Searching Shortest Path in a Network Using Modified A^* Search Algorithm

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***Abstract*—The aim of this work is to explore the application of Trapezpoidal Intuitionistic Fuzzy Numbers (TrIFNs) in afuzzy environment to find the least cost path problem of any network. A New Intuitionistic Trapezoidal Fuzzy A* Algorithm (NITFAA) is proposed to solve Intuitionistic Fuzzy Shortest Path Problem (IFSP) in a directed network using the Intuitionistic Fuzzy Ordered Weighted Geometric (ITFOWG) aggregation operator. The parameters associated with each nodes and edges are represented as TrIFNs which is the most generalized form of Trapezoidal Fuzzy Numbers (TrFNs) that contains degree of acceptance as well as degree of rejection.**

Keywords-intuitionistic fuzzy sets; trapezoidal intuitionistic fuzzy numbers; A* algorithm.

I. INTRODUCTION

The shortest path problem concentrates on finding a shortest distance between a pair of nodes with certain parameters like cost, time etc. In literature there are many approaches to solve Shortest Path Problems (SPP) in fuzzy graph **[3-6, 9-16, 18]**. In this paper, we have proposed a new A^* algorithm in uncertain environment (intuitionistic fuzzy) to find the least cost path or shortest path with respect to total intuitionistic fuzzy cost of a given network from selected node to goal node. After the introduction of fuzzy sets by Zadeh **[20]**, a number of applications of fuzzy set theory came into existence. The notion of intuitionistic fuzzy sets (IFSs) was introduced by Attanssov **[1]**. The main advantage of IFS is its property to cope with the uncertainty that must exist due to the information acheived. This is done by considering a second function named as non-membership function, along with the membership function of conventional fuzzy sets. Thus apart from the degree of the belongingness, the IFSs also combine the idea of non-belongingness in order to describe the real status of the information in an effective way. IFSs as a generalization of fuzzy sets can be used in situations when description of a problem are represented by linguistic variable(s), given in term of a membership function only, seems to be insufficient.

In search methodology, A^* algorithm **[8]** is one type of informed search technique to find a least cost path in a network while traversing from source node to goal node. However, in the literature there is no investigation on SPP using A^* search technique with data in the form of trapezoidal intuitionistic fuzzy numbers. We proposed an improved algorithm for solving the SPP using A^* search technique with intuitionistic fuzzy (IF) heuristic and IF edge parameters.

II. PROPOSED METHOD

In this paper, we have considered directed Intuitionistic Fuzzy (IF) network $\tilde{G}$ having crisp vertex and edge set where $\tilde{G}_v$ is the vertex set and $\tilde{G}_E$ is the edge set. We expressed the heuristic cost associated with every $v_i(\in V)$ and weights w_{ij} associated with every $e_{ij}(\in E)$ connecting node v_i and $v_j$in the form of TrIFNs. In our problem we represent the heuristic cost which is associated with every node v_k and the cost associated with every edge (e_{kj}) of the considered network in the form of TrIFNs $([a,b,c,d];\mu,)$. The heuristic cost is denoted by $\widetilde{H}_{v_k}$ and the edge cost is represented as $\widetilde{w}_{kj}$. Heuristic of every node is calculated based on the opinion of experts and every individual expert's opinion is represented in the form of TrIFN. We use Intuitionistic Trapezoidal Fuzzy Ordered Weighted Geometric $(ITFOWG)$ aggregation operator **[17]** to calculate the aggregated heuristic cost, $Agg_{\widetilde{H}_{v_k}}$form a set of $\widetilde{H}_{v_k}$ where $\tilde{v}_k$ is the k^{th} node of the network under consideration and $\widetilde{H}_{v_k}$is the i^{th} expert opinion about the heuristic cost for vertex v_k. We have assumed that aggregated heuristic cost, $Agg_{\widetilde{H}_{v_k}}$associated with each node $\tilde{v}_k$ has the monotone property. A monotone heuristic is such, that for any node v_n which is the successor of node v_m on a path to the goal node then$(Agg_{\widetilde{H}_{v_m}} - Agg_{\widetilde{H}_{v_n}}) \leq \widetilde{\boldsymbol{w}}_{\boldsymbol{m}\,\boldsymbol{n}}$**;** where $\widetilde{w}_{m\,n}$ is the intuitionistic fuzzy (IF) weight associated with the edge e_{mn} from v_m to v_n. For a node v_i the IF heuristic cost, $\widetilde{H}_{v_i}$ is represented in the form of $([a_i, b_i, c_i, d_i];\ \mu_{i},{}_{i})$ and the IF weight $\widetilde{w}_{kj}$, associated with every edge e_{kj} is represented as ,$\left([a_{kj}, b_{kj}, c_{kj}, d_{kj}];\ \mu_{kj},{}_{kj}\right)$. For each expert's opinion $\widetilde{H}_{v_i}$, the minimum cost incurred while traversing from v_i through terminal node T , lies between $[a_i, d_i]$. μ_i of $\widetilde{H}_{v_i}$ represent the degree of acceptance that the cost incurs while travelling from v_i through T, will be accepted as minimum with respect to the total cost for travelling from v_i to T . $_i$ of $\widetilde{H}_{v_i}$represent the degree of rejection that the cost incurs while travelling from v_k to T will be rejected as minimum with respect to the total cost of travelling from v_k to T . The cost $(\widetilde{w}_{kj})$ incurred while traversing e_{kj} , lies between $[a_{kj}, d_{kj}]$. μ_{kj} of $\widetilde{w}_{kj}$ represents the degree of acceptance that the edge e_{kj} will be included in the shortest path from source to goal (terminal) node of the network with respect to the total cost incurred while travelling from source node to T. Similarly $_{kj}$ of $\widetilde{w}_{kj}$ represents the degree of rejection that the edge e_{kj} will be rejected from being accepted in the shortest path from source to goal node of the network with respect to the total cost incurred while travelling from source node to T. Maximum aggregated degree of acceptance μ_{ij} and minimum aggregated degree of rejection $_{ij}$ corresponds to the minimum total IF cost while traversing from v_i to T in $\tilde{G}$.

A. Intuitionistic Trapezoidal Fuzzy A* Algorithm (ITFAA)

Input: A simple weighted fuzzy network $\tilde{G} = (\tilde{G}_v, \tilde{G}_E)$ with crisp vertex and edge sets with intuitionistic fuzzy cost associated with the edges and the nodes. S is the source node and T is the goal node.

Output: Minimum cost path with respect to total intuitionistic fuzzy cost path from selected node to goal node while traversing through a network. We consider $\widetilde{H}_{v_x}$ as the heuristic cost for a node v_x, which is the opinion of i^{th} expert. $\widetilde{H}_{v_x}$, represents the minimal IF cost incurred while travelling from v_x to T. We consider T does not have any heuristic cost as it is a terminal node, hence $\widetilde{H}_{v_T} = \langle[0\ 0\ 0\ 0];\ 1.0, 0.0\rangle$. $Agg_{\widetilde{H}_{v_x}}$ is represented as aggregated heuristic cost calculated from the opinion of experts, which is associated with every node v_x of $\tilde{G}$. ω, represents the weight vector, $\omega = (\omega_1, \omega_2, \dots, \omega_i, \dots \omega_n)^T$;where each ω_j is related to $\widetilde{H}_{v_x}$ with $0 \leq \omega_j \leq 1$ and $\sum_{j=1}^{n} \omega_j = 1\ \forall j$.

Step 1: Set $OPEN = \{S\}, CLOSED = \{\emptyset\}, \tilde{C}_{opt}(v_j) = \tilde{0}$ and $\tilde{f}(v_j) = Agg_{\widetilde{H}_{v_j}}$ where S is the set of source node and $v_j \in \tilde{G}_v$.

Step 2: Calculate the aggregated heuristic cost for each vertex v_k of $\tilde{G}$ as, $Agg_{\widetilde{H}_{v_k}} = ITFOWG(\widetilde{H}_{v_{k_1}}, \widetilde{H}_{v_{k_2}}, \dots, \widetilde{H}_{v_{k_p}}, \dots, \widetilde{H}_{v_{k_t}})$ where $\widetilde{H}_{v_{k_p}}$ is the heuristic cost of k^{th} node as per the opinion of p^{th} expert.

Step 3: If $OPEN = \{\varphi\}$ or $(Agg_{\widetilde{H}_{v_m}} - Agg_{\widetilde{H}_{v_n}}) > \widetilde{\boldsymbol{w}}_{mn}$ where v_n is the successor node of v_m for all nodes of the network then the algorithm terminates with no result.

Step 4: If $T \in CLOSED$ then terminate with success.

Step 5: For each successor v_k of v_j if $v_k \notin [OPEN \cup CLOSED]$. Set $\tilde{C}_{opt}(v_k) = \tilde{C}_{opt}(v_j) + \widetilde{w}_{jk}$; where $v_j, v_k \in \tilde{G}_v; \tilde{C}_{opt}(v_j)$ is the sum of optimal edge cost from S to v_j and $\widetilde{w}_{jk}$ is the cost of traversing the edge e_{jk} from v_j to v_k.

Step 6: Set $\tilde{f}(v_k) = \tilde{C}_{opt}(v_k) + Agg_{\widetilde{H}_{v_k}}$.

Step 7: Set $OPEN = OPEN \cup \{v_k\}$.

Step 8: If $v_k \in [OPEN \cup CLOSED]$, set $\tilde{C}_{opt}(v_k) = max\{\tilde{C}_{opt}(v_k),\ \tilde{C}_{opt}(v_j) + \widetilde{w}_{jk}\}$.

Step 9: Set $\tilde{f}(v_k) \coloneqq \tilde{C}_{opt}(v_k) + Agg_{\widetilde{H}_{v_k}}$.

Step 10: For any node v_i of $\tilde{G}$ with $\tilde{f}(v_i) = max(\tilde{f}(v_1), \dots, \tilde{f}(v_i), \dots, \tilde{f}(v_n))\ \forall v_i \in OPEN$. Set $OPEN = OPEN \setminus \{v_i\}\ and\ CLOSED = CLOSED \cup {v_i}^{v_m}$; where v_m is the predecessor node of v_i.

Step 11: Goto Step 4 until $T \in CLOSED$.

Step 12: Trace the path P starting from T in $CLOSED$ by backtracking the label of every node from T.

B. Proof of Admissibility of ITFAA

ITFAA is admissible if it uses the aggregated heuristic cost possessing monotone property. Then $\tilde{f}^*$ cost never decreases where $\tilde{f}^*$ is the actual cost of the optimal solution path of $\tilde{G}$.

We prove the admissibility of ITFAA with the help of some lemmas. We use the following notations while proving the lemmas.

P_{opt}: $(S, v_1, \dots, v_j, \dots, T)$ is the optimal path with S as source node and T as terminal node.

$\tilde{f}(v_m)$ determines the estimated cost of a path P (not optimal) from node S to T via v_m in a network $\tilde{G}$.

$\tilde{f}^*(v_m)$ determines the actual but unknown cost of optimal solution path P_{opt} from node S to T via v_m in a network $\tilde{G}$.

$\tilde{f}^*$ determines the actual but unknown cost of optimal solution path P_{opt} from node S to T in a network $\tilde{G}$.

$\tilde{C}_{opt}(v_m)$ determines the estimated cost of a path P (not optimal) from S to v_m in a network $\tilde{G}$.

$\tilde{C}^*_{opt}(v_m)$ determines the actual cost of the sub optimal solution path of P_{opt} from S to v_m in a network $\tilde{G}$.

$Agg_{\widetilde{H}_{v_m}}$ is the aggregated heuristic cost from node v_m to T associated with the vertex v_m as estimated by a group of experts.

$\widetilde{H}^*_{v_m}$ is the actual cost of the sub optimal solution path of P_{opt} from node v_m to T.

It is to be noted that $\tilde{C}^*_{opt}(v_m)$, $\widetilde{H}^*_{v_m}$ and $\tilde{f}^*(v_m)$ will have maximum degree of acceptance and minimum degree of acceptance as compared to $\tilde{C}_{opt}(v_m)$, $Agg_{\widetilde{H}_{v_m}}$ and $\tilde{f}(v_m)$ with respect to the total cost incurred while traversing P_{opt} in $\tilde{G}$.

Lemma 1: If a path exists from source to goal node then any intermediate node v_j of the optimal path should relocate itself from $OPEN$ to $CLOSED$ state. Furthermore, $\tilde{f}^*$ cost of v_j is not greater than optimal cost.

Proof: For an optimal path, P_{opt}: $(S, v_1, \dots, v_j, \dots, T)$, according to our algorithm S will enter in $OPEN$. Let v_1 is the immediate successor of S. When S relocates from $OPEN$ to $CLOSED$, v_1 is placed in $OPEN$. Continuing in this manner whenever a node from the above path is moved from $OPEN$ to $CLOSED$, the immediate successor of that relocated node is placed in $OPEN$ and if T is relocated from $OPEN$ to $CLOSED$, then ITFAA terminates with the optimal path P_{opt}.

Moreover, since v_j is one of the node which constitute the optimal path P_{opt}, then $\tilde{f}(v_j) = \tilde{C}_{opt}(v_j) + Agg_{\tilde{H}_{v_j}}$

Now as v_j is in P_{opt}, $\tilde{C}_{opt}(v_j) = \tilde{C}^*_{opt}(v_j)$ this implies

$\tilde{f}^*(v_j) = \tilde{C}^*_{Opt}(v_j) + Agg_{\tilde{H}_{v_j}} \leq \tilde{C}^*_{Opt}(v_j) + \tilde{H}^*_{v_m} = \tilde{f}^*(v_j)$. Now, as v_j is a node in P_{opt}, then we have, $\tilde{C}^*_{Opt}(v_j) + \tilde{H}^*_{v_m} = \tilde{f}^*(v_j) \leq \tilde{f}^*(T)$ where,

$$\tilde{f}^*(T) = \tilde{C}^*_{Opt}(T) + \tilde{H}^*_T$$

Lemma 2: If there exists a path from start to goal node, ITFAA finds a path. This is true even if the graph is infinite.

Proof: ITFAA always select a node $v_k$such that $\tilde{f}(v_k)$ has a maximum scoring function. This node v_k eventually relocates from $OPEN$ to $CLOSED$ state. ITFAA then concentrates on all the immediate successor of v_k and compare all their respective $\tilde{f}$ costs. This process continues till the goal node T migrates on to the $CLOSED$ state. ITFAA always explore the successors of particular node v_k with $\tilde{f}(v_k)$ having maximum rank value among all the nodes in $OPEN$ rather than exploring all the successor of the nodes, until T is selected from $OPEN$ to $CLOSED$. In other words, ITFAA always extends the partial solution by increasing the $\tilde{C}^*_{Opt}$ value to a finite positive value, also there is only finite number of partial solutions cheaper than the cost of P_{opt} and these are compared in finite time duration.

This implies ITFAA does not necessarily visits every node in order to search P_{opt} in $\tilde{G}$ but eventually explores those nodes that are responsible for constituting the optimal path P_{opt} from source S to goal node Tin finite amount of times before it terminates even if the size and order of $\tilde{G}$are infinite.

Lemma 3: ITFAA finds the least cost path to the goal node.

Proof: Let us assume that ITFAA terminate with the node $v_m(\neq T)$ with a cost $\tilde{f}^*(v_m) > \tilde{f}^*(T)$. Now when our algorithm tries to explore v_m as per Lemma 1, there must exist a node v_r which constitute P_{opt} such that $\tilde{f}^*(v_r) \leq \tilde{f}^*(T)$. Therefore $\tilde{f}^*(v_r) < \tilde{f}^*(v_m)$ and ITFAA would pick v_r instead of v_m. Thus our assumption is wrong. From this contradiction, ITFAA will always terminate by finding a path having optimal cost.

III. CONCLUSION

In this paper, we have proposed a new algorithm, ITFAA which meets the objective of classical A^* algorithm in uncertain environment. This type of shortest path problem using IF heuristic costs and IF edge cost is unique in the literature. Since there is no other work on A^* algorithm using intuitionistic fuzzy parameters for heuristic cost as well as the edge cost for a network, numerical comparison of this work with other could not be done.In future we will like to extend our algorithm in type-2 fuzzy environment. Also some other methodologies can be proposed to solve this problem and the results may be compared. Computer programs can be developed to implement the proposed methodology for networks with higher order and size.

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