Study on Cooperation Mechanism of Closed-loop Supply Chain Based on Revenue Distribution with Modified Shapley-value

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Abstract

A closed-loop supply chain of revenue distribution model consists of a manufacturer, a Third-Part Logistics and a retailer is founded, revenue of each cooperative coalition in different cooperative ways is given based on cooperative game theory. Results indicate that cooperation will bring each cooperative coalition a bigger revenue. The feasibility of using shapley-value method in this model is proved, and then a reasonable equilibrium is solved by using a modified shapley-value method with considering the different risks each member undertook, Finally, an example is given to show the feasibility and availability of the developed mechanism.

Keywords: game theory, closed-loop supply chain, shapley-value, risk factors, allocation mechanism

1. Introduction

Closed Loop Supply Chain (CLSC) is a new logistic concept which is proposed in 2003. CLSC is a complete supply chain consists of both forward logistics and reverse logistics, and it starts form the production and ends in the recovery of the product. Not only the related policies and regulations but also the saved cost from the recovery make for its emerge. Enterprises which applying CLSC may establish its social image, and the profit of the firm can be promoted by reducing its production cost at the same time [1,2]. With the increasing development of the economy, more and more enterprises decide to apply CLSC. CLSC has become a new topical issue of supply chain management. Applying CLSC will be a inexorable trend in the development of green economy.

The recent research on CLSC mainly focuses on the design of recovery network [3-6], pricing of CLSC [7] and the inventory control [8-9] and so on. But there is little work about the coordination problem of CLSC, especially the revenue distribution problem when the members of CLSC work cooperatively. The existing distribution method mainly involves contract sharing mechanism, Nash equilibrium model and least core method [10-11]. Yet, all these method have their subjectivity, and it's hard for the CLSC members to accept. So the coordination of CLSC is difficult to operate, and the partnership among the supply chain will be difficult to maintain and control. Ma and Wang [12] used the Shapley-value method to solve the revenue distribution problem among cooperative supply chain members, Zhang and Luo [13] applied shapley-value on the profit distribution of corporate economic entity, but all these are on the precondition that the supply chain members' revenue are explicit. There is no mathematical proof on the feasibility of using shapley-value when the revenue condition is fuzzy.

This paper combines forward logistics with reverse logistics to establish a three-step CLSC revenue distribution model including a manufacturer (M), a retailer (R) and a Third-Part Logistics (TPL). This model gives the revenue condition of each cooperative union under different cooperative ways and proposes a equilibrium revenue distribution solution when all the CLSC members work cooperatively. The innovation of this paper is that the distribution model we proposed is under illegible condition, and we prove the feasibility to use shapley-value while the revenue condition of CLSC member is stochastic. We also take the risk which each CLSC member assumed into account to modify the shapley-value revenue distribution model.

2. Model formulation

In this model, the manufacturer is responsible to make new products and recycle the recovered products. In the middle of the CLSC, the Third-Part Logistics is responsible to transport the new and recovered products. The retailer which is the lowest stream of CLSC is responsible to sail the new products and recover the used products. There are some assumptions of the model:

The unit product cost of using new material is c_m. The unit cost to manufacture a product by using recovered parts is c_r. We assume that $c_m \succ c_r$, and we define $\Delta(\Delta = c_m - c_r)$ to represent the saved cost from the recycling. The wholesale price which the manufacturer charges form the retailer is w. b_m represents the recovered cost which the retailer charges form the manufacturer. The retailer sells a unit product for a p purchase. The Third -Part Logistics transport the new products and the recovery products to and from the retailer, and the transportation price are p_3 and p_4. Accordingly, the transport costs are c_3 and c_4. The consumer's demand function is $D(p) = \alpha - \beta p$. α and β are two positive constants. β denotes the consumer's demand sensitivity correspond to the price of the product. The consumer's supply function is $G(b_r) = k + hb_r$, k and h are two positive constants. h denotes consumer's supply sensitivity correspond to the price of the recovery product. C represents the recovery cost which undertook by the retailer. V indicates the revenue eigenfunction.

Provided the assumptions above, the target function of the CLSC members are:

$$v\{M\} = D(p)(w - c_m) + G(b_r)(\Delta - b_m) \tag{1}$$

$$v\{TPL\} = D(p)(p_3 - c_3) + G(b_r)(p_4 - c_4) \tag{2}$$

$$v\{R\} = D(p)(p - w) + G(b_r)(b_m - b_r) - C \tag{3}$$

In order to make the expressions above have practical sense, we assume that:

$$p > c_m, \quad p_3 > c_3, \quad p_4 > c_4, \quad \alpha - \beta c_m \geq \alpha - \beta p = D(p) \geq 0$$

3. Model structure

There are three ways of cooperation for the supply chain members in this three-step CLSC: they don't cooperate form each other, they cooperate partially and they all cooperate. In the first instance, it's a three-stage Stackelberg game through the entire CLSC; in the second, there is a two-stage Stackelberg game between part of participants with the others. In the third, the problem is to give a best decision when all the members work cooperatively. Now we give a solution for the eigenfunction under three ways of cooperation above based on Stackelberg game theory as follows.

3.1. Model of no cooperation

Assumed that the relation ship of the manufacturer and the retailer is a Stackelberg game when they don't cooperate. The manufacturer is the leader of this game. In the first stage of the game, the manufacturer decide the wholesale price w and recovery price b_m so as to maximize his profit. In the first stage Third - Part Logistics decide his service price p_3 and p_4. In the last stage of the game, the retailer decide the product's unit sale price p and b_r to maximize his profit.

According to the backward induction, we get the first-order condition of p and b_r of equation (3) as

$$\begin{cases} p* = \dfrac{\alpha + \beta w}{2\beta} \\ b_r^* = \dfrac{-k + hb_m}{2h} \end{cases} \tag{4}$$

Equation (4) is the reaction equation of p and b_r to w and b_m. Replace p and b_r in target equation (1) with p^* and b_r^* to produce the first-order condition of w and b_m as

$$\begin{cases} p^* = \dfrac{3\alpha + \beta c_m}{4\beta}, b_r^* = \dfrac{-3k + \pi h}{4h} \\ w^* = \dfrac{\alpha + \beta c_m}{2\beta}, b_m^* = \dfrac{-k + \pi h}{2h} \end{cases} \tag{5}$$

According to equation (5) we rewrite equation (1), (2) and (3) to obtain the profit of the manufacturer, Third-Part Logistics and retailer as

$$v^*\{M\} = \frac{1}{4}\left[\frac{(\alpha - \beta c_m)^2}{2\beta} + \frac{(k + \pi h)^2}{2h}\right] \tag{6}$$

$$v^*\{TPL\} = \frac{1}{4}[(\alpha - \beta c_m)(p_3 - c_3) + (k + \pi h)(p_4 - c_4)] \tag{7}$$

$$v^*\{R\} = \frac{1}{4}\left[\frac{(\alpha - \beta c_m)^2}{4\beta} + \frac{(k + \pi h)^2}{4h}\right] - C$$

(8)

3.2. Model of complete cooperation

In this condition, all the members of CLSC work cooperatively. Now the profit target function of the entire CLSC is

$$\begin{aligned} v\{M, TP, R\} &= v\{M\} + v\{R\} + v\{TP\} \\ &= D(p)(p - c_m + p_3 - c_3) + \\ &\quad G(b_r)(\pi - b_r + p_4 - c_4) - C \end{aligned} \tag{9}$$

We get the first-order condition of p and b_r of equation (9) as

$$\begin{cases} p^{**} = \dfrac{\alpha + \beta(c_m - p_3 + c_3)}{2\beta} \\ b_r^{**} = \dfrac{-k + h(\pi + p_4 - c_4)}{2h} \end{cases} \tag{10}$$

Transform equation (9) with (10) to rewrite the profit target function as

$$v\{M,TP,R\}=\frac{[\alpha-\beta(c_m-p_3+c_3)]^2}{4\beta}+\frac{[k+h(\pi+p_4-c_4)]^2}{4h}-C \tag{11}$$

3.3. Model of partial cooperation

There are three ways of cooperation when the three members cooperate partially: M ally with R, M ally with TPL and R ally with TPL. First we concern the ally consist of the manufacturer and the retailer. According to equation (1) and (2), the target profit function of the ally $\{M,R\}$ is

$$\begin{aligned} v\{M,R\}&=v\{M\}+v\{R\} \\ &=D(p)(p-c_m)+G(b_r)(\pi-b_r)-C \end{aligned} \tag{12}$$

By using the similar idea in the previous section, we get the first-order condition of p and b_r of equation (12) as

$$\begin{cases} p_1=\dfrac{\alpha+\beta c_m}{2\beta} \\ b_{r1}=\dfrac{-k+\pi h}{2h} \end{cases} \tag{13}$$

Transform equation (12) with (13) to rewrite the profit target function as

$$v\{M,R\}=\frac{(\alpha-\beta c_m)^2}{4\beta}+\frac{(k+\pi h)^2}{4h}-C \tag{14}$$

By using the same method we obtain the ally $\{M,TP\}$'s profit target function as

$$v\{M,TP\}=\frac{[\alpha-\beta(c_m-p_3+c_3)]^2}{8\beta}+\frac{[k+h(\pi+p_4-c_4)]^2}{8h} \tag{15}$$

Note that the target function of the ally $\{R,TP\}$ when missing the manufacturer is the sum of equation (7) and (8). That is

$$v\{R,TP\}=v^*\{R\}+v^*\{TP\} \tag{16}$$

Based on the analysis and calculations above, the profits of each ally in different cooperate ways are shown in table 1.

4. Equilibrium solution for revenue distribution

When the game players begin to cooperate, their most cared problems are not the actions of others but the profits they will obtain in the cooperation. The cooperation would fail if the profit gained in a cooperation is lower than that in a noncooperation. So as to improve the stability of the cooperation, we introduce the Shapley-value to solve this problem. The revenue distributed by Shapley-value is proportional to the contribution made by the member. So, it is fair and acceptable for the game players. This section begins with a brief description of the Shapley-value and proposes a equilibrium solution for the revenue distribution, and the solution based on Shapley-value is under the premise of stochastic revenue condition.

4.1. Basic principles of Shapley-value

Shapley-value is proposed by American professor Shapley to solve N-person cooperative game. It realizes that the overall interests of each member in the coalition to be fairly and effectively distributed [14]. When n players take part in a business action in a cooperative way, every coalition would receive a revenue in a specific cooperation way. The increase of participants would not decrease the total revenue, that is what we called Pareto Improvement [15]. Thus, the complete cooperation of the game players will bring a biggest profit, and Shapley-value is one of the solutions to distribute the biggest 'cake'. The definitions of Shapley-value are given as follows.

Cooperate ways		Profits of each ally
No cooperation	$v\{M\}$	$\frac{1}{4}\left[\frac{(\alpha-\beta c_m)^2}{2\beta}+\frac{(k+\pi h)^2}{2h}\right]$
	$v\{TP\}$	$\frac{1}{4}\left[(\alpha-\beta c_m)(p_3-c_3)+(k+\pi h)(p_4-c_4)\right]$
	$v\{R\}$	$\frac{1}{4}\left[\frac{(\alpha-\beta c_m)^2}{4\beta}+\frac{(k+\pi h)^2}{4h}\right]-C$
Complete cooperation	$v\{M,TP\}$	$\frac{[\alpha-\beta(c_m-p_3+c_3)]^2}{8\beta}+\frac{[k+h(\pi+p_4-c_4)]^2}{8h}$
	$v\{M,R\}$	$\frac{(\alpha-\beta c_m)^2}{4\beta}+\frac{(k+\pi h)^2}{4h}-C$
	$v\{R,TP\}$	$v\{R\}+v\{TP\}$
Partial cooperation	$v\{M,TP,R\}$	$\frac{[\alpha-\beta(c_m-p_3+c_3)]^2}{4\beta}+\frac{[k+h(\pi+p_4-c_4)]^2}{4h}-C$

Table 1: Profits of each ally in different cooperate ways.

Let $I=\{1, 2, \cdots, n\}$ be the set of n, and for any subset s of I (any cooperation of the n players) there is a real-valued function $v(s)$ which satisfies

$$v(\Phi)=0 \tag{17}$$

$$v(s_1 \cup s_2) \geq v(s_2)+v(s_2) \tag{18}$$

where $s_1 \cap s_2 = \Phi$.

Define $v(s)$ the eigenfunction of set I to denote subset s 's revenue. Equation (17) and (18) denote that '1+1>2', which means that the benefits of cooperation are bigger than that when there is no cooperation. Cooperation will not damage the individual's benefit, and the total benefit becomes maximum when all the players cooperate mutually. Let $v(I)$ be the maximum benefit.

Assume that $\phi_i(v)(i=1,2,...n)$ denotes the deserved revenue of the i th player of set I in the cooperation. Then the revenue distribution of the maximum benefit could be expressed as $\Phi(v)=(\phi_1(v), \phi_2(v), \cdots, \phi_n(v))$. Obviously, the precondition to cooperate successively is

$$\sum_{i=1}^{n}\phi_i(V)=V(I)$$

and

$$\phi_i(v)\geq v(i),\quad i=1,2,...,n \tag{19}$$

The revenue of the i th player distributed by Shapley-value is

$$\phi_i(v)=\sum_{s\in s(i)} w(|s|)\left[v(s)-v(s\setminus i)\right],\quad i=1,2,...,n \tag{20}$$

$$w(|s|)=\frac{(n-|s|)!(|s|-1)!}{n!} \tag{21}$$

where $s(i)$ is a subset of I . $|s|$ and n is the number of element contained in subset s and set I . $w(|s|)$ is weight coefficient. $v(s)$ is the revenue of subset s while $v(s\setminus i)$ is the revenue of subset s removed of member i .

4.2. The premise of using Shapley-value

The total profit created by the cooperative supply chain is greater than or equal to the sum of their profit when there is no cooperation. Thus , it is wise for the CLSC members to cooperate with each other. Equation (18) denotes the superadditivity of cooperative game, and it is also the premise for the supply chain members to cooperate with each other. Only equation (18) would make the cooperation to reach a 'win-win' effect. It is the precondition to use Shapley-value. So the proposed revenue distribution model must fulfill equation (18), from table 1 we can see

$v\{M,R\}\succ v\{M\}+v\{R\}$, $v\{R,TPL\}=v\{R\}+v\{TPL\}$. Now we give the proof of $v\{M,TPL\}\succ v\{M\}+v\{TPL\}$ and $v\{M,TPL,R\}\succ v\{M\}+v\{TPL\}+v\{R\}$ as follows

$$v\{M,TPL\}-v\{M\}+v\{TPL\}=\frac{\beta(p_3-c_3)^2}{8}$$

$$\Rightarrow v\{M,TPL\}\succ v\{M\}+v\{TPL\}$$

$$v\{M,TPL,R\}-v\{M\}+v\{TPL\}+v\{R\}$$
$$=\frac{(\alpha-\beta c_m)^2+4(\alpha-\beta c_m)(p_3-c_3)+2(p_3-c_3)^2}{2\beta}$$
$$+\frac{(k+\pi h)^2+4(k+\pi h)(p_4-c_4)+2(p_4-c_4)^2}{2h}$$

According to the constraint condition we can derive that $\alpha-\beta c_m\geq\alpha-\beta p=D(p)\geq 0$, $p_3-c_3>0$ and $p_4-c_4>0$, then there is $v\{M,TPL,R\}>v\{M\}+v\{TPL\}+v\{R\}$. With the calculations above, we can see that the total profit created by the cooperative supply chain is greater than the sum of their profit when there is no cooperation, which means it is feasible to apply Shapley-value in this model.

4.3. Shapley-value: Equilibrium solution for distribution

According to equation (20) and Table 1, we give the Shapley-value of the manufacturer which is also the manufacturer's deserved revenue as below.

In a similar way, we can calculate the Shapley-value of the Third-Party Logistics $\phi_{TPL}(v)$ and the Shapley-value of the retailer $\phi_R(v)$.

$$\begin{aligned}\phi_M(v) &= \sum_{s\in s(i)} w(|s|)\left[v(s)-v(s\setminus i)\right] \\ &= \frac{1}{3}\left[v\{M\}-v\left(\{M\}-\{M\}\right)\right]+ \\ &\frac{1}{6}\left[v\{M,R\}-v\left(\{M,R\}-\{M\}\right)\right]+ \\ &\frac{1}{6}\left[v\{M,TPL\}-v\left(\{M,TPL\}-\{M\}\right)\right]+ \\ &\frac{1}{3}\left[v\{M,TPL,R\}-v\left(\{M,TPL,R\}-\{M\}\right)\right]\end{aligned} \tag{22}$$

5. The shortcoming and modification of Shapley-value

Shapley-value acquiesce that the risk which all the participants took is the same as $1/n$, but the practical conditions may not fit in with the premise. For the members of different node in the supply chain takes different risks, thus The bigger the risk a member takes the higher the revenue he will to be paied. So the risk problem should be taken into consideration when using Shapley-value. We apply the Analytical Hierarchy Process to give a risk vector $T=\{t_1,t_2,\ldots,t_n\}$ to describe the risk took by the CLSC members, where $\sum_{i=1}^{n} t_i = 1$. Then, the modifying risk factor of Shapley-value is $(1+\Delta t_i)=1+\left(t_i-\frac{1}{n}\right)$ [11], which make up for the flaw of Shapley-value. The modified Shapley-value is

$$\phi_i'(v)=(1+\Delta t_i)\phi_i(v)=\left[1+\left(t_i-\frac{1}{n}\right)\right]\phi_i(v) \tag{23}$$

In order to make the distribution plan more closing to reality, some factors such as the amount capital contributions, spillover effect and so on should be taken into account too. This paper only consider the risk of the CLSC members. Equation (21) is the distributed revenue which have considered the risk factor.

6. Example analysis

Assume that $c_m=30$, $c_r=10$, $\alpha=100$, $\beta=1$, $k=20$, $h=2$, $p_3=3.5$, $p_4=2$, $c_3=c_4=1.5$, $C=75$, risk vector $T=\{0.3,0.2,0.5\}$, Based on the above parameters and equation (20), now we give the Shapley-value which is the deserved revenue of the manufacturer. The detailed calculation process is described in Table 2.

Add up the data of the last row of Table 2 and we can obtain the deserved revenue which is also the Shapley-value of the manufacturer $\phi_M(v)=1061.51$, and in a similar way the Shapley-value of the Third-Part Logistics and the retailer can be solved as $\phi_{TPL}(v)=57.18$ and $\phi_R(v)=567.52$. The revenue distribution can be expressed as $\{\phi_M(v),\phi_{TPL}(v),\phi_R(v)\}=\{1061.51, 57.18, 567.52\}$.

From the risk vector $T=\{0.3,0.2,0.5\}$ we can obtain the modified factor vector $\Delta t_i=\{-0.03,-0.13,0.16\}$. Then the modified Shapley-value of the manufacturer is

$\phi'_M(v) = (1+\Delta t_i)\phi_M(v) = 1029.66$, and the Shapley-value of the retailer and Third-Part Logistics can be resolved in a same way. They are expressed and compared with basic Shapley-value in Table 3.

S	{M}	{M,TPL}	{M,R}	{M,TPL,R}
$v(s)$	837.50	880.56	1600	1686.125
$v(s \setminus M)$	0.00	42.50	343.75	386.25
$v(s) - v(s \setminus M)$	837.50	838.06	1256.25	1299.86
$\lvert s \rvert$	1	2	2	3
$w(\lvert s \rvert)$	1/3	1/6	1/6	1/3
$w(\lvert s \rvert)[v(s) - v(s \setminus M)]$	279.17	139.67	209.38	433.29

Table 2: The calculation of manufacturer's reserved revenue.

Results	M	TPL	R
Shapley-value	1061.51	57.18	567.52
Modified value	31.85	7.43	-90.80
Modified Shapley-value	1029.66	49.75	658.32

Table 3: Final distribution based on modified Shapley-value.

Modified Shapley-value revenue distribution model is fair and reasonable for it takes the risks of the players into account in the game. Clearly, there is $\phi_M(v) \succ v\{M\}$, $\phi_{TP}(v) \succ v\{TP\}$ and $\phi_R(v) \succ v\{R\}$ Similarly. So, it verify that the profit of the member gained in a cooperation is bigger than that in a noncooperation.

7. Conclusions

In a closed-loop supply chain, each participant could have a bigger profit if they cooperate with each other, thus there is a stimulant for all the CLSC members to establish a cooperative federation. After that, the critical problem is whether the revenue distribution plan is fair or not which also decides the success or failure of the cooperation. Each parterner play games for its own interest. Cooperative game is a good solution to find a reasonable revenue distribution and a stable cooperation. Shapley-value gives a sole and equilibrium solution, and the solution is based on the contribution which he makes for the cooperative coalition. So the solution is fair and acceptable by the CLSC members.

The recent researches on Shapley-value stay on the premise of explicit revenue condition. It is needed that the feasibility of applying Shapley-value should be testified when the revenue condition is stochastic. This paper gives a three-step CLSC revenue distribution model under different situation, and the feasibility of using Shapley-value has been proved. Anyhow, Shapley-value has its own flaw such as the omission of the risks took by the supply chain members. In the final distribution, this paper takes the risks into account to modify the Shapley-value. Example show that the proposed distribution model is practical and feasible.

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References

[1] Y. Y. Wang, B.Y. Li and L. Shen. Analysis onPrice Decision of Closed-Loop Supply Chain Based on Game Theory[J]. Journal of Nanjing University of Aeronautics & Astronautics, 2008, 40(2): 275-278.
[2] Z. Q. Huang and Q. L. Da. Study on Efficiency of Serial Supply Chains with Remanufacture [J]. Journal of Management Sciences in China, 2006, 9(6): 51-57.
[3] M. Fleischmann , H. R. Krikke , et al. A Characterization of Networks for Product Recovery [J]. Omega, 2000, 28: 653-666.
[4] D. Francas and S. Minner. Manufacturing Network Configuration in Supply Chains with Product Recovery [J]. Omega, 2009, (37): 757-769.
[5] M. S. Pishvaee, R. Z. Farahani and W. Dullaert. A Memetic Algorithm for Bi-objective Integrated Forward/Reverse Logistics Network Design [J]. Computers & Operations Research, 2010, (37): 1100-1112.
[6] M. El-Sayed, N. Afia and A. El-Kharbotly. A Stochastic Model for Forward – Reverse Logistics Network Design Under Risk [J]. Computers & Industrial Engineering, 2010, (58): 423 – 431.
[7] R. C. Savaskan, S. Bhattacharya and L. N. Van Wassenhove. Closed -loop Supply Chain Models With Product Remanufacturing [J]. Management Science , 2004 , 50 (2) :239 - 252.
[8] J. X. You, M. G. Sui, and J. Du. Review on Optimal Models and Algorithms of Repairable Inventory in Closed-loop Supply Chain [J]. Journal of TongJi university(national science), 2004, 32(11): 1534-1539.
[9] S. L. Chung, H. M. Wee and P. C. Yang. Optimal Policy for A Closed-Loop Supply Chain Inventory System with Remanufacturing [J]. Mathematical and Computer Modelling 2008, (48): 867 – 881.
[10] F. Ye and Y. N. Li. Research on Incentive Mechanism of Three-layer Supply Chain Coordination [J]. Journal of Systems & Management2007, 16(2): 130-134.
[11] T. J. Xiao, K. R. Shi and D. Q. Yang. Coordination of A Supply Chain with Consumer Return under Demand Uncertainty [J]. International Journal of Production Economics 2010, (124): 171 – 180.
[12] S. H. Ma and P. Wang. The Study of Profit Allocation among Partners in Supply Chain Based on the Shapley Value[J]. Industrial Engineering and Management, 2006, (4): 43-49.
[13] R. H. Zhang and R. G. Luo. Research on Benefit Assignment of Common Delivery Based on Shapley Value Method [J]. Journal of Wuhan University of Technology, 2008, 30(1): 150-153.
[14] D. Y. Ji. The Shapley Value Method of N-person Cooperative Counter measure [J]. Journal of Shenyang University, 2003, 15(1): 22-23.
[15] Fel Tkamp V. Alternative Axiomatic Characterizations of the Shapley and Banzhaf Values [J]. Internat J Game Theory, 1995, 24: 179-186.