A Method of Moving Average Filtering for Non-uniformly Sampled Signals

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Abstract. This paper presents two new methods for the analysis and the process of non-uniformly sampled signals. During the process of practical sampling, the spectrum estimation from non-uniformly sampled signals is obtained by continuous Fourier transform. A method of moving average filtering for non-uniformly sampled signals is given. In the research non-uniformly sampled signals is represented as the algebraic addition of impulse function. The spectrum character of non-uniformly sampled signals is gotten through unevenly sampling.The moving average filtering method for non-uniformly sampled signals is based on different sampling interval. The validity of this method has been verified by simulation experiments. This developed technique can be applied in the processing of laser gyroscope output signal and the laser gyroscope bias error is reduced by 10.1% compared with traditional average filter.

1 Introduction

Currently digital signal processing (DSP) are almost based on uniform sampling to describe signal features. In many practical data collection system, sampling interval is uniform due to integral period sampling, the instability of clock pulse or different parameters of the A/D converter. In spectrum analysis and digital signal processing, the sampling sequence of independent variables is given in the form of an integer by introducing the time normalization process, there is no information about the non-uniform sampling interval, which inevitably leads to the error in spectrum analysis and signal processing result. And in the whole frequency range non-uniform sampling will produce noise spectrum, even submerges mall amplitude spectrum, and makes it difficult to detect and process the small amplitude signal.

In engineering technology point of the view,scholars at home and abroad conduct a deep study on technological realization of non-uniform sampling and reconstruction as well as applications. Y.C.Jenq first investigated the problem of orthogonal signal of spectrum[1] with the ideal non-uniformly extraction by analysis, the basic principle is to decompose non-uniformly sampled sequence into the uniform sampled sequence M subsequences composition, and construct the relationship of non-uniformly sampled sequence of discrete spectrum and analog spectrum. Based on documents [2], it discusses the analytical method of non-uniform sampled signal spectrum, which proves the validity of this method from the point of view of mathematical expectation. The documents [3-5] have a further study on non-uniform sampled signal digital spectrum, and give us the formula of non-uniform sampled signal digital spectrum. Although these fruitful research achievements greatly promoted the development of the non-uniform sampled signal analysis theory, there is a limit in the applicability of theory. The limitation is the case of the case of Superposition of multiple cycle sampling. At the same time, the reconstruction of general non-uniform sampled signal has to be studied further.

This paper studies the non-uniform sampled signal digital spectrum in depth, obtains the formula of non-uniform sampled signal digital spectrum, and proposes moving average filtering method of non-uniform sampled signal. Apply the spectrum method and moving average filtering method of non-uniform sampled signal in this paper. This paper analyses the spectrum characteristic of laser gyroscope output signal, and effectively reduces the zero bias signal of laser gyroscope.

2 The analysis of Non-uniform sampled signal spectrum

The continuous time signal x(t) of the spectrum x is usually defined as:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \qquad (1)$$

We cannot get the true value of the signal from —∞ to ∞, so we do not use the method which is proposed by definition (1). The actual processing is the process that a window function truncates the signal.Without loss of generality, the time is limited to the width of T0 in this paper, and assuming a window function truncates the signal. In order to calculation, x(t) needs to be discretization, the limited data length of sampled values N replace the continuous time signal x (c), if the observation time is uniform sampling, the sample frequency is fN=N/T0. For non-uniform N point sampling, the set {t1,t} reprensents the sampling time, and making use of Dirac impulse function, regarding the sampled signal spectrum as continuous signal spectrum,

$$X_S(f) = \int_0^T \sum_{n=0}^{N} x(t_n)\delta(t-t_n)e^{-j2\pi ft}dt = \sum_{n=1}^{N} x(t_n)e^{-j2\pi ft_n} \qquad (2)$$

The spectrum in frequency domain is continuous from formula（2）,it is not appropriate to conduct it by computer, therefore frequency domain needs to be discretization.

In time domain, N-point non-uniform sampling proceeds cyclic continuation with period T0, namely the waveform in this range shifts nT0 overlay the original waveform, which forms cycle waveforms. The convolution of $\delta(t-nT_0)$ can realize the shifting of cyclic continuation,so the waveforms after cyclic continuation in mathematics can be expressed as the convolution between original waveform and Impulse sequence, the results possess discrete spectrum.The cyclic continuation signal is

$$\tilde{x}_s(t) = [\sum_{n=0}^{N-1} x(t_n)\delta(t-t_n)] * [T_0 \sum_{v=-\infty}^{\infty}\delta(t-vT_0)] = T_0 \sum_{v=-\infty}^{\infty}\sum_{n=0}^{N-1} x(t_n)\delta(t-t_n-vT_0) \qquad (3)$$

$\tilde{x}_s(t)$ is periodic signal, which can be expanded into the form of Fourier series

$$\tilde{x}_s(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T_0} \qquad (4)$$

The Fourier coefficient is $a_k = \frac{1}{T_0}\int_0^{T_0} \tilde{x}_s(t)e^{-j2\pi kt/T_0}dt = \sum_{n=0}^{N-1} x(t_n)e^{-j2\pi kt_n/T_0}$

Taking the Fourier transform of periodic signal (4), we can get the n sampling values of continuous function x(t), k=0,1,…,N-1. Taking the Fourier transform of n points,

$$X_s(\frac{k}{T_0}) = \sum_{n=0}^{N-1} x(t_n)e^{-j2\pi kt_n/T_0} \qquad (5)$$

After time domain sampling, truncation and cyclic continuation, it builds the relationship between the non-uniform time sampling values and frequency sampling values. And the Fourier transform of the new signal Xs(k/T0) can be seen as the Fourier transform of the original singal.

3 The Method of Moving Average Filtering for Non-uniformly

To sampled signal sequence of uniformly spaced time ΔT, the Method of Moving Average Filtering takes the arithmetic mean of the number of N of sampling values, the math expression is

$$y(t_n) = \frac{1}{N\cdot\Delta T}\sum_{i=n-N+1}^{n} x(t_i)\cdot\Delta T = \frac{1}{N}\sum_{i=n-N+1}^{n} x(t_i) \qquad (6)$$

In formula（6）, $y(t_n)$ is the arithmetic mean of the number of N of sampling values in a $N\Delta T$ time, $x(t_i)$ is the number of i of sampling value, ΔT is the interval of sampling time.

To non-uniform sampling sequence $\{x(t_0), x(t_1), x(t_2),..., x(t_{N-1}),..., x(t_n)\}$, the internal of sampling time is different,the non uniform moving average filtering with the length of time T can be redefined as

$$y(t_n)=\frac{1}{T}\sum_{i=n-N+1}^{n}x(t_i)\cdot(t_{i+1}-t_i) \tag{7}$$

Non-uniform sampled signal can be regarded as the algebraic sum, it is formed by the shifting and companding of rectangular pulse function

$$g_\tau(t)=\begin{cases}1,|t|<\frac{\tau}{2}\\ 0,|t|>\frac{\tau}{2}\end{cases} \tag{8}$$

The spectrum function is

$$Y(j\omega)=\int_{-\infty}^{\infty}g_\tau(t)e^{-j\omega t}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}1\cdot e^{-j\omega t}dt=\frac{2\sin(\frac{\omega\tau}{2})}{\omega} \tag{9}$$

According to the time shifting of FFT, Taking the Fourier transform of rectangular pulse function after the time shifting t0 is $g_\tau(t-t_0)\leftrightarrow e^{-j\omega t_0}G(j\omega)$, so the spectrum of non-uniform sampled signal after processing moving average filtering is as follow.

$$Y(j\omega)=\frac{2}{T}\sum_{i=0}^{N-1}e^{-jwt_i}\frac{\sin\frac{\omega(t_{i+1}-t_i)}{2}}{\omega}x(t_i) \tag{10}$$

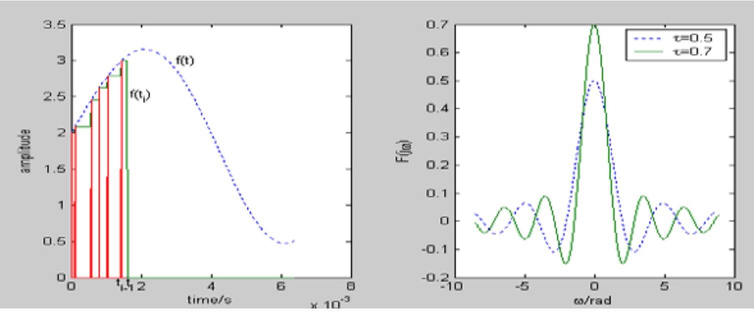

Fig.1 The spectrum of non-uniform sampled signal moving average filtering computing method

In Fig.1 the solid line depicts the width of sliding window, and the dash line depicts the curve of amplitude function. As shown in Fig.1 ,the spectrum calculation of non-uniform sampled moving average filtering., moving average filtering is equivalent to lowpass filter, According to this feature, setting the appreciate width of the window can suppress the noise and save signals as much as possible, in so as to improve the SNR. From another perspective,when solving the average for the date of sliding window, the noise will not be accumulated because the noise is random and the signal will be highlighted for its accumulate.

4 Simulation

This paper uses MATLAB data processing function to analyze digital spectrum of the non-uniform sampled signal, and verify the effectiveness of the signal reconstruction method based on Inverse discrete Fourier Transform the sampling interval is $\tau_n=\{1.007,0.996,...\}$, 1024 points sinusoidal sequence(milliseconds) as is shown in Fig. 2(a)

$$y(t_n)=0.2\sin(2\pi\times125\times t_n)+0.2\sin(2\pi\times126\times t_n)+v(t_n),0<t_n<1024ms \tag{11}$$

This set of data sampling period is 1 ms， less than a percent on the sampling time fluctuation, because the frequency resolution is $\Delta f\approx 1/\sum t_n$, Applying FFT uniform sampling cannot distinguish $f_1=125Hz$ sinusoidal signal from $f_2=125Hz$ sinusoidal signal in spectrum. You can adopt non-uniform sampled signal spectrum analysis method, which is proposed in this paper, do 20 times the continuous spectrum of non-uniform sampled signal, and find mathematical expectation. As is shown inFig.2 (b), you can easily distinguish 125Hz sinusoidal signal from 126Hz sinusoidal signal. In order to improve frequency resolution, the traditional fast Fourier transform will decrease the sampling interval, increase the sampling points, but it will increase the data redundancy and computational complexity.

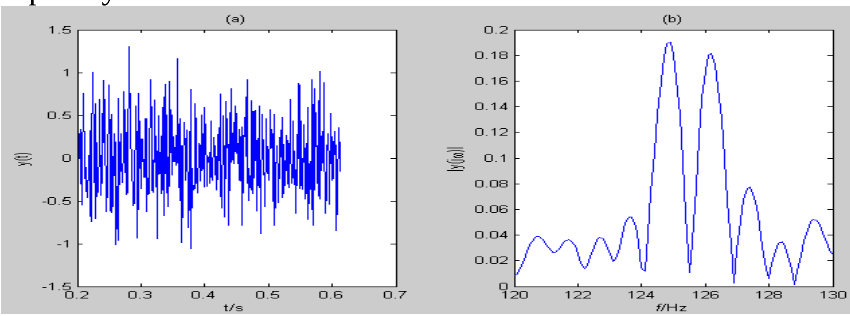

Fig.2Non-uniform sampling sine signal sampled sequence and spectrum diagram

Therefore, based on the spectrum of non-uniform sampling algorithm in dealing with the sampled data of the same data length, we can improve the frequency resolution of the spectrum.

Using formula（7）to deal with the mixed noise signal（9）with 16 points moving average filtering, its mean value and variance are 0.0013 and 0.0111, but if using the conventional average filtering to deal with non-uniform sampled signal, its mean value and variance are 0.0028 and 0.0125.This verifies the effectiveness of the moving average filtering method.

5 The application of Non-uniform sampled signal spectrum analysis and its reconstruction in engineering practice

Laser gyroscope is a kind of digital sensor, which is in the form of quantitative pulse output angle increment, its output signal is a sinusoidal Signal in a non-uniform sampled time. The laser gyroscope bias error is the main technical indicators to measure its performance, its size is directly related to the stability and navigation precision of the strapdown inertial navigation system [6-7].

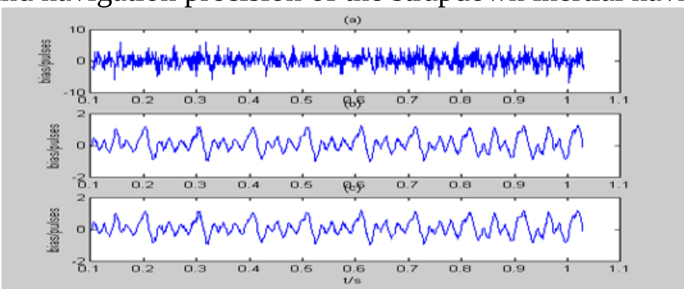

Fig.3（a）A certain type of the laser gyroscope zero bias (b) The zero bias with traditional average filtering method (c) The zero bias with moving average filtering method

Fig.3 (a) shows us a certain type of zero drift signal of laser gyroscope whole cycle sampling demodulation, its mean value and standard deviation are 0.0913 pulse/s and 4.3364 pulse/s. The zero bias signal is processed by the conventional average filter and non-uniform moving average filtering method, its mean values respectively are 0.1075 pulse/s, 0.1007 pulse/s, the rate of standard deviation respectively are 0.2453 pulse/s and 0.2204 pulse/s. As the experimental results suggest, the laser gyroscope bias error is reduced effectively after processing the non-uniform moving average filtering, the laser gyroscope bias error is reduced by 10% compared with traditional average filter. The random walk coefficient of the laser gyroscope is $0.0064^{\circ}/\sqrt{h}$, which can be accurately estimated to calculate after processing the moving average filtering.

6 Conclusion

This paper presents two new methods that are for the analysis and the process of nonuniformly sampled signals. In order to improve frequency resolution, the traditional fast Fourier transform will decrease the sampling interval, increase the sampling points, but it will increase the data redundancy and computational complexity. And based on the spectrum of non-uniform sampling algorithm in dealing with the sampled data of the same data length, we can improve the frequency resolution of the spectrum. Non-uniform sampled moving average filtering method solves the problem of non-uniform sampled interval, attenuates the high frequency signal, and plays an important role in smoothing the data. This developed technique can be applied in the processing of the laser gyroscope output signal and the laser gyroscope bias error is reduced effectively.

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