Price Decision Models of a Manufacturer-retailer Supply Chain Based on Game Theory

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***Abstract*—This paper focuses on the pricing decision problem of a two-echelon supply chain consisting of one manufacturer and two retailers in which the manufacturer acts as a leader. The price decision models are developed by considering the sensitivity of the retail quantity to the wholesale price of the manufacturer and the sale prices of the retailers. The optimal decisions are discussed and analyzed under retailers' different game behavior: Cournot, Collusion and Stackelberg. The research indicates that the duopolistic retailers could charge the highest price to customers and gain the greatest profit in the Collusion solution and it is reverse in the Cournot solution, but the leader's profit may be fewer than the follower in the Stackelberg situation**

Keywords-manufacturer-retailer supply chain; pricing; game behaviour.

I. INTRODUCTION

An important issue of Supply Chain Management is how the competitive behavior affects the pricing decisions in a two-echelon supply chain. Many models have been developed on this research. Most of the research paid close attention to the following Stackelberg structure: a manufacture acts as a Stackelberg leader and a retailer acts as a Stackelberg follower. Quantity discount scheme is added into the two-echelon system which is composed with one supplier and one retailer under the manufacturer-Stg situation in [1]. The two-echelon system is further extended to study with a single supplier and several different buyers when the incremental quantity discount policies are considered in [2]. It is found that the discount policy is impartial to both the supplier and the retailer. Then it is pointed out that a suitable quantity discount scheme could be offered by the manufacturer to the retailer in the manufacturer-Stg process in [3]. It is presented that the scheme benefits the manufacturer. It is studied that different demand affects on the optimal solution in a two-echelon supply chain in the manufacturer-Stg situation. It is confirmed that the profit of manufacturer is the double of the retailer's when a price-versus-demand relationship exists of a two-echelon supply chain in [4]. Other related researches in recent years are in [5-9].

However, most of the literature considered only one single retailer or different independent retailers when the market demand function is linear. Some significant factors are seldom explored, such as the influence from the wholesale price of the manufacturer and the sale prices of the retailers, and the situation which retailers play Stackelberg. Thus, it is researched that a supply chain formting of one manufacturer and two retailers in this paper. It is the main objective that the profits are maximized of the manufacturer and retailers on the optimal wholesale price and sale price. The following contents would be fully considered: (i) The retailer's retail quantity is not only related with market demand but sensitive to the wholesale price of the manufacturer and the sale prices of the retailers. (ii) Competition of retailers is divided in three situations: Cournot situation, Collusion situation and Stackelberg situation.

The rest of this paper is organized as follows. The basic profit models are developed of the manufacturer and retailers in section 2. In section 3, the solutions of the models are obtained. In section 4, the optimal policy is analyzed. A conclusion is presented of the whole paper in section 5.

II. BASIC MODEL

In this paper, a supply chain composed of one manufacturer and two retailers is considered in which the manufacturer sells a single product to the retailers. The mechanism in two-echelon supply chain assumes the manufacturer as a leader and sets the wholesale prices to the two retailers which is called the manufacturer-Stg process. Then, the two retailers set their retail price and corresponding order quantity independently competing in a common market. The retail quantity is determined by the market demand, the wholesale price of the manufacturer and the sale prices of the retailers. For simplification, we ignore the inventory cost and transportation cost of the retailers.

According to the above assumptions, the model is built using the following notation:

G_{R_i} retailer-i's profit, i=1, 2;

G_M manufacture's profit;

p_i the sale price charged to customers by retailer-i, decision variable;

w the wholesale price per unit charged to retailers by the manufacturer;

c production cost per unit;

Q_i retailer-i's retail quantity (or order quantity), decision variable.

Q sum of all retailers' order quantities, i.e. $Q=\sum_{i=1}^{N} Q_i$.

The order quantity of retailer-i is defined as

$$Q_i = D_i - \alpha_i w - \beta_i p_i + \theta p_j \quad i \neq j, \quad i,j = 1,2 \tag{1}$$

The parameter $D_i > 0$, $\alpha_i > 0$, and $0 < \theta < \beta_i$. D_i denotes the market demand of retailer-i's when retail prices are zero and $w = w_0$. α_i is the sensitivity that the wholesale price affects each retail quantity. β_i is the sensitivity that the retailer-i's price affects its sales quantity. θ is the substitution degree between retailers, which reflects the impact on customer demand in the retail marketing mix decision.

Thus, the retailer- i 's profit and the manufacture's profit could be generated through the following formula

$$G_{R_i} = (p_i - w)Q_i = (p_i - w)[D_i - \alpha_i w - \beta_i p_i + \theta p_j] \tag{2}$$

$$G_M = (w-c)Q = (w-c)(Q_1 + Q_2) \tag{3}$$

III. Solution of Model

In this section, the above model will be optimized when the retailers compete in different way. The following three situations are mainly focused on.

(1) Cournot situation, i.e., the sale price and quantity are independently set by each retailer merely assuming his competitor's sale price as a parameter.

(2) Collusion situation, i.e., to maximize the downstream retail market's total profit, both retailers are glad to determine their sale prices together.

(3) Stackelberg situation.

A. Optimization of model when the two retailers act in the Cournot situation.

In this subsection, a supply chain is consider when the two retailers act in the Cournot solution. The retailer-i's profit only depends on his sale price when the manufacturer's wholesale unit price is fixed. Then, retailer-i can maximize G_{R_i} with regard to p_i, taking p_j as a parameter, $i \neq j, \quad i,j = 1,2$. Thus, the optimal sale prices (p_1^* and p_2^*) could be obtained of retailer-1 and retailer-2 through solving $dG_{R_1} / dp_1 = 0$ and $dG_{R_2} / dp_2 = 0$.

$$p_1^* = \frac{2\beta_2[D_1 + (\beta_1 - \alpha_1)w] + \theta[D_2 + (\beta_2 - \alpha_2)w]}{4\beta_1\beta_2 - \theta^2} \tag{4}$$

$$p_2^* = \frac{2\beta_1[D_2 + (\beta_2 - \alpha_2)w] + \theta[D_1 + (\beta_1 - \alpha_1)w]}{4\beta_1\beta_2 - \theta^2} \tag{5}$$

When (4) and (5) is fed into (1), the optimal retail quantities Q_1^* and Q_2^* could be easily obtained.

(4)-(5) show the optimal reaction functions of the two duopolistic retailers for any w set by the manufacturer. The retailers' reaction functions can be known by the manufacturer for any w -value that she has set. Hence from (3), R_1 is easily obtained which is the sign of the second-order derivative of G_M with respect to w, where

$$R_1 = (\beta_1 + \beta_2)(\theta^2 - \beta_1\beta_2) + \beta_1\beta_2(2\theta - \beta_1 - \beta_2) - 2\beta_1\beta_2(\alpha_1 + \alpha_2) - \theta(\alpha_1\beta_2 + \alpha_2\beta_1) \tag{6}$$

Since $0 < \theta < \beta_i$ (i=1, 2), it is very obviously that R_1 is below zero. It implies that G_M is a concave function of w. So the optimal wholesale price set by the manufacturer can be obtained by solving $dG_M / dw = 0$

$$w^* = \frac{c}{2} - \frac{\beta_2(2\beta_1 + \theta)D_1 + \beta_1(2\beta_2 + \theta)D_2}{2R_1} \tag{7}$$

Then the optimal policies and the corresponding profits of the retailers and the manufacturer will be easily yield.

B. Optimization of model when the duopolistic retailers act in the collusion situation.

It is assumed that the retailers want to act in alliance in the interest of maximizing their profit in this subsection. The downstream retail market's total profit is

$$\begin{aligned} G_R &= G_{R_1} + G_{R_2} \\ &= (p_1 - w)[D_1 - \alpha_1 w - \beta_1 p_1 + \theta p_2] + (p_2 - w)[D_2 - \alpha_2 w - \beta_2 p_2 + \theta p_1] \end{aligned} \tag{8}$$

One can easily obtain the optimal sale prices of the two retailers by solving equations $dG_{R_1} / dp_1 = 0$ and $dG_{R_2} / dp_2 = 0$

$$p_1^* = \frac{\beta_2 D_1 + \theta D_2 + (\alpha_1\beta_2 + \alpha_2\theta + \beta_1\beta_2 - \theta^2)w}{2(\beta_1\beta_2 - \theta^2)} \tag{9}$$

$$p_2^* = \frac{\theta D_1 + \beta_1 D_2 + (\alpha_2\beta_1 + \alpha_1\theta + \beta_1\beta_2 - \theta^2)w}{2(\beta_1\beta_2 - \theta^2)} \tag{10}$$

The optimal order quantities of the two retailers Q_1^* and Q_2^* can be obtained by Substituting the above results into (1).

Thus, the profit of manufacturer is

$$G_M = (w-c)[D_1 + D_2 - (\alpha_1 + \alpha_2)w - (\beta_1 + \beta_2 - 2\theta)w]/2 \tag{11}$$

It can be observed that G_M is a concave function with respect to w. So the optimal wholesale price could be obtained by Solving $dG_M / dw = 0$

$$w^* = \frac{c}{2} + \frac{D_1 + D_2}{2(\alpha_1 + \alpha_2 + \beta_1 + \beta_2 - 2\theta)} \tag{12}$$

At last, the optimal pricing policies can be obtained by combining (12) and with (9), (10). And the corresponding profits of the duopolistic retailers and the manufacturer are achieved.

C. Optimization of model when the two retailers act in the Stackelberg situation.

In this subsection, it is supposed that one of the two retailers (e.g., retailer-1) acts as a leader and the other (i.e., retailer-2) acts as the follower in the Stackelberg situation. The following reaction function can be get of Retailer-2 by $dG_{R_2} / dp_2 = 0$.

$$p_2 = [D_2 + (\beta_2 - \alpha_2)w + \theta p_1]/2\beta_2 \tag{13}$$

To maximize the profit, the retailer-2 adjusts his sale price on the basis of the retailer-1's pricing decision.

Retailer-1’s profit can be maximized based on the reaction function of retailer-2. Substituting (13) into profit function of retailer-1, It can be get

$$G_{R_1}=\frac{(p_1-w)[2\beta_2 D_1+\theta D_2-(2\alpha_1\beta_2+\theta\alpha_2-\theta\beta)w-(2\beta_1\beta_2-\theta^2)p_1]}{2\beta_2} \quad (14)$$

From (14),it can be derived that the profit of retailer-1 merely is a function with respect to p_1 and the function is concave. So, the optimal sale price of retailer-1 can be obtained by solving $dG_{R_1}/dp_1=0$.

$$p_1^*=\frac{2\beta_2 D_1+\theta D_2+(\beta_2\theta+2\beta_1\beta_2-\theta^2-2\alpha_1\beta_2-\alpha_2\theta)w}{2(2\beta_1\beta_2-\theta^2)} \quad (15)$$

Then the retailer-2’s optimal sale price can be easily determined.

$$p_2^*=\frac{2\beta_2\theta D_1+(4\beta_1\beta_2-\theta^2)D_2+(4\beta_1\beta_2^2+2\beta_1\beta_2\theta+\alpha_2\theta^2-\beta_2\theta^2-2\alpha_1\beta_2\theta-4\alpha_2\beta_1\beta_2-\theta^3)w}{4\beta_2(2\beta_1\beta_2-\theta^2)} \quad (16)$$

The retailers’ reaction functions is known by the manufacturer, which is given by (15) and (16) for any set w-value, so the profit of the manufacturer will be

$$\begin{aligned} G_M&=(w-c)(Q_1^*+Q_2^*) \\ &=(w-c)[D_1+D_2-(\alpha_1+\alpha_2)w-(\beta_1-\theta)p_1^*-(\beta_2-\theta)p_2^*] \end{aligned} \quad (17)$$

Then R_2 can be obtained which is just contrary to the sign of the second-order derivative of (17) with respect to w, where

$$\begin{aligned} R_2=&4\alpha_1\beta_1\beta_2^2+4\alpha_2\beta_1\beta_2^2+2\alpha_1\beta_2^2\theta-2\alpha_1\beta_2\theta^2+2\alpha_2\beta_1\beta_2\theta-\alpha_2\beta_2\theta^2 \\ &-\alpha_2\theta^3+4\beta_1^2\beta_2^2+4\beta_1\beta_2^3-4\beta_1\beta_2^2\theta-4\beta_1\beta_2\theta^2-3\beta_2^2\theta^2+2\beta_2\theta^3+\theta^4 \end{aligned} \quad (18)$$

Due to $0<\theta<\beta_i$ (i=1, 2), it can be easily deduced $R_2>0$. Thus, G_M is also a concave function of w. Then, we can obtaine the optimal wholesale price by solving $dG_M/dw=0$

$$w^*=\frac{c}{2}+\frac{2\beta_2(2\beta_1\beta_2+\beta_2\theta-\theta^2)D_1+(4\beta_1\beta_2+2\beta_1\beta_2\theta-\beta_2\theta^2-\theta^3)D_2}{2R_2} \quad (19)$$

At last, the optimal pricing policies can be get by combining (19) with (15), (16). Then the two retailers and the manufacturer’s maximum profits can be inferred.

IV. ANALYSIS OF THE OPTIMAL SOLUTIONS IN THE THREE COMPETITIVE SITUATION

In this section, the optimal solutions are compared which are presented above in three competitive situation.

A. A special case with $D_1=D_2=D$, $\alpha_1=\alpha_2=\alpha$ *and* $\beta_1=\beta_2=\beta$.

In this subsection, a special case is first considered with $D_1=D_2=D$, $\alpha_1=\alpha_2=\alpha$ and $\beta_1=\beta_2=\beta$. It can be described that the market demand of the two retailers are similar. The optimal solutions are showed in table 1 in the special case.

TABLE I. THE OPTIMAL SOLUTIONS UNDER THE CASE WITH $D_1=D_2=D$, $\alpha_1=\alpha_2=\alpha$ AND $\beta_1=\beta_2=\beta$

	Cournot	Collusion	Stackelberg
p_1^*	$\frac{(\beta-\alpha)(\alpha+\beta-\theta)c+(\alpha+3\beta-2\theta)D}{2(\alpha+\beta-\theta)(2\beta-\theta)}$	$\frac{(\alpha+\beta-\theta)c+3D}{4(\beta-\theta)}$	$\frac{(2\alpha\beta+\alpha\theta+6\beta^2-\beta\theta-3\theta^2)D+(\alpha+\beta-\theta)[(2\beta-\theta)(\beta-\theta)-\theta^2]c}{4(\alpha+\beta-\theta)(2\beta^2-\theta^2)}$
p_2^*	$\frac{(\beta-\alpha)(\alpha+\beta-\theta)c+(\alpha+3\beta-2\theta)D}{2(\alpha+\beta-\theta)(2\beta-\theta)}$	$\frac{(\alpha+\beta-\theta)c+3D}{4(\beta-\theta)}$	$\frac{(2\alpha\beta\theta-7\beta\theta^2+12\alpha\beta^2+12\beta^3-2\beta^2\theta-3\alpha\theta^2+\theta^3)D+(\alpha+\beta-\theta)[4\alpha\beta^2-2\alpha\beta\theta-\alpha\theta^2+4\beta^3+2\beta^2\theta-\beta\theta^2-\theta^3]c}{8\beta(\alpha+\beta-\theta)(2\beta^2-\theta^2)}$
w^*	$\frac{(\alpha+\beta-\theta)c+D}{2(\alpha+\beta-\theta)}$	$\frac{(\alpha+\beta-\theta)c+D}{2(\alpha+\beta-\theta)}$	$\frac{(\alpha+\beta-\theta)c+D}{2(\alpha+\beta-\theta)}$
$G_{R_1}^*$	$\frac{\beta[(\alpha+\beta-\theta)c+D]^2}{4(2\beta-\theta)^2}$	$\frac{[(\alpha+\beta-\theta)c-D]^2}{16(\alpha+\beta-\theta)^2}$	$\frac{(2\beta+\theta)^2[(\alpha+\beta-\theta)c-D]^2}{32\beta(2\beta^2-\theta^2)}$
$G_{R_2}^*$	$\frac{\beta[(\alpha+\beta-\theta)c+D]^2}{4(2\beta-\theta)^2}$	$\frac{[(\alpha+\beta-\theta)c-D]^2}{16(\alpha+\beta-\theta)^2}$	$\frac{(4\beta^2+2\beta\theta-\theta^2)[(\alpha+\beta-\theta)c-D]^2}{64\beta(2\beta^2-\theta^2)}$
G_M^*	$\frac{\beta[(\alpha+\beta-\theta)c+D]^2}{(2\beta-\theta)(\alpha+\beta-\theta)}$	$\frac{[(\alpha+\beta-\theta)c-D]^2}{4(\alpha+\beta-\theta)^2}$	$\frac{(8\beta^3+6\alpha^2\theta+4\beta^2\theta+3\beta\theta^2-\theta^3)[(\alpha+\beta-\theta)c-D]^2}{16\beta(\alpha+\beta-\theta)(2\beta^2-\theta^2)}$

Then we can obtain the following inferences from table 1.

(1) The optimal pricing policy of the manufacturer is not affected by the duopolistic retailers' different game behaviors, but the duopolistic retailers' optimal pricing policies have rather obvious changes.

(2) Among different situations, Collusion can make the duopolistic retailers charge the highest price to customers and gain the greatest profit. Cournot leads to the lowest retail price setting from the duopolistic retailers which earns the lowest profit.

(3) As the Stackelberg leader, the manufacturer's profit will always be more than that of the duopolistic retailers' in the supply chain.

(4) Facing similar market demand, the duopolistic retailers' action in union is beneficial only to themselves, whereas their non-cooperation will be profitable to the manufacturer.

B. The general case with $D_1\neq D_2$, $\alpha_1\neq\alpha_2$ *or* $\beta_1\neq\beta_2$.

This case can be explained that the market demand of the duopolistic retailers is dissimilar. Because the calculation is very complicated, it is difficult to present analytical results. So, in this subsection, three numerical examples will be directly done. In table 2, the assumed parameters are listed. It is described in Example 1 that the retailer-2 has less sensitivity to the wholesale price than retailer-1 and the other parameters have the equal values. It is represented in Example 2 that the retailer-2 has less market demand than retailer-1. It is showed in Example 3 that the retailer-2 has greater sensitivity to his retail price than retailer-1. The numerical results are present in table 3.

From table 3, the following inferences can be observed.

(1) The retailers' different competitive behaviors lead the manufacturer charge different wholesale prices. And

the variation of wholesale price mainly depends on the demand and has no regularity.

(2) Among all three kinds of game behaviors that the retailers may choose, the collusion behavior will lead to the highest pricing of the retailer-1 and the lowest pricing of the retailer-2.

(3) Collusion makes the duopolistic retailers charge the highest price to customers and gain the greatest profit. Cournot results in the lowest retail price and retailers obtain the lowest profit.

(4) In the Stackelberg case, the leader (retailer-1) may not gain greater profit than the follower (retailer-2). The retailer-1's profit is greater than the retailer-1's only when the value of D_1/β_1 is greater than D_2/β_2. This proves that Stackelberg may not benefit the leader.

TABLE II. DATA FOR NUMERICAL EXAMPLES

examples	D_1	D_2	α_1	α_2	β_1	β_2	θ	c
I	30	30	3	2	2	2	1	1
II	30	25	3	3	2	2	1	1
III	30	30	2	2	1.5	2	1	1

TABLE III. THE OPTIMAL SOLUTION UNDER COURNOT, COLLUSION AND STACKELBERG SITUATIONS

		p_1^*	p_2^*	w^*	G_{R1}^*	G_{R2}^*	G_M^*
Cournot	I	9.26	10.81	9.79	0	10.81	8.79
	II	8.96	7.99	8.13	1.39	0	9.50
	III	13.24	11.81	12.17	1.72	0	9.90
Collusion	I	13.08	11.67	5.50	15.17	59.38	93.38
	II	12.48	8.65	4.69	28.24	31.58	72.83
	III	18.02	12.21	6.23	20.24	19.7	114.3
Stackelberg	I	9.63	10.91	8.63	1.74	10.35	48.05
	II	9.29	8.20	7.50	5.61	0.98	29.47
	III	13.85	11.96	10.50	14.03	4.28	67.57

V. CONCLUSION

By considering the duopolistic retailers' three kinds of game behavior: Cournot, Collusion and Stackelberg, our work further extended the existing two-echelon supply chain models. The manufacturer-retailer supply chain system is composed of one manufacturer and two competitive retailers where the manufacturer leads the two retail followers. Then, the optimal pricing and quantity decisions were discussed under different game behaviors of the duopolistic retailers. The results shows that (i) if act in collusion, the duopolistic retailers should charge higher sale prices than under other situations and it is reverse if the retailers pursue the Cournot solution; (ii) In collusion situation, the duopolistic retailers could archieve the highest profit and it is reverse if the retailers pursue the Cournot solution; (iii) when the duopolistic retailers play Stackelberg game in the downstream market, the leader's profit may not be greater than the follower's, and it may exceed that of the follower's only when the leader's value of D_i/β_i is greater than that of the follower's.

In this paper, a simplified model of supply chain is studied. However, in practice, there may be several manufacturers and retailers, and the inventory cost and transportation cost should also be considered. These problems may need for further research.

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