A New Similarity Measurement Formula between Vague Sets

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Abstract—Vague sets can denote and solve more complicated fuzzy and indefinite information. Similarity measure is an important, effective and widely used method on data processing and analysis. Based on the concept of Vague sets and the theory of conversion from single data to Vague data, this research put forward a new similarity measurement formula between Vague sets, to develop the existing Vague sets similarity measurements.

Keywords-vague sets; similarity measurement; similarity measurement formula

I. INTRODUCTION

The Vague sets theory brought forward by Gau and Buehrer [1] is an extension of Fuzzy sets theory created by Zadeh [2]. Compared with Fuzzy sets theory, Vague sets theory has the ability to denote fuzzy information much more comprehensive.

II. THE DEFINITION OF VAGUE SETS

On the assumption that the discourse domain is not empty, to any $u \in U$, define interval $[t_L(u), 1-f_L(u)]$ as the Vague membership function or Vague value for Vague set L on point u, meantime, $0 \le t_L(u) \le 1$, $0 \le f_L(u) \le 1$ and $t_L(u) + f_L(u) \le 1$ .And define $t_L(u)$, $f_L(u)$ and $\pi_L(u)(=1-t_L(u)-f_L(u))$ as real subordinate function, unreal subordinate function and uncertain function of Vague set L individually. Meanwhile, $t_L(u)$ and $f_L(u)$ indicate the subordinate extent of supporting and objecting $u \in L$.

When $U = \{u_1, u_2, \cdots, u_n\}$ is discrete domain, the Vague set L upon that can be expressed as $L = \sum_{i=1}^{n}[t_L(u_i), 1-f_L(u_i)]/u_i$, or $L = \sum_{i=1}^{n}[t_{l_i}, 1-f_{l_i}]/u_i$, or $L = \{[t_{l_1}, 1-f_{l_1}], [t_{l_2}, 1-f_{l_2}], \cdots, [t_{l_n}, 1-f_{l_n}]\}$.

III. CONVERTING SINGLE VALUE DATA INTO VAGUE DATA

Building-up a Vague environment is essential to research and solve practical problems through the application of Vague sets theory. And the method is to converting single value data into Vague value data.

Definition 1 on the assumption that $U = \{u_1, u_2, \cdots, u_n\}$ is the discourse domain, and group $L_i (i = 1, 2, \cdots m)$ is based upon that. Nonnegative single value data u_{ij} are supposed to $u_j (j = 1, 2, \cdots n)$ upon L_i [3].

a. Vague axiom, $0 \le t_{ij} \le 1 - f_{ij} \le 1$.

b. Benefit axiom, if $0 \le u_{kj} < u_{ij}$, $t_{kj} \le t_{ij}$ and $1-f_{kj} \le 1-f_{ij}$, single value data u_{ij} and u_{kj} can be converted into Vague data $L_i(u_j) = u_{ij} = [t_{ij}, 1-f_{ij}]$ and $L_k(u_j) = u_{kj} = [t_{kj}, 1-f_{kj}]$.

If it is satisfied with Vague axiom and Benefit axiom, then the Vague data transformation formula $L_i(u_j) = u_{ij} = [t_{ij}, 1-f_{ij}]$ towards nonnegative single value data u_{ij} is the benefit-type transformation formula from non-negative single value data w_{ij} to Vague data $V_i(w_j)$.

c. Consumption axiom, if $0 \le u_{kj} < u_{ij}$, when $t_{kj} \ge t_{ij}$, $1-f_{kj} \ge 1-f_{ij}$, single value data u_{ij} and u_{kj} can be converted into Vague data $L_i(u_j) = u_{ij} = [t_{ij}, 1-f_{ij}]$ and $L_k(u_j) = u_{kj} = [t_{kj}, 1-f_{kj}]$.

If it is satisfied with Vague axiom and Benefit axiom, then the Vague data transformation formula $L_i(u_j) = u_{ij} = [t_{ij}, 1-f_{ij}]$ towards nonnegative single value data u_{ij} is the consumption-type transformation formula from non-negative single value data w_{ij} to Vague data $V_i(w_j)$.

Above all, the benefit-type transformation formula is fit for project of high extent on both value and preference, while, the consumption transformation formula is fit for project of high extent on value and low extent on preference.

IV. SIMILARITY MEASURE BETWEEN VAGUE SETS

Definition 2 to any Vague value $l = [t_l, 1-f_l]$, define $t_l^{(0)} = t_l$, $f_l^{(0)} = f_l$, $\pi_l^{(0)} = \pi_l$; $t_l^{(m)} = t_l \cdot (1 + \pi_l + \pi_l^2 + \cdots + \pi_l^m)$, $f_l^{(m)} = 1 + \pi_l + \pi_l^2 + \cdots + \pi_l^m)$, $\pi_l^{(m)} = \pi_l^{m+1}$; $\alpha_l^{(m)} = t_l^{(m)} - f_l^{(m)}$, $\beta_l^{(m)} = t_l^{(m)} + f_l^{(m)}$, $(m = 0, 1, 2, \cdots)$ [4].

Because high differentiate ratio, Vague data mining is widely applied to form similarity measurement formula between Vague sets. The similarity measurement formula between Vague sets below is formed by definition 2 through application of Vague data mining method.

Definition 3 on the assumption that Vague value is $l = [t_l, 1-f_l]$, $k = [t_k, 1-f_k]$, then the formula $M(l,k)$ is the similarity measure between Vague value l and k. Then if $M(l,k)$ is satisfied with the axioms below [5].

a. Commonness axiom, $0 \le M(l,k) \le 1$;

b. Symmetry axiom, $M(l,k) = M(k,l)$;

c. Reflexivity axiom, $M(l,l) = 1$;

d. Minimum axiom, whenever $l = [1,1]$ and $k = [0,0]$, or $l = [0,0]$ and $k = [1,1]$, and $M(l,k) = 0$.

$M(l,k)$ means the similarity extent between Vague value l and k, the bigger $M(l,k)$ value is, the higher similar level between Vague value l and k is, when $M(l,k)$ value equals to one mean that Vague value l and k are same.

For the other side, the smaller $M(l,k)$ value is, the lower similar level between Vague values l and k is, when $M(l,k)$ value equals to zero mean that Vague value l and k are totally different.

Similar to definition 3, on the basis of similarity measurement $M(L,K)$ between Vague set L and K, then the weighted similarity measurement can be defined as $WM(L,K)$.

V. NEW SIMILARITY MEASUREMENT FORMULA BETWEEN VAGUE SETS

Theorem 1 If $u_{j\max} = \max\{u_{1j}, u_{2j}, \cdots, u_{mj}\}$, then

$$L_i(u_j) = u_{ij} = [t_{ij}, 1-f_{ij}] = \left[\left(\frac{u_{ij}}{u_{j\max}}\right)^3, \frac{u_{ij}}{u_{j\max}}\right] \quad (1)$$

is the benefit-type transformation formula from non-negative single value data u_{ij} to Vague data.

While

$$L_i(u_j) = u_{ij} = [t_{ij}, 1-f_{ij}] = \left[1-\left(\frac{u_{ij}}{u_{j\max}}\right), 1-\left(\frac{u_{ij}}{u_{j\max}}\right)^3\right] \quad (2)$$

is the consumption-type transformation formula from non-negative single value data u_{ij} to Vague data.

Theorem 2 on the assumption that Vague value is $l = [t_l, 1-f_l]$, $k = [t_k, 1-f_k]$, $m = 0, 1, 2, \cdots$, then

$$M_m(l,k) = \frac{3 - \left|f_l^{(m)} - f_k^{(m)}\right| + \left|\alpha_l^{(m)} - \alpha_k^{(m)}\right|}{3 + \left|f_l^{(m)} - f_k^{(m)}\right| + \left|\alpha_l^{(m)} - \alpha_k^{(m)}\right|} \quad (3)$$

is the similarity measure formula between Vague value l and k .

Theorem 3 on the assumption that discourse domain $U=\{u_1,u_2,\cdots,u_n\}$, upon which there are Vague sets $L=\sum_{i=1}^{n}[t_L(u_i),1-f_L(u_i)]/u_i$ and $K=\sum_{i=1}^{n}[t_K(u_i),1-f_K(u_i)]/u_i$, also can be abbreviated as $L=\sum_{i=1}^{n}[t_{u_i},1-f_{u_i}]/u_i$ and $K=\sum_{i=1}^{n}[t_{k_i},1-f_{k_i}]/u_i$, meanwhile $m=0,1,2,\cdots$.

Then Formula

$$T_m(L,K)=\frac{1}{n}\sum_{i=1}^{n}\frac{3-\left|f_{l_i}^{(m)}-f_{k_i}^{(m)}\right|-\left|\alpha_{l_i}^{(m)}-\alpha_{k_i}^{(m)}\right|}{3+\left|f_{l_i}^{(m)}-f_{k_i}^{(m)}\right|-\left|\alpha_{l_i}^{(m)}-\alpha_{k_i}^{(m)}\right|} \quad (4)$$

is the similarity measure formula between Vague sets L and K .

Theorem 4 on the assumption that $w_i\in[0,1]$ is the weight of element $u_i(i=1,2,\cdots,n)$, and $\sum_{i=1}^{n}w_i=1$, when $m=0,1,2,\cdots$.

Then upon the assumption of theorem 3, formula

$$WT_m(L,K)=\sum_{i=1}^{n}w_i\cdot\frac{3-\left|f_{l_i}^{(m)}-f_{k_i}^{(m)}\right|-\left|\alpha_{l_i}^{(m)}-\alpha_{k_i}^{(m)}\right|}{3+\left|f_{l_i}^{(m)}-f_{k_i}^{(m)}\right|-\left|\alpha_{l_i}^{(m)}-\alpha_{k_i}^{(m)}\right|} \quad (5)$$

is the weighted similarity measure formula between Vague sets L and K .

VI. CONCLUSION

The above-mentioned similarity measurement formula between Vague sets can be applied to select product series as per the selector's desire.

The applicable steps are as follows. First of all, the products to be selected and the indexes upon which should be found. Secondly, the candidates to be selected need to be decided. Thirdly, the ideal product should be decided. Then raw data needed to be converted into Vague data, meanwhile, the Vague sets between the ideal product and candidates are established. Soon after, the weight of each index should be decided as per the selector's desire. After this, the weighted Vague sets similarity measure between the ideal product and candidates need to be calculated. Finally, the most desirable product is found.

This similarity measurement formula between Vague sets is a special example of Vague pattern recognition, can be used to make decisions between multiple targets.

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