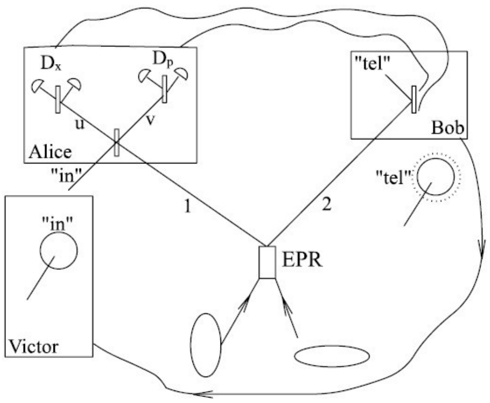

Fig. 1. Scheme for quantum teleportation

The quantum channel for quantum teleportation is the two-mode squeezed vacuum state which is described by the Wigner function[12]

$$W_{EPR}(\alpha_1, \alpha_2) = \frac{4}{\pi^2} \exp\{-e^{-2r}[(x_1 - x_2)^2 + (p_1 + p_2)^2] - e^{-2r}[(x_1 + x_2)^2 + (p_1 - p_2)^2]\} \, , \tag{5}$$

r is the squeezing factor, $\alpha_{1,2} = x_{1,2} + ip_{1,2}$ 。 $W_{in}(\alpha_{in})$ signs the Wigner function of the unknown input state and $\alpha_{in} = x_{in} + ip_{in}$ 。 Then the Wigner function of the (three mode) overall system reads $W(\alpha_{in}, \alpha_1, \alpha_2) = W_{in}(\alpha_{in}) W_{EPR}(\alpha_1, \alpha_2)$.

As shown schematically in Fig. 1, at first , Alice combine mode 1 with mode "in" at a 50/50 beam splitter . $\alpha_u = (\alpha_{in} - \alpha_1)/\sqrt{2}$ and $\alpha_v = (\alpha_{in} + \alpha_1)/\sqrt{2}$ are the two output modes. After the beam splitter, the total Wigner function is written as

$$W(\alpha_u, \alpha_v, \alpha_2) = W_{in}\left(\frac{(x_u + x_v)}{\sqrt{2}}, \frac{(p_u + p_v)}{\sqrt{2}}\right) W_{EPR}\left(\frac{(x_v - x_u) + i(p_v - p_u)}{\sqrt{2}}, \alpha_2\right) . \tag{6}$$

Then Alice performs a bell measurement on her modes 1 and "in". Alice's Bell detection yields certain classical values x_u and p_v , which describes via integration over x_v and p_u . Assuming $(x_u + x_v)/\sqrt{2} = x$ and $(p_u + p_v)/\sqrt{2} = p$, then

$$\int\int dx_v dp_u W(\alpha_u, \alpha_v, \alpha_2) = 2\int\int dxdp W_{in}(x, p) W_{EPR}[x - \sqrt{2}x_u + i(\sqrt{2}p_v - p), \alpha_2] . \tag{7}$$

Next Alice transfers her measurement of classical results to Bob. In order to better recover the input variables x_{in} and p_{in} , Bob magnify his receiving classical information $\sqrt{2}$ times, and translate the mode 2. Bob's displacements are now incorporated α_2 by the substitution $\alpha_2 = x_b - \sqrt{2}x_u + i(p_b - \sqrt{2}p_v)$. Finally, integration over x_u and p_v yields the teleported Wigner function

$$\begin{aligned} W_{out}(\beta) &= 2\int\int\int\int dx_u dp_v dxdp W_{in}(x, p) \\ &\times W_{EPR}[x - \sqrt{2}x_u + i(\sqrt{2}p_v - p), x_b - \sqrt{2}x_u + i(p_b - \sqrt{2}p_v)] \\ &= \frac{2}{\pi e^{-2r}} \int\int dxdp W_{in}(x, p) \exp\left(-\frac{|x_b - x|^2 + |p_b - p|^2}{e^{-2r}}\right) \\ &= \frac{2}{\pi e^{-2r}} \int d^2\alpha W_{in}(\alpha) \exp\left(-\frac{|\beta - \alpha|^2}{e^{-2r}}\right) \\ &= W_{in} * G_\sigma \end{aligned} \, , \tag{8}$$

where “*” is the sign of convolution. So the teleported state is a convolution of the input state with the complex Gaussian $G_\sigma(\alpha) = \frac{1}{\pi\sigma}\exp(-\frac{|\alpha|^2}{\sigma})$, with $\sigma = e^{-2r}$ and $\beta = x_b + ip_b$.

According to Eqs. (8) and (4), we obtain several representation for the fidelity

$$\begin{aligned} F &= \pi\int d^2\beta W_{out}(\beta)W_{in}(\beta) \\ &= \pi\int d^2\beta d^2\alpha G_\sigma(\beta-\alpha)W_{in}(\alpha)W_{in}(\beta) \\ &= \frac{1}{\pi}\int d^2\xi\chi_{out}(\xi)\chi_{in}^*(\xi) \\ &= \int d^2\xi G_\sigma(\xi)|\chi_{in}(\xi)|^2 \end{aligned}, \quad (9)$$

obviously for the squeezing factor $r \to \infty$, $\sigma = 0$, $W_{out} = W_{in}$, $F = 1$。

The Quantum Teleportation Fidelity of the Squeezed Vacuum State

The fidelity calculated by Wigner function of quantum teleportation for an arbitrary state are applicable. For example, we use two-mode squeezed vacuum state with the squeezing factor r as a quantum channel to teleportate the quantum squeezed vacuum state with the squeezing factor s. The quantum squeezed vacuum state is[12]

$$\hat{S}(s)|0\rangle = (\cosh s)^{-1/2}\sum_{n=0}^{\infty}\frac{1}{n!}(-\frac{\tanh s}{2})^n\sqrt{(2n)!}|2n\rangle, \quad (10)$$

whose Wigner function can be given by

$$W(\alpha) = \frac{2}{\pi}\exp(-2e^{2s}x^2 - 2e^{-2s}p^2), \quad (11)$$

and the Characteristic function is

$$\chi(\xi) = \exp(-\frac{e^{2s}}{2}\xi_1^2 - \frac{e^{-2s}}{2}\xi_2^2), \quad (12)$$

where $\xi = \xi_1 + i\xi_2$. According to Eqs. (9) - (11), the fidelity of the quantum teleportation of the quantum squeezed vacuum state is calculated

$$F = \frac{1}{\sqrt{(1+\sigma e^{-2s})(1+\sigma e^{2s})}} . \quad (13)$$

For the certain r, when $s = 0$, which means a squeezed vacuum state degenerated into a vacuum state, the fidelity F is the maximum value. $F = \frac{1}{1+\sigma} = \frac{1}{1+e^{-2r}}$ is the same as that of the teleportation of a coherent state. When the squeezing factor $s \to \infty$时，the fidelity $F \to 0$. Thus the result shows that squeezing factor is smaller, the quantum characteristics of the state are more easily retained in the transmission process.

Summary

The fidelity of quantum information science is an important physical quantities, it can measure the performance of quantum teleportation. In this paper, We use two-mode squeezed vacuum state as a quantum channel to teleportate the quantum squeezed vacuum state and analyze the fidelity of the teleportation by the Wigner function. We discuss the problem of the preservation of the quantum features on the teleportation process, which may provide a theoretical basis for the experiment of quantum teleportation of the squeezed vacuum state.

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