A Fusion Method of Rocket Launchers' Testing Information Based on the Improved D-S Evidence Theory

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Abstract. In order to make more effective decision level analysis on the testing information of rocket launchers, this paper analyzes the existing disadvantages of evidence theory, draws into the concept of weight coefficient integrated with existing improved method, proposes new improvement of the combination rule of evidence theory and makes discussion through examples. Besides, comparing with results of other algorithms, this paper verifies the advantages of its new method on processing conflict evidence as well as making effective use of the evidence set.

Introduction

In the course of processing testing information of rocket launchers, the same device status may lead to different fault symptoms, while the same fault symptom may be caused by different device status due to the complex relation between symptom space and device status. That is where the advantage of processing testing information of rockets with multi-sensor data fusion of evidence theory stands out. However, traditional processing method of evidence theory has its own defects: "veto with a single vote" appears easily; and the size of subsets is unable to be distinguished. This paper puts forward a new method of evidence combination, attaching great importance to the interrelation among evidences. Meanwhile, the new method can effectively solve the above defects and achieve satisfactory results.

D-S evidence theory and its weak points

Fundamental concepts.

1.Basic probability assignment function[1]. Under θ, a given frame of discernment, set up a set function m: $2^\theta \rightarrow [0,1]$. If m satisfies m(Ø)=0 and $\sum_{A\in\theta} m(A)=1$, then m(A) is a basic probability assignment function.

2.Belief function. Set θ as a frame of discernment. Set function m: $2^\theta \rightarrow [0,1]$ is a basic probability assignment function of θ. Define function Bel: $2^\theta \rightarrow [0,1]$ as

$$\mathrm{Bel}(A) = \sum_{B\subset A} m(B) \qquad (\forall A \subset \theta) \tag{1}$$

where Bel is a belief function of θ.

3.Plausibility function. Under θ, a frame of discernment, if $\mathrm{Pl}(A)= 1 - \mathrm{Bel}(\bar{A})$, Pl is a plausibility function of θ. It can be got that.

$$\mathrm{Bel}(A) \le \mathrm{Pl}(A) \tag{2}$$

Call [Bel(A), Pl(A)] a confidence interval, or an uncertain interval.

Combination rule

1.The combination rule of two belief functions. Set Bel_1 and Bel_2 as two belief functions of θ, a frame of discernment, and m_1 and m_2 as corresponding basic probability assignments. Their focal elements are A_1, …, A_k and B_1, …, B_k. If $\sum_{A\cap B=\emptyset} m_1(A_i)\, m_2(B_j) < 1$, then define basic probability assignment function m：$2^\theta \to [0,1]$ as

$$m(A) = \begin{cases} 0 & A = \emptyset \\ \frac{\sum_{A\cap B=A} m_1(A_i) m_2(B_j)}{1-\sum_{A\cap B=\emptyset} m_1(A_i) m_2(B_j)} & A \neq \emptyset \end{cases} \quad (3)$$

where $m=m_1 \oplus m_2$. Set $K=\sum_{A\cap B=\emptyset} m_1(A_i)\, m_2(B_j)$. K in this formula represents the degree of conflict between two evidences. When K=1, the conflict between two evidences is complete, meaning that the evidence theory cannot be used. When $K< 1$, the conflict between two evidences is not complete, meaning that the evidence theory can be used, but the larger the K is, the higher the degree of conflict grows, and the greater the possibility of producing an erroneous result becomes.

2.The combination rule of multiple belief functions. Probability assignment functions of multiple belief functions' combinations are $m=m_1 \oplus \ldots \oplus m_i$. The corresponding focal element of m_i is A_i, similar to the combination rule of two belief functions. Set $U=A_1 \cap A_2 \cap \ldots \cap A_i$, $\forall A \subset \theta$, m：$2^\theta \to [0,1]$which can be defined as

$$m(A) = \begin{cases} 0 & A = \emptyset \\ \frac{\sum_{U=A} m_1(A_1) m_2(A_2) \ldots m_i(A_i)}{1-\sum_{U=\emptyset} m_1(A_1) m_2(A_2) \ldots m_i(A_i)} & A \neq \emptyset \end{cases} \quad (4)$$

Disadvantages of evidence theory

Though evidence theory holds many advantages, it also has its internal limitations, which tend to cause various problems during practice. This paper summarizes the disadvantages of evidence theory by analyzing the following examples.

Ex 1: Assume that the frame of discernment is θ={A,B,C}, and the basic probability assignments of two sensors are:

D1：$m_1(A)=0.99$，$m_1(B)=0.01$，$m_1(C)=0$

D2：$m_2(A)=0$，$m_2(B)=0.01$，$m_2(C)=0.99$

These probability assignments show that these two probability assignments respectively supported by two sensors are quite contradictory; and the evidences are strongly conflicting with each other as well. In that case, it can be calculated that K=0.9999，m(A)=0，m(B)=1，m(C)=0. The output results of these two sensors demonstrate a low support degree towards B. However, B is definite to happen after combination, which apparently makes no sense.

In this example, if the amount of sensors is changed from two to n with D1=D3=…=Dn, all the other n-1 evidences hold a rather high support degree towards A except D2. Nevertheless, B happens again inevitably. This is the typical "veto with a single vote" in evidence theory. Therefore, the first disadvantage of evidence theory is that no reasonable results can be obtained when evidences are conflicting.

Ex 2: Conditions remain the same as those in Ex 1, except that D1 is changed into:

D1：$m_1(A)=0.98$，$m_1(B)=0.01$，$m_1(C)=0.01$

The probability assignment of A decreases by 1% and that of B increases by 1%, which are the only two changes in this example compared with those conditions in Ex 1. After being combined by evidence theory, the result is calculated as K=0.99，m(A)=0，m(B)=0.01, m(C)=0.99, directly opposite to that of Ex 1. A subtle adjustment on probability assignment leads to such a considerable change on decision results, indicating that the second disadvantage of evidence theory is its instability and sensitivity towards subtle changes[3].

Ex 3: Assume that the frame of discernment is θ={A,B,C}, so the basic probability assignments of two sensors are:

$$\text{D1}：m_1(A)=0.8，m_1(\theta)=0.2$$
$$\text{D2}：m_2(A,B)=0.6，m_2(\theta)=0.4$$

The combination results are

$$m(A) = \frac{0.8\times0.6+0.8\times0.4}{0.8\times0.6+0.8\times0.4+0.2\times0.6+0.2\times0.4} = 0.8$$

$$m(A, B) = \frac{0.6\times0.2}{0.8\times0.6+0.8\times0.4+0.2\times0.6+0.2\times0.4} = 0.12$$

$$m(\theta) = \frac{0.2\times0.4}{0.8\times0.6+0.8\times0.4+0.2\times0.6+0.2\times0.4} = 0.08$$

If evidence D2 is changed into D2: $m_2(A,B,C)=0.6$， $m_2(\theta)=0.4$, the combination result of $m(A,B,C)=0.12$ remains the same. This is the third disadvantage of evidence theory: the sizes of subsets are indistinguishable.

Improved D-S evidence theory

Determination of evidence reliability

Definition 4.1 θ is a frame of discernment, containing n propositions. P(θ) is the space composed by all subsets of θ. Set U as a space composed of elements in P(θ). If the elements of U remain in U after linear combination, U is proved to be evidence focal element vector space with elements $\{A_1, A_2, \ldots, Am\}$ of P(θ) as its basis. If $v \subset U$, then

$$v = \sum_{i=1}^{m} a_i A_i，(a_i \in R，i = 1,2, \ldots m) \tag{5}$$

Definition 4.2 θ is a frame of discernment, containing n propositions. P(θ) is the space composed by all subsets of θ. Taking $m(A_i)$ as its coordinate, vector m in P(θ) is a basic probability assignment. Therefore, m can be represented as

$$m=[m(A_1), m(A_2), \ldots, m(A_i)] \text{ and } \sum_{i=1}^{m} m(A_i) = 1$$

Definition 4.3 θ is a frame of discernment, containing n propositions. m_i and m_j are two basic probability assignments of θ. Therefore, the distance between m_i and m_j can be represented as

$$d_{ij} = \sqrt{\frac{(m_i\text{-}m_j)^T D(m_i\text{-}m_j)}{2}} \tag{6}$$

D is an m×m matrix, whose elements are:

$$D(Ai，Aj)=\frac{|A_i \cap A_j|}{|A_i \cup A_j|}，\ i,j=1,2\ldots m \tag{7}$$

Then it can be obtained that

$$d_{ij} = \sqrt{\frac{\|m_i\|^2+\|m_j\|^2-2\langle m_i,m_j\rangle}{2}} \tag{8}$$

$\|m_i\|^2=\langle m_i, m_i\rangle$， and $\langle m_i, m_i\rangle$ is the inner product of vector m_i and vector m_j.

$$\langle m_i, m_j\rangle = \Sigma\Sigma m(A_i)m(A_j)\ \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \tag{9}$$

Set the number of evidence sources as n. The distance between evidences can by calculated by using above formulas. The larger the distance value between two evidences is, the larger their conflict value becomes. On the contrary, the smaller the distance value is, the smaller the conflict value becomes. Distance value can be represented by a matrix:

$$D = \begin{bmatrix} 0 & d_{12} & \dots & d_{1n} \\ d_{21} & 0 & \dots & d_{2n} \\ \vdots & \vdots & & \vdots \\ d_{n1} & d_{n2} & \dots & d_{nn} \end{bmatrix} \quad (10)$$

Assume that the average Euclidean distance between an evidence set and an evidence is S. Thus, S reflects the difference degree between the evidence and other evidences in the evidence set. That is

$$S = \frac{1}{n-1}\sum_{j=1}^{n} d_{ij}\ (0 \le S \le 1) \quad (11)$$

When evidence A_i and other evidences in the evidence set strongly conflict with each other, the value of S becomes larger. The reliability and combination weight coefficient of A_i decline compared with other evidences. Set ai as the weight coefficient of A_i. According to a similar method on defining reliability (Sun et al. 2000), the weight set composed by all weight coefficients of evidence set can be concluded as

$$a=(a_1, a_2 \cdots a_i),\ \ 0 < a_i \le 1$$

The relation between ai and S is represented by function f(S), $0 < f(S) \le 1$..

$$a_i=f(S)=(1-S)b^{-s} \quad (12)$$

b is a constant coefficient.

If you take a derivative of f(S), you can get

$$f'(S)=(S\ln b-\ln b-1)b^{-s} \quad (13)$$

If you take the second derivative of f(S), you can get

$$f''(S)=b^{-s}\ln b(2-S\ln b+\ln b) \quad (14)$$

The reliability $\varepsilon = e^{-k}$, within which k represents the degree of conflict.(Sun et al. 2000) From this it is clear that weight coefficient a_i and average Euclidean distance S also have a similar relation, i.e. $f'(S) < 0$ and $f''(S) < 0$. Therefore, the inequalities are

$$\begin{cases} (S\ln b - \ln b - 1)b^{-s} < 0 \\ b^{-s}\ln b(2 - S\ln b + \ln b) < 0 \end{cases} \quad (15)$$

Solve them and get $e^{-1} \le b < 1$

Take $b=e^{-1}$ and substitute the value of b into formula 8. Get

$$a_i=f(S)=(1-S)e^{s} \quad (16)$$

Determination of combination algorithm

According to definition 4.1, the conflict evidence can be preprocessed after obtaining the weights of all evidences. Set mi as the basic probability assignment function of conflict evidence, m_i' as the basic probability assignment function after preprocess, then

$$m_i'=a_i m_i \quad (17)$$

Meanwhile, the combination rule is changed into

$$\begin{cases} m_i'(A_i) = m_i(A_i) \times a_i \\ m_i'(\theta) = 1 - \sum_{i=1}^{n} m_i(A_i) \times a_i \end{cases} \quad (18)$$

Analysis of examples

During the process of monitoring the current of class I solid-state relay of rockets, data collected from former experiments can be used as evidence to determine current state of the relay. Letters A, B and C stand for normal, low and high current states, and compose a frame of discernment θ={A,B,C}. Therefore, evidence set D, consisted of two groups of historical data, is used to monitor the current.

D_1： $m_1(A)=0.98$， $m_1(B)=0.01$， $m_1(C)=0.01$

D_2： $m_2(A)=0$， $m_2(B)=0.01$， $m_2(C)=0.99$

With D-S evidence theory, it can be calculated that

$m(A)=0$， $m(B)=0.01$， $m(C)=0.99$， $m(\theta)=0$

According to the improved D-S evidence theory, it can be calculated that

Weight coefficients: $a_1=0.0533$， $a_2=0.0533$

$$m(A)=0.0495，m(B)=0.01，m(C)=0.0506，m(\theta)=0.8987$$

From the above results, it can be known that evidences D_1 and D_2 conflict with each other and the results directly obtained by D-S evidence theory apparently support D_2, which is unreasonable. While the improved D-S evidence theory shows that two weight coefficients are only 0.0533, indicating both of them have a little impact on the decision. Besides, the final results also illustrate that the uncertain component takes up 89.87% while the probabilities on supporting A and C only occupy about 5% each. Therefore, it is not a suitable time to make decision until more evidences are added.

In that case, introduce evidence D_3: $m_2(A)=0.90$，$m_2(B)=0$，$m_2(C)=0.10$. For the purpose of illustrating the advantages of improved algorithm proposed in this paper, compare it with another similar improved algorithm by applying a different improved method (Sun et al. 2000)

According to D-S evidence theory, it can be calculated that

$$m(A)=0，m(B)=0.01，m(C)=0.99，m(\theta)=0$$

According to another improved method (Sun et al. 2000), it can be calculated that

$$m(A)=0.3206，m(B)=0.0034，m(C)=0.1876，m(\theta)=0.4875$$

According to the algorithm of this paper, it can be calculated that

$$\text{Weight coefficients: } a_1=0.7960，a_2=0.1595，a_3=0.8323$$

$$m(A)=0.9308，m(B)=0.0015，m(C)=0.0311，m(\theta)=0.0376$$

It can be seen that evidence D_3's probability assignment on A is 0.9, implying its high degree of support to A. But the results directly obtained through evidence theory show little changes compared with those before the introduction of D_3. The results obtained by another improved method (Sun et al. 2000) hold 32% to the support for A, but 48.75% to the uncertain component. It is still inappropriate to use these results as final decision results. The results obtained through the algorithm proposed by this paper hold 93.08% to the support for A, and show that the uncertainty interval is only 0.0376. Moreover, due to strong support between evidence D_1 and D_2, their weights in evidence set are also high, while the reliability of D_2 is comparatively low. Therefore, these results can be regarded as final decision results.

Furthermore, introduce D_4: $m_2(A)=0.91$，$m_2(B)=0$，$m_2(C)=0.09$. Evidence D_4 also supports A. The results calculated directly by evidence theory are:

$$m(A)=0，m(B)=0.01，m(C)=0.99，m(\theta)=0$$

Obviously, no matter how many new evidences are added, the conflicting results of existing evidences cannot be solved. Then let's see the results obtained through another improved method (Sun et al. 2000):

$$m(A)=0.4037，m(B)=0.0029，m(C)=0.1758，m(\theta)=0.4139$$

And the results calculated by the method in this paper are

$$\text{Weight coefficients: } a_1=0.9046，a_2=0.1930，a_3=0.9334，a_4=0.9323$$

$$m(A)=0.9959，m(B)=0，m(C)=0.0036，m(\theta)=0.0005$$

Compared with the results before the introduction of D_4, the support for A calculated by another improved method (Sun et al. 2000) increases a little, but the convergence speed is rather slow because of lack of evidences. As for the algorithm of this paper, its results show that evidence D_3 and evidence D_4 both support A, so they possess nearly the same weight coefficients as D_1, indicating all these three evidences occupy equally important positions. Meanwhile, D_2 conflicts with all the other three evidences, consequently holding a lower weight coefficient and a weaker impact on decision results. This method not only confirms the existence and impact of conflict evidence D_2, but also sets limits to D_2 to prevent it from affecting decision results. From the perspective of basic probability assignment, the support for A is increased to 0.9959 with D_4 being added into this case, while the uncertainty interval is decreased to 0.0005. In accordance with such results, it is completely reasonable to make decision now. This method enjoys a fast convergence speed as well as a strong anti-interfering ability, enabling itself to provide more reliable decision results even with a few evidences.

Conclusion

Based on the analysis and summary of the disadvantages of evidence theory, this paper puts forward a new evidence combination method, starting from the source of evidence. This method neither denies nor neglects the conflict evidence's impact on final results, but sets weight coefficients of all evidences, limits or utilizes conflict evidences in an objective way and makes decision even with a few evidences. In addition, this method attaches great importance to the relation among evidences. It connects every individual evidence together and finds out their degrees of importance in evidence set by determining their weight coefficients. During this whole process, no subjective factor is involved, ensuring the final results to be objective and practical.

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