Dynamic Inverse Based Controller for a Hypersonic Flight Vehicle

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Abstract. A dynamic inverse based controller is designed for the longitudinal dynamics of a generic hypersonic vehicle. This model is strong nonlinear, multivariable coupling and includes uncertain parameters based on its high speed, lager aerodynamic load and rapid changing flight area. After feedback linearization of original model to get dynamic inverse linear model, optimal control method is used to develop controller to follow height and velocity commands. Simulation studies demonstrate that this controller is capable to follow height and speed change commands rapidly and accurately with certain robustness.

Introduction

Hypersonic vehicle has roused widespread concern in the 21st century for its huge military and civilian use prospects and for its representative capacity of the commanding heights of aviation technology. Hypersonic vehicle is designed for flight vehicle witch velocity is faster than 5 mach number [1]. As results of high flight speed, rapid changes in aerodynamic parameters and the large flight envelope, there are many strong coupling factors within flight model and control model. Previous studies shows that serious nonlinear also exist over all the model and control loops, and conventional control technology is difficult to meet the flight and control requirements [2].

Dynamic inverse method is common way to adopt nonlinear system, through which a input/output linear system can be developed [3]. And then many major linear system control ways can be used. Within this article, feedback linearization is used on the nonlinear longitudinal model and a optimal control is established with consideration of parametric uncertain. In demonstration part, a generic hypersonic wing-cone model is used to evaluate the control performance.

Hypersonic vehicle model

A generic hypersonic model coming from NASA is used here, known as wing-cone which is a standard model for hypersonic control study. The control values are pith aerodynamic rudder and engine throttle, and the model is descrambled as following [4,5].

$$\dot{V} = \frac{T\cos\alpha - D}{m} - \frac{\mu\sin\gamma}{r^2} \tag{1}$$

$$\dot{\gamma} = \frac{L + T\sin\alpha}{mV} - \frac{(\mu - V^2 r)\cos\gamma}{Vr^2} \tag{2}$$

$$\dot{h} = V\sin\gamma \tag{3}$$

$$\dot{\alpha} = q - \dot{\gamma} \tag{4}$$

$$\dot{q} = \frac{M_y}{I_y} \tag{5}$$

The engine model is [6]

$$T=\frac{1}{2}\rho V^2 S C_T \quad C_T=\begin{cases}0.02576\beta & if\ \beta<1\\ 0.0224+0.00336\beta & if\ \beta>1\end{cases} \tag{6}$$

Where, β is the engine throttle value and β_c is its command value. The dynamic characteristics of the engine is descrambled as

$$\ddot{\beta}=-2\zeta\omega_n\dot{\beta}-\omega_n^2\beta+\omega_n^2\beta_c \tag{7}$$

The uncertain parameters are:

$m=136820(1+\Delta m)$, $I_y=9.49(1+\Delta I_y)\times 10^6$, $S=334.73(1+\Delta S)$, $\bar{c}=24.38(1+\Delta\bar{c})$, $\rho=\rho_h(1+\Delta\rho)$.

Feedback linearization model

The longitude model of hypersonic vehicle is strongly nonlinear, and direct method of controller designing is very difficult. Considering the relation between the input and output states, feedback linearization can be used to obtain linear model.

Take states as $\mathbf{x}=[V\quad\gamma\quad\alpha\quad\beta\quad h]^{\mathrm{T}}$, inputs as δ_e and β_c, outputs as V and h, and we could obtain [7]

$$\begin{bmatrix}\ddot{V}\\ h^{(4)}\end{bmatrix}=\begin{bmatrix}f_v\\ f_h\end{bmatrix}+\mathbf{K}\begin{bmatrix}\beta_c\\ \delta_e\end{bmatrix} \tag{8}$$

Where

$$f_v=\frac{\omega_1\ddot{x}_0+\dot{x}^{\mathrm{T}}\Omega_2\dot{x}}{m} \tag{9}$$

$$\begin{aligned}f_h&=3\ddot{V}\dot{\gamma}\cos\gamma-3\dot{V}\dot{\gamma}^2\sin\gamma+3\dot{V}\ddot{\gamma}\cos\gamma-3V\dot{\gamma}\ddot{\gamma}\sin\gamma-V\dot{\gamma}^3\cos\gamma+\frac{\omega_1\ddot{x}_0+\dot{x}^{\mathrm{T}}\Omega_2\dot{x}}{m}\sin\gamma\\&+V\left[\pi_1\ddot{x}_0+\dot{x}^{\mathrm{T}}\Pi_2\dot{x}\right]\cos\gamma\end{aligned} \tag{10}$$

$$\mathbf{K}=\begin{bmatrix}\frac{1}{m}\frac{\partial T}{\partial\beta}\omega_n^2\cos\alpha & -\frac{c_e\rho V^2 S\bar{c}}{2mI_y}\left(T\sin\alpha+\frac{\partial D}{\partial\alpha}\right)\\ \frac{\partial T}{\partial\beta}\frac{\omega_n^2\sin(\alpha+\gamma)}{m} & \frac{c_e\rho V^2 S\bar{c}}{2mI_y}\left[T\cos(\alpha+\gamma)+\frac{\partial L}{\partial\alpha}\cos\gamma-\frac{\partial D}{\partial\alpha}\sin\gamma\right]\end{bmatrix} \tag{11}$$

As shown in Eq. 8, an exact linear model between inputs and outputs is established.

Optimal controller design

Linear quadratic optimal control method is used to develop the controller to track flight height command h_d and velocity command V_d.

Define track errors as: $e_V=V-V_d$, $e_h=h-h_d$, and define $\boldsymbol{\eta}_1=[e_V\quad\dot{e}_V\quad\ddot{e}_V]^{\mathrm{T}}$, $\boldsymbol{\eta}_2=[e_h\quad\dot{e}_h\quad\ddot{e}_h\quad\dddot{e}_h]^{\mathrm{T}}$, and then we obtain the following linear system

$$\begin{cases}\dot{\boldsymbol{\eta}}_1=\boldsymbol{A}_1\boldsymbol{\eta}_1+\boldsymbol{B}_1U_1\\ \dot{\boldsymbol{\eta}}_2=\boldsymbol{A}_2\boldsymbol{\eta}_2+\boldsymbol{B}_2U_2\end{cases} \tag{12}$$

Where, $\boldsymbol{A}_1=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}$, $\boldsymbol{B}_1=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $U_1=\ddot{V}-\ddot{V}_d$, $A_2=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$, $B_2=\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$, $U_2=h^{(4)}-h_d^{(4)}$.

Define optimal performance function as

$$\begin{cases} J_1 = \frac{1}{2}\int_0^{\infty}[\boldsymbol{\eta}_1^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{\eta}_1 + U_1^{\mathrm{T}}\boldsymbol{R}_1 U_1]dt \\ J_2 = \frac{1}{2}\int_0^{\infty}[\boldsymbol{\eta}_2^{\mathrm{T}}\boldsymbol{Q}_2\boldsymbol{\eta}_2 + U_2^{\mathrm{T}}\boldsymbol{R}_2 U_2]dt \end{cases} \tag{13}$$

Where, $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are non-negative symmetric matrixes, $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are positive symmetric matrixes.

According to the linear quadratic regulator theory, the optimal control comes as

$$\begin{cases} U_1^* = -\boldsymbol{R}_1^{-1}\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{\eta}_1 \\ U_2^* = -\boldsymbol{R}_2^{-1}\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{P}_2\boldsymbol{\eta}_2 \end{cases} \tag{14}$$

Where, $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are positive symmetric constant matrixes, which can be obtain through the following equations

$$\begin{cases} \boldsymbol{P}_1\boldsymbol{A}_1 + \boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_1 - \boldsymbol{P}_1\boldsymbol{B}_1\boldsymbol{R}_1^{-1}\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{P}_1 + \boldsymbol{Q}_1 = 0 \\ \boldsymbol{P}_2\boldsymbol{A}_2 + \boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{P}_2 - \boldsymbol{P}_2\boldsymbol{B}_2\boldsymbol{R}_2^{-1}\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{P}_2 + \boldsymbol{Q}_2 = 0 \end{cases} \tag{15}$$

And then, we get

$$\begin{bmatrix} \dddot{V} \\ h^{(4)} \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} + \begin{bmatrix} \dddot{V}_d \\ h_d^{(4)} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{R}_1^{-1}\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{\eta}_1 \\ -\boldsymbol{R}_2^{-1}\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{P}_2\boldsymbol{\eta}_2 \end{bmatrix} + \begin{bmatrix} \dddot{V}_d \\ h_d^{(4)} \end{bmatrix} \tag{16}$$

Take Fig. 16 to Fig. 8 and optimal control for hypersonic vehicle as

$$\begin{bmatrix} \beta_c \\ \delta_e \end{bmatrix} = K^{-1}\left(\begin{bmatrix} -\boldsymbol{R}_1^{-1}\boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{\eta}_1 \\ -\boldsymbol{R}_2^{-1}\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{P}_2\boldsymbol{\eta}_2 \end{bmatrix} + \begin{bmatrix} \dddot{V}_d \\ h_d^{(4)} \end{bmatrix} - \begin{bmatrix} f_v \\ f_h \end{bmatrix} \right) \tag{17}$$

Simulation and analysis

The simulation is took at the follow Balance condition: $V_0 = 4546.95\, m/s$, $h_0 = 32000m$, $\gamma_0 = 0^\circ$, $q_0 = 0\,^\circ/s$, $\alpha_0 = 1.7797^\circ$, $\beta_{c0} = 0.1759$, $\delta_{e0} = -0.3962^\circ$. The optimal control values are chosen as:

$$\boldsymbol{Q_1} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 100 \end{bmatrix},\ \boldsymbol{Q_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix},\ R_1 = R_2 = 1.$$

Flight height and speed commands are $h_d = 800m$ ($\dot{h}_d = \ddot{h}_d = \dddot{h}_d = 0$) and $V_d = 50\,m/s$ ($\dot{V}_d = \ddot{V}_d = 0$). The simulation is implemented base on Matlab&Simulink program. Simulation continues for 100 seconds and the results are shown in Fig. 1 and Fig. 2.

As shown in Fig. 1 and Fig. 2, the flight height and velocity commands of hypersonic vehicle are tracked rapidly and exactly under uncertain parameters, and the coupling between height and velocity control is very small.

Conclusions

Dynamic inverse method is used in the nonlinear model of hypersonic vehicle, based on which an optimal controller is designed to track flight height and velocity commands under uncertain parameters. Quadratic optimal control has nice stability margin and this advantage is just be used on hypersonic vehicle to meet the control challenges and improve system robustness. The simulation results show that the optimal controller works quite well and fulfills control requirements of hypersonic vehicle.

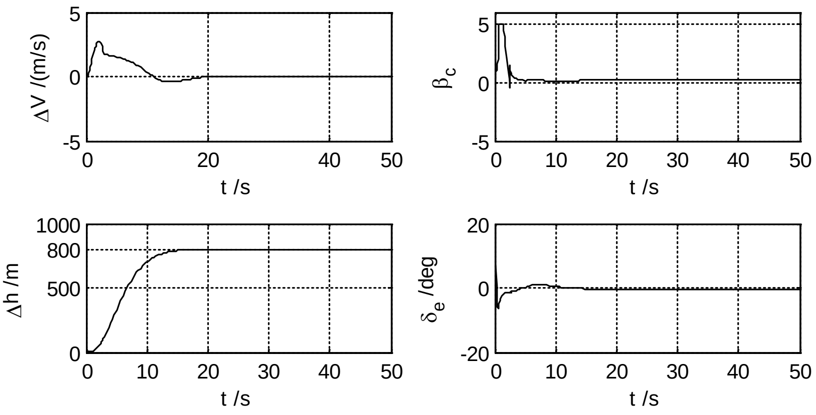

Fig. 1. Response graphs for $h_d = 800m$

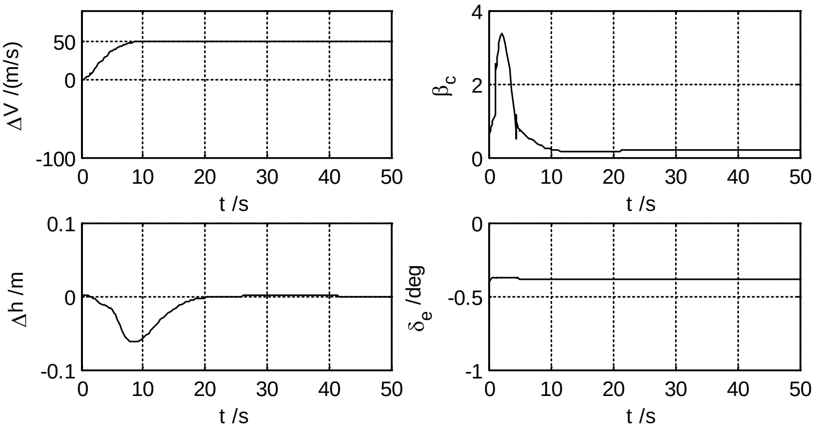

Fig. 2. Response graphs for $V_d = 50m/s$

References

[1] Fidan B, Mirmirani M, Ioannou P A: AIAA International Space Planes and Hypersonic Systems and Technologies Conference and Exhibit. Norfolk, USA: AIAA, (2003), p. 1.

[2] Marrison C I, Stengel R F: Journal of Guidance, Control and Dynamics, Vol. 21-1 (1998), p. 58.

[3] Wan Jing, Wang Qian and Ai Jianliang: 18th AIAA/3AF International Space Planes and Hypersonic Systems and Technologies Conference, USA: AIAA, (2012), p. 1.

[4] John D. Shaughnessy, S. Zane Pinckney, John D.McMinn et.al: NASA TM2102610, (1991).

[5] L. Fiorentini, A. Serrani, Michael A. Bolender and Dabid B. Doman: AIAA 2007-6329.

[6] Wang Q, Stengel R F: Journal of Guidance, Control and Dynamics, Vol. 23-4 (2000), p. 577.

[7] Haojian Xu, Maj D. Mirmirani and Petros A. Ioannou: Journal of Guidance, Control and Dynamics, Vol. 27-5 (2004), p. 829.