

# A Study of Reservoir Dynamic Analysis Method Based on Different Injection Profile in Petroleum Engineering

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**Abstract**—At present, the physical phenomenon of injection profile plays an important role to improve oil-driving efficiency and oil recovery. A mathematical method is studied and applied to solve related production problems based on Buckley–Leverett theory. In addition, some different injection profile is shown under the condition of the reservoir permeability which always obeys some special distributions. Finally, the phenomenon can be shown by grid modeling method and visualization technology in our research, which can provide a reference for reservoir performance analysis and production estimation.

**Keywords**—energy studies; mathematical modeling; injection profile; Buckley–Leveret theory

## I. INTRODUCTION

In petroleum engineering in order to research injection profile, most studies rely on physical experiments and exploration technology for an answer such as isotope logging, water flooding experiment, and logging curve interpretation [1–3].

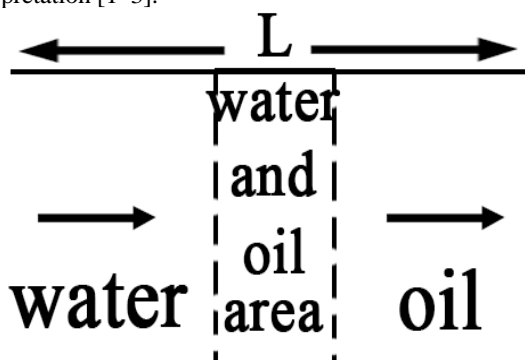


Figure 1. One dimensional linear flow problem.

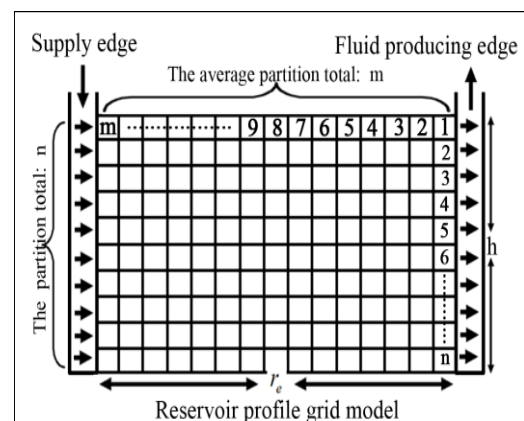


Figure 2. Reservoir profile grid model.

As a result, we must face so many complex technologies and high cost that more time and resources will be devoted to study, but above all, sometimes a large error will be found between calculation water production rate and history water production rate in the progress of history matching and performance analysis [4, 5].

Although many scholars have studied some mathematical methods to draw injection profile, a lot of restrictions will appear such as only solving one dimensional problem (such as Figure 1) and can't calculate oil and water production.

In this paper, with constructing the grid model method, a mathematic model was modeled based on oil-water two-phase flow which can not only research injection profile but also realize oil and water production analysis, moreover, implementing the research of two-dimensional problem with the multi-layer grid modeling technology. The grid profile model, is shown in Figure 2.

The research contains Buckley–Leverett theory [6] and establishes the oil-water two-phase plane radial flow function for each longitudinal grid layers, in addition, which shows the movement rules of isosaturation surface. According to related model, we can study oil production, water production, and water drive front position.

In the end, oil and water production, water drive front position can be calculated in this model and injection profile will be drawn based on water drive front position of the longitudinal layers and the water saturation of every grid at different time. In addition, a distribution about  $\{K_1, K_2, \dots, K_n\}$  will be given by a mathematic problem research.

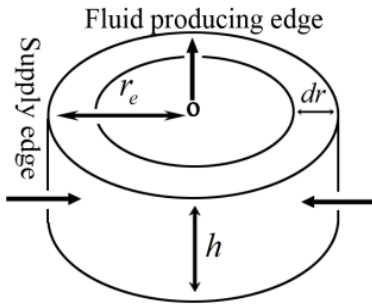


Figure 3. The radial flow model.

## II. OIL-WATER TWO-PHASE PLANE RADIAL FLOW MODEL BASED ON BUCKLEY–LEVERETT THEORY

### A. Assumptions of the oil-water two-phase model

At present, the numerical simulation method has been very widely used, but analytical solution still plays an important role in two phase flow research because of simple and quick characteristic. Therefore, in order to study the oil-water two-phase plane radial flow, the following assumptions must be provided based on analytical solution in our research model.

- (i) Supposing liquid flow rule is a plane radial flow (as shown in Figure 3) from reservoir boundary to well production area.
- (ii) During the water-oil displacement progress, rock is wetted by water and the displacement process is a self-priming process.
- (iii) Two fluids: water and oil which are incompressible.
- (iv) There are not capillary pressure and gravity effects.
- (v) The oil-water two-phase region is considered.

### B. Derivation of the oil-water two-phase model

Relying on the seepage principle in oil-water two-phase flow [7], we can have some equations based on liquid production capacity and water production rate [8]. Finally, the oil-water two-phase plane radial flow mathematical model can be obtained:

$$r_e^2 - r^2 = \frac{f_w' \cdot Q^t}{\phi \cdot \pi h} \quad (1)$$

In addition, the plane radial flow model of the each grid can be shown by the expression (2) in the profile model (in Figure 2):

$$r_e^2 - r_j^{i2} = \frac{f_w^t(S_{wj}^i)' Q_j^t}{\phi_j \cdot \pi h_j} \quad (2)$$

Where  $i = 1, 2, \dots, m$ ,  $j = 1, 2, \dots, n$ .  $m$  is the horizontal grid number,  $n$  is the layer number.

### C. The relevant definitions of water drive front

The definition of a special water saturation, determining the point of the irreducible water saturation as a fixed point  $(S_{wl}, f_w(S_{wl}))$  and choosing any other point  $(S_w, f_w(S_w))$  on the curve of the water saturation and water production rate function  $f_w = f_w(S_w)$  [9] and based on the following function for obtaining the value of  $Max\{k(S_w)\}$ .

$$k(S_w) = (f_w(S_w) - f_w(S_{wl})) / (S_w - S_{wl})$$

When  $Max\{k(S_w)\}$  can be found, the corresponding water saturation  $S_w$  can be called the frontier saturation which can be expressed by  $S_{wf}$ . Finally, according to expression (2), a corresponding displacement  $r_{wf}$  in each layer will be shown in Eq. (3).

$$r_{wfj}^2 = r_e^2 - \frac{f_w^t(S_{wfj}^i)' Q_j^t}{\phi_j \cdot \pi h_j} \quad (3)$$

Therefore, the water drive displacement can be calculated in each layer according to the expression (3) at different times.

## III. THE FLOW DISTRIBUTION MODEL IN FLUID PRODUCING EDGE PRODUCTIVITY ANALYSIS OF RESERVOIR PROFILE MODEL

In reservoir simulation there is a parameter can represent the mobility of the fluid in each grid, which can be called conductivity related to oil-water relative permeability, oil and water viscosity. And in general, the conductivity can be obtained by the expression (4) in each grid. In this paper, we can conductivity use to calculate liquid production capacity in each layer.

$$T_j^i = \frac{2\pi h_j K_j}{\ln \frac{r}{r_j^i}} (K_{roj}^i / \mu_o + K_{rwj}^i / \mu_w) \quad (4)$$

Where  $kro_j^i = kro(S_{wj}^i(t))$  ,  $krw_j^i = krw(S_{wj}^i(t))$  ,  
 $K_j \in \{K_1, K_2, \dots, K_n\}$

And the liquid-producing capacity can be obtained by the expression (5) at every point in time.

$$T_j = 1 / \sum_{i=1}^m \frac{1}{T_j^i} \quad (5)$$

Therefore, we can have the total liquid-producing capacity as shown in Eq. (6).

$$T = \sum_{j=1}^n T_j \quad (6)$$

Finally the total fluid production can be shown by the expression (7) in each layer at every point in time.

$$Q_j^t = \frac{T_j}{T} \cdot Q_t \quad (7)$$

#### IV. THE PRODUCTIVITY ANALYSIS IN THE FLUID PRODUCING EDGE

To calculate oil and water production of the fluid-producing edge at every point in time in reservoir profile model, we can use these steps:

**Step 1:** Obtaining the derivative of the water cut of the fluid producing edge in each layer from (3) when  $r_e \leq r_{wfj}^i$ . As follows:

$$f_w^t(S_{wj}^1) = \frac{(r_e^2 - r_j^{1^2}) \cdot \phi_j \cdot \pi \cdot h_j}{Q_j^t} \quad (8)$$

**Step 2:** Relying on the functional relation:  $S_w \sim f_w'(S_w)$  or solving inverse function:  $S_{wj}^i(t) = F^{-1}(r_j^i, \phi_j, h_j, Q_j^t)$ , to obtain the value of the  $S_{wj}^1$ .

**Step 3:** Relying on the functional relation:  $f_w(S_w) \sim S_w$  to obtain the value:  $f_w(S_{wj}^1)$ .

**Step 4:** When we have the value of the  $f_w(S_{wj}^1)$ , we can calculate oil production and water production in each grid layer from Eqs. (9) and (10)

The partition total:  $n = 40$  (Table 1).

$$K_1 = 2mD, K_2 = 4mD, K_3 = 6mD, \dots, K_{39} = 78mD, K_{40} = 80mD$$

Example 2(permeability distribution date):

$$K_1 = 2mD, K_2 = 4mD, K_3 = 6mD, \dots, K_{39} = 78mD, K_{40} = 100mD$$

$$Q_{oj}^t = Q_j^t \cdot (1 - f_w^t(S_{wj}^1)) \quad (9)$$

$$Q_{wj}^t = Q_j^t \cdot f_w^t(S_{wj}^1) \quad (10)$$

Table 1: Relative Permeability Experimental Data  
 $(kro(S_w) \quad krw(S_w))$ .

Sw	Kro	Krw
0.22	1	0
0.38	0.56	0.03
0.50	0.29	0.05
0.60	0.15	0.08
0.68	0.07	0.15
0.7	0	0.29

Example 1 (permeability distribution date):

**Step 5:** The total oil production and calculated water cut of the fluid producing edge can be calculated as follows:

$$Q_w^t = \sum_{j=1}^n Q_{wj}^t, \quad Q_o^t = \sum_{j=1}^n Q_{oj}^t \quad (11)$$

$$f_w(t) = Q_w^t / (Q_w^t + Q_o^t) \quad (12)$$

#### V. COMPUTATIONAL EXAMPLES AND ANALYSIS

In order to research the production capacity of the different injection profile in oilfield production in a reservoir model, we construct two examples according to examples in literature. We solve them by two groups of different permeability distribution model:  $\{K_1, K_2, \dots, K_n\}$  and the other is the same date. Reservoir description and production data:

Reservoir radius:  $r_e = 400m$  ; Oil viscosity:  $\mu_o = 2.4 \text{ mpa s}$  ; Water viscosity:  $\mu_w = 0.9 \text{ mpa s}$  ; Reservoir thickness:  $h = 20m$  ; The original water saturation:  $S_{wc} = 0.22$  .Total computing time:  $Time = 4200d$  .Average liquid producing:  $q = 600 \text{ m}^3/d$  , Porosity:  $\phi = 0.2$

In the reservoir profile grid model in Figure 2, Time steps:  $\Delta T = 60d$  ; The average partition total :  $m = 50$  ;

The injection profiles and production curves of the two examples are shown at the previous moment of water producing as follows:

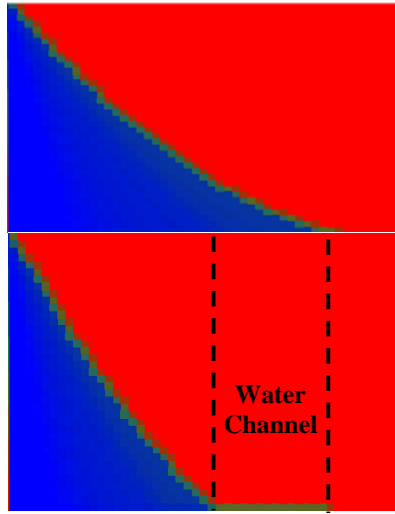


Figure 4. Injection profile pictures of the two examples. a) Example 1: Time=1560 days (b) Example 2: Time=1260 days.

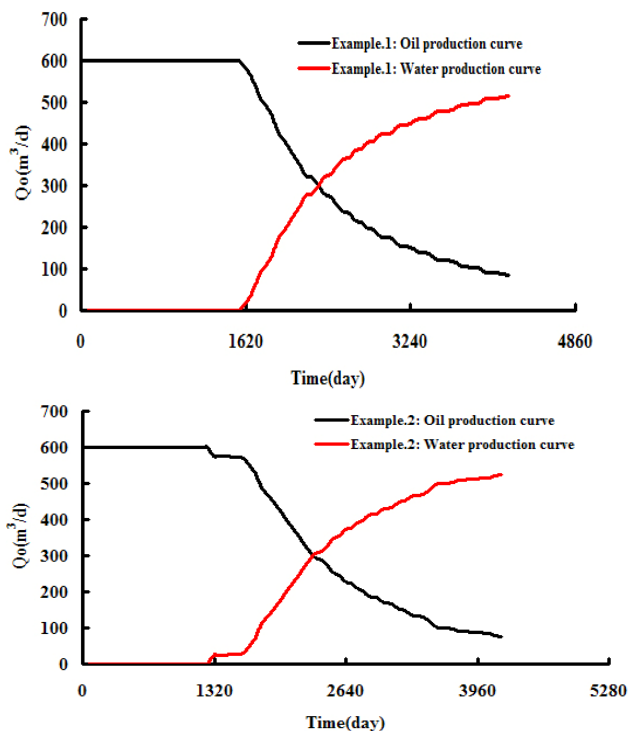


Figure 5. Oil and water production curves of the two examples. (a) Example 1: Production curves (b) Example 2: Production curves.

As can be seen from Figures 4 and 5 in the same reservoir grid model, the water production time of the example 1 is 1620 days and the second one is 1320 days, moreover the water channel have been found at that time according to Figure 5(b) in the example 2, because the different permea-

bility in the fortieth layer (Examples 1:  $K_{40} = 80mD$ , Examples 2:  $K_{40} = 80mD$ ). Therefore, we can also know that the water channel forming is earlier, the oil production drawdown is earlier, as a result, it will have a negative impact oilfield production.

## VI. CONCLUSIONS

Finally, in this paper a reservoir dynamic analysis method was constructed according to the grid modeling method and Buckley–Leverett theory based on two-phase flow. This facilitates not only research into the permeability distribution and the physical phenomenon of injection profile but also oil-water production analysis and cost savings. This approach can provide a reference for heterogeneity by researching whether the vertical permeability distribution always obeys some distribution model. According to the mathematics model, we can analyze water-yielding capacity and the production of fracture reservoir, moreover; it can allow to the researchers to avoid large systematic errors.

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